EXISTENCE OF SOLUTIONS WITH ASYMPTOTIC BEHAVIOR OF EXTERIOR PROBLEMS OF HESSIAN EQUATIONS

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ABSTRACT. In this paper, we use the Perron method to prove the existence of viscosity solutions with asymptotic behavior at infinity to Hessian equations.

1. INTRODUCTION

In this paper, we study the Hessian equation

$$(1.1) \quad \sigma_k(\lambda(D^2u)) = 1 \quad \text{in } \mathbb{R}^n \setminus \partial\Omega,$$

where $\Omega \subset \mathbb{R}^n$ is any strictly convex bounded domain,

$$\sigma_k(\lambda(D^2u)) = \sum_{i_1 < \dots < i_k} \lambda_{i_1} \cdots \lambda_{i_k}, \quad k = 1, \dots, n$$

is the k th elementary symmetric function of $\lambda(D^2u) = (\lambda_1, \dots, \lambda_n)$, the eigenvalues of the Hessian matrix D^2u .

The Hessian equation (1.1) is an important class of fully nonlinear elliptic equations. For $k = 1$ (1.1) is the Poisson equation $\Delta u = 1$, and for $k = n$, (1.1) is the Monge-Ampère equation $\det(D^2u) = 1$. There exist many excellent results in bounded domains of Hessian equations; see [3]–[11] and the references therein. For instance, Caffarelli, Nirenberg and Spruck in [3] established the classical solvability of the Dirichlet problem of Hessian equations. Trudinger in [9] demonstrated the existence and uniqueness of weak solutions and Urbas in [11] proved the existence of viscosity solutions.

In [1], Caffarelli and Li investigated the existence of solutions with asymptotic behavior to Monge-Ampère equations in exterior domains and entire solutions to Monge-Ampère equations. In this paper, we study the solutions with asymptotic behavior of exterior problems of Hessian equations (1.1) using the Perron method.

To work in the realm of elliptic equations, we have to restrict the solutions to the following class of functions. Let

$$\Gamma_k = \{\lambda \in \mathbb{R}^n \mid \sigma_j(\lambda) > 0, j = 1, 2, \dots, k\}.$$

Γ_k is a symmetric cone, that is, any permutation of λ is in Γ_k if $\lambda \in \Gamma_k$. When $k = 1$, Γ_k is the half-space $\{\lambda \in \mathbb{R}^n \mid \sum_{i=1}^n \lambda_i > 0\}$. When $k = n$, Γ_k is the positive

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cone $\Gamma^+ = \{\lambda \in \mathbb{R}^n \mid \lambda_i > 0, i = 1, \dots, n\}$. Following [3], we give the definition of k -convex functions.

Definition 1.1. A function $u \in C^2(\mathbb{R}^n \setminus \partial\Omega)$ is called k -convex if $\lambda \in \overline{\Gamma_k}$, where $\lambda = \lambda(D^2u) = (\lambda_1, \lambda_2, \dots, \lambda_n)$ are the eigenvalues of the Hessian matrix D^2u .

Definition 1.2. A function $u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ is called a viscosity subsolution of (1.1) if for any $y \in \mathbb{R}^n \setminus \partial\Omega$, $\xi \in C^2(\mathbb{R}^n \setminus \partial\Omega)$ satisfying

$$u(x) \leq \xi(x), \quad x \in \mathbb{R}^n \setminus \partial\Omega \quad \text{and} \quad u(y) = \xi(y),$$

we have

$$\sigma_k(\lambda(D^2\xi(y))) \geq 1.$$

A function $u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ is called a viscosity supersolution of (1.1) if for any $y \in \mathbb{R}^n \setminus \partial\Omega$ and any k -convex function $\xi \in C^2(\mathbb{R}^n \setminus \partial\Omega)$ satisfying

$$u(x) \geq \xi(x), \quad x \in \mathbb{R}^n \setminus \partial\Omega \quad \text{and} \quad u(y) = \xi(y),$$

we have

$$\sigma_k(\lambda(D^2\xi(y))) \leq 1.$$

A function $u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ is called a viscosity solution of (1.1) if u is both a viscosity subsolution and a viscosity supersolution of (1.1).

u is a viscosity solution of (1.1) if u is a k -convex classical solution of (1.1). Conversely, u is a k -convex classical solution of (1.1) if u is a viscosity solution of (1.1) and u is of class C^2 ; see [11].

Definition 1.3. A function $u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ is called k -convex if in the viscosity sense $\sigma_j(\lambda(D^2u)) \geq 0$ in $\mathbb{R}^n \setminus \partial\Omega$, $j = 1, 2, \dots, k$.

$u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ is k -convex if and only if u is C^0 subharmonic; u is n -convex if and only if u is convex.

The main result of this paper is the following existence theorem.

Theorem 1.1. Let $k \geq 3$. For any $c \in \mathbb{R}$, there exists a constant $\beta_0 \in \mathbb{R}$ such that for any $\beta > \beta_0$ there exists a k -convex viscosity solution $u \in C^0(\mathbb{R}^n \setminus \partial\Omega)$ of (1.1) which satisfies

$$(1.2) \quad \limsup_{|x| \rightarrow \infty} \left(|x|^{k-2} \left| u(x) - \left[\frac{c_*}{2} |x|^2 + c \right] \right| \right) < \infty,$$

$$(1.3) \quad u(x) = -\beta, \quad x \in \partial\Omega,$$

where $c_* = (\frac{1}{C_n^k})^{\frac{1}{k}}$ and Ω is any convex domain in $\mathbb{R}^n$.

Remark 1.1. For $k = n$, Theorem 1.1 corresponds to the results in [1], and for entire solution problems of Hessian equations, Theorem 1.1 should also hold.

2. PRELIMINARIES

In this section, we prove some lemmas which will be used later.

Lemma 2.1. Let D be a domain in $\mathbb{R}^n$ and $f \in C^0(\mathbb{R}^n)$ be nonnegative. Assume that the k -convex functions $v \in C^0(\overline{D})$, $u \in C^0(\mathbb{R}^n)$ satisfy respectively

$$\sigma_k(\lambda(D^2v)) \geq f(x), \quad x \in D,$$

$$\sigma_k(\lambda(D^2u)) \geq f(x), \quad x \in \mathbb{R}^n.$$

Moreover,

$$(2.1) \quad \begin{aligned} u &\leq v, x \in \overline{D}, \\ u &= v, x \in \partial D. \end{aligned}$$

Set

$$w(x) = \begin{cases} v(x), & x \in D, \\ u(x), & x \in \mathbb{R}^n \setminus D. \end{cases}$$

Then $w \in C^0(\mathbb{R}^n)$ is a k -convex function and satisfies in the viscosity sense

$$\sigma_k(\lambda(D^2 w)) \geq f(x), x \in \mathbb{R}^n.$$

Proof. Let $y \in \mathbb{R}^n, \xi \in C^2(\mathbb{R}^n)$ satisfy $w(y) = \xi(y)$,

$$(2.2) \quad w(x) \leq \xi(x), x \in \mathbb{R}^n.$$

If $y \in D$, we have

$$v(y) = w(y) = \xi(y), v(x) = w(x) \leq \xi(x), x \in D.$$

Therefore

$$\begin{aligned} \sigma_j(\lambda(D^2 \xi(y))) &\geq 0, 1 \leq j \leq k, \\ \sigma_k(\lambda(D^2 \xi(y))) &\geq f(y). \end{aligned}$$

If $y \in \mathbb{R}^n \setminus D$, we have

$$u(y) = w(y) = \xi(y), u(x) = w(x) \leq \xi(x), x \in \mathbb{R}^n \setminus D.$$

By (2.1), (2.2),

$$u(x) \leq \xi(x), x \in \mathbb{R}^n.$$

Therefore

$$\begin{aligned} \sigma_j(\lambda(D^2 \xi(y))) &\geq 0, 1 \leq j \leq k, \\ \sigma_k(\lambda(D^2 \xi(y))) &\geq f(y). \end{aligned}$$

This completes the proof of Lemma 2.1. $\square$

The following existence theorem for viscosity solutions on a ball can be found in [4].

Lemma 2.2. *Let B be a ball in $\mathbb{R}^n$ and $f \in C^0(\overline{B})$ be nonnegative. Suppose $\underline{u}, \overline{u} \in C^0(\overline{B})$ are respectively the viscosity subsolution and supersolution of*

$$\sigma_k(\lambda(D^2 u)) = f(x), x \in B,$$

and satisfy $\underline{u}|_{\partial B} = \overline{u}|_{\partial B} = \varphi \in C^0(\partial B)$. Then there exists a unique k -convex function $u \in C^0(\overline{B})$ satisfying

$$\begin{aligned} \sigma_k(\lambda(D^2 u)) &= f(x), \quad x \in B, \\ u &= \varphi(x), \quad x \in \partial B. \end{aligned}$$

Lemma 2.3. *Let B be a ball in $\mathbb{R}^n$ and $f \in C^0(\overline{B})$ be nonnegative. Suppose $\underline{u} \in C^0(\overline{B})$ satisfies in the viscosity sense $\sigma_k(\lambda(D^2\underline{u})) \geq f(x)$ in B . Then the Dirichlet problem*

$$(2.3) \quad \sigma_k(\lambda(D^2u)) = f(x), \quad x \in B,$$

$$(2.4) \quad u = \underline{u}(x), \quad x \in \partial B$$

has a unique k -convex viscosity solution $u \in C^0(\overline{B})$.

Proof. Clearly, $\underline{u}$ is a viscosity subsolution of (2.3), (2.4). From Lemma 2.2, we only need to prove that (2.3), (2.4) has a viscosity supersolution $\overline{u} \in C^0(\overline{B})$ satisfying $\overline{u} = \underline{u}$ on ∂B .

Let $v \in C^2(B) \cap C^0(\overline{B})$ satisfy

$$\Delta v = 0, \quad x \in B,$$

$$v = \underline{u}, \quad x \in \partial B.$$

We claim that v is a viscosity supersolution of (2.3). Indeed, suppose v is not a viscosity supersolution of (2.3). Then there exist $y \in B$ and some k -convex function $\xi \in C^2(B)$ such that

$$(2.5) \quad v(x) \geq \xi(x), \quad x \in B, \quad v(y) = \xi(y),$$

but

$$\sigma_k(\lambda(D^2\xi(y))) > f(y).$$

By the k -convexity of ξ and the Newton-Maclaurin inequality

$$\sigma_1(\lambda) \geq n \left(\frac{\sigma_k(\lambda)}{C_n^k} \right)^{\frac{1}{k}}, \quad \lambda \in \overline{\Gamma}_k,$$

we know that

$$\Delta \xi(y) \leq n \left(\frac{f(y)}{C_n^k} \right)^{1/k} > 0.$$

But from (2.5), we get

$$D^2v(y) \geq D^2\xi(y).$$

Hence

$$\Delta \xi(y) \leq \Delta v(y) = 0.$$

This is a contradiction. Lemma 2.3 is proved. $\square$

The following lemma can be found in [1] or [2].

Lemma 2.4. *Let $\Omega \subset \mathbb{R}^n$ be a bounded strictly convex domain, $\partial\Omega \in C^2$, $v \in C^2(\overline{\Omega})$. Then there exists a constant c only dependent of $n, \Omega, \|v\|_{C^2(\Omega)}$ such that for any $\zeta \in \partial\Omega$ there exists $\overline{x}(\zeta) \in \mathbb{R}^n$ satisfying*

$$|\overline{x}(\zeta)| \leq c, \quad w_\zeta < v, \quad x \in \overline{\Omega} \setminus \{\zeta\},$$

where

$$w_\zeta(x) := v(\zeta) + \frac{1}{2}(|x - \overline{x}(\zeta)|^2 - |\zeta - \overline{x}(\zeta)|^2), \quad x \in \mathbb{R}^n.$$

3. PROOF OF THEOREM 1.1

In this section, we establish the existence theorem of solutions with prescribed asymptotic behavior at infinity to Hessian equations (1.1).

We divide the proof into three steps.

In the first step, we construct a viscosity subsolution of (1.1).

Suppose $\Phi \in C^\infty(\overline{\Omega})$ is a k -convex function satisfying

$$\begin{aligned}\sigma_k(\lambda(D^2\Phi)) &= c_0 > 1, \quad x \in \Omega, \\ \Phi &= 0, \quad x \in \partial\Omega.\end{aligned}$$

By the comparison principle, $\Phi \leq 0$ in Ω , and by Lemma 2.4, for each $\zeta \in \partial\Omega$, there exists $\overline{x}(\zeta) \in \mathbb{R}^n$ such that

$$w_\zeta(x) < \Phi(x), \quad x \in \overline{\Omega} \setminus \{\zeta\},$$

where

$$w_\zeta(x) := \frac{1}{2}(|x - \overline{x}(\zeta)|^2 - |\zeta - \overline{x}(\zeta)|^2), \quad x \in \mathbb{R}^n,$$

and $\sup_{\zeta \in \partial\Omega} |\overline{x}(\zeta)| < \infty$. Therefore

$$\begin{aligned}w_\zeta(\zeta) &= 0, \quad w_\zeta(x) \leq \Phi(x) \leq 0, \quad x \in \overline{\Omega}, \\ \sigma_k(\lambda(D^2w_\zeta(x))) &= C_n^k > 1, \quad x \in \mathbb{R}^n.\end{aligned}$$

Thus

$$w(x) := \sup_{\zeta \in \partial\Omega} w_\zeta(x), \quad x \in \mathbb{R}^n$$

satisfies

$$(3.1) \quad w(x) \leq \Phi(x), \quad x \in \Omega.$$

From the reference [7], w satisfies

$$(3.2) \quad \sigma_k(\lambda(D^2w)) \geq 1, \quad x \in \mathbb{R}^n.$$

Define

$$V(x) = \begin{cases} \Phi(x), & x \in \Omega, \\ w(x), & x \in \mathbb{R}^n \setminus \Omega. \end{cases}$$

Then $V \in C^0(\mathbb{R}^n)$. By (3.1) and Lemma 2.1, V is a k -convex function satisfying in the viscosity sense

$$(3.3) \quad \sigma_k(\lambda(D^2V)) \geq 1, \quad x \in \mathbb{R}^n.$$

Fix some $R_1 > 0$ such that

$$\Omega \Subset B_{R_1},$$

and let

$$R_2 := 2R_1\sqrt{c_*}.$$

For $a > 1$, define

$$w_a(x) := \inf_{B_{R_1}} V + \int_{2R_2}^{|\sqrt{c_*}x|} (s^k + a)^{\frac{1}{k}} ds, \quad x \in \mathbb{R}^n.$$

Then

$$D^2w_a = (R^k + a)^{\frac{1}{k}-1} \begin{pmatrix} R^{k-1}c_* & 0 & \cdots & 0 \\ 0 & (R^{k-1} + \frac{a}{R})c_* & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & (R^{k-1} + \frac{a}{R})c_* \end{pmatrix},$$

where $R = |y|$. So $\lambda(D^2w_a) \in \Gamma_k, 0 < |x| < \infty$, and

$$\begin{aligned} & \sigma_k(\lambda(D^2w_a)) \\ &= (R^k + a)^{\frac{1}{k}-1} \left(R^{k-1}c_*C_{n-1}^{k-1} \left(R^{k-1} + \frac{a}{R} \right)^{k-1} c_*^{k-1} + C_{n-1}^k \left(R^{k-1} + \frac{a}{R} \right)^k c_*^k \right) \\ &= \frac{1}{C_n^k} \left(C_{n-1}^{k-1} + C_{n-1}^k \left(1 + \frac{a}{R^k} \right) \right) \\ &\geq \frac{1}{C_n^k} (C_{n-1}^{k-1} + C_{n-1}^k) \\ &= 1, 0 < |x| < \infty. \end{aligned}$$

By the definition of R_2 ,

$$|\sqrt{c_*}x| \leq \frac{R_2}{2}, \quad |x| \leq R_1.$$

Consequently

$$\begin{aligned} (3.4) \quad w_a(x) &\leq \inf_{B_{R_1}} V + \int_{2R_2}^{\frac{R_2}{2}} (s^k + a)^{\frac{1}{k}} ds \\ &< \inf_{B_{R_1}} V \leq V(x), \quad |x| \leq R_1. \end{aligned}$$

Fix some $R_3 > 3R_2$ satisfying

$$R_3\sqrt{c_*} > 3R_2.$$

We choose $a_1 > 1$ such that for $a \geq a_1$,

$$w_a(x) > \inf_{B_{R_1}} V + \int_{2R_2}^{3R_2} (s^k + a)^{\frac{1}{k}} ds \geq V(x), \quad |x| = R_3.$$

Then by (3.4),

$$(3.5) \quad R_3 \geq R_1.$$

By the definition of w_a ,

$$\begin{aligned} w_a(x) &= \inf_{B_{R_1}} V + \int_{2R_2}^{|\sqrt{c_*}x|} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds + \int_{2R_2}^{|\sqrt{c_*}x|} s ds \\ &= \inf_{B_{R_1}} V + \int_{2R_2}^{|\sqrt{c_*}x|} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds + \frac{c_*}{2}|x|^2 - 2R_2^2 \\ &= \frac{c_*}{2}|x|^2 + c + \inf_{B_{R_1}} V + \int_{2R_2}^{\infty} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds - c \\ &\quad - 2R_2^2 - \int_{|\sqrt{c_*}x|}^{\infty} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds, \quad x \in \mathbb{R}^n. \end{aligned}$$

Let

$$\mu(a) = \inf_{B_{R_1}} V + \int_{2R_2}^{\infty} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds - c - 2R_2^2.$$

Then $\mu(a)$ is continuous and monotonic increasing for a and when $a \rightarrow \infty$, $\mu(a) \rightarrow \infty$. Also,

$$w_a(x) - \mu(a) \leq \frac{c_*}{2}|x|^2 + c, \quad a \geq a_1, x \in \mathbb{R}^n.$$

Moreover,

$$(3.6) \quad w_a(x) = \frac{c_*}{2}|x|^2 + c + \mu(a) + O(|x|^{2-k}), \quad \text{when } |x| \rightarrow \infty.$$

For $a \geq a_1$, set $\beta_0 = \mu(a)$ and define, for any $\beta > \beta_0$,

$$\underline{u}_a(x) = \begin{cases} \max\{V(x), w_a(x)\} - \beta, & |x| \leq R_3, \\ w_a(x) - \beta, & |x| \geq R_3. \end{cases}$$

Then

$$\underline{u}_a(x) = -\beta, \quad x \in \partial\Omega,$$

and by (3.6),

$$\underline{u}_a(x) \leq \frac{c_*}{2}|x|^2 + c + O(|x|^{2-k}), \quad \text{when } |x| \rightarrow \infty.$$

Choose $a_2 \geq a_1$ sufficiently large such that for $a \geq a_2$,

$$\begin{aligned} V(x) - \beta &\leq V(x) - \beta_0 \\ &= V(x) - \inf_{B_{R_1}} V - \int_{2R_2}^{\infty} s \left(\left(1 + \frac{a}{s^k}\right)^{\frac{1}{k}} - 1 \right) ds + c + 2R_2^2 \\ &\leq c \leq \frac{c_*}{2}|x|^2 + c, \quad |x| \leq R_3. \end{aligned}$$

Therefore

$$\underline{u}_a(x) \leq \frac{c_*}{2}|x|^2 + c, \quad a \geq a_2, x \in \mathbb{R}^n.$$

By Lemma 2.1, $\underline{u}_a \in C^0(\mathbb{R}^n)$ is k -convex and satisfies in the viscosity sense

$$\sigma_k(\lambda(D^2 \underline{u}_a)) \geq 1, \quad x \in \mathbb{R}^n.$$

In the second step, we define the Perron solution of (1.1).

For $a \geq a_2$, let $\mathbb{S}_a$ denote the set of k -convex functions $v \in C^0(\mathbb{R}^n)$ satisfying

$$\begin{aligned}\sigma_k(\lambda(D^2v)) &\geq 1, & x \in \mathbb{R}^n \setminus \partial\Omega, \\ v(x) &= -\beta, & x \in \partial\Omega, \\ v(x) &\leq \frac{c_*}{2}|x|^2 + c, & x \in \mathbb{R}^n.\end{aligned}$$

Clearly, $\underline{u}_a \in \mathbb{S}_a$. Hence $\mathbb{S}_a \neq \emptyset$. Define in $\mathbb{R}^n$,

$$u_a(x) := \sup\{v(x) | v \in \mathbb{S}_a\}.$$

Then u_a is a viscosity subsolution of (1.1) and satisfies

$$\begin{aligned}u_a(x) &= -\beta, & x \in \partial\Omega, \\ u_a(x) &\leq \frac{c_*}{2}|x|^2 + c, & x \in \mathbb{R}^n.\end{aligned}$$

In the following, we prove that u_a is a viscosity supersolution of (1.1).

In the third step, we prove that u_a is a viscosity solution of (1.1).

For any $x_0 \in \mathbb{R}^n \setminus \partial\Omega$, choose a ball $B = B_\varepsilon(x_0) \subset \mathbb{R}^n \setminus \partial\Omega$. By Lemma 2.3, the Dirichlet problem

$$(3.7) \quad \begin{aligned}\sigma_k(\lambda(D^2\tilde{u})) &= 1, & x \in B, \\ \tilde{u} &= u_a, & x \in \partial B\end{aligned}$$

has a k -convex viscosity solution $\tilde{u} \in C^0(\overline{B})$. From the comparison principle,

$$u_a \leq \tilde{u}, \quad x \in B.$$

Define

$$\psi(x) = \begin{cases} \tilde{u}(x), & x \in B, \\ u_a(x), & x \in \mathbb{R}^n \setminus (B \cup \partial\Omega). \end{cases}$$

By Lemma 2.1,

$$\sigma_k(\lambda(D^2\psi(x))) \geq 1, \quad x \in \mathbb{R}^n \setminus \partial\Omega.$$

Because

$$\begin{aligned}\sigma_k(\lambda(D^2\tilde{u})) &= 1 = \sigma_k(\lambda(D^2g)), & x \in B, \\ \tilde{u} &= u_a \leq g, & x \in \partial B,\end{aligned}$$

where $g(x) = \frac{c_*}{2}|x|^2 + c$, from the comparison principle,

$$\tilde{u} \leq g, \quad x \in B.$$

Hence $\psi \in \mathbb{S}_a$.

By the definition of u_a , $u_a \geq \psi$ in $\mathbb{R}^n$. Consequently $\tilde{u} \leq u_a$ in B . As a result,

$$(3.8) \quad \tilde{u} = u_a, \quad x \in B.$$

Because x_0 is arbitrary, we know that u_a is a viscosity supersolution of (1.1).

By the definition of u_a ,

$$\underline{u}_a \leq u_a \leq g, \quad x \in \mathbb{R}^n,$$

so u_a satisfies (1.2). Theorem 1.1 is proved. $\square$

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