MAPPING SPACES BETWEEN MANIFOLDS AND THE EVALUATION MAP

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ABSTRACT. Let $f : M \rightarrow N$ be a map between simply connected n -dimensional manifolds. We suppose that $\deg f \neq 0$. Then the injection of $\text{aut}_1(N)$ into the component $\text{Map}(M, N; f)$ of the mapping space containing f induces an injection on the rational homotopy groups, and the evaluation at the base point $\text{map}(M, N; f) \rightarrow N$ is zero on the rational homotopy groups of even dimension.

This paper concerns a generalization of a theorem of Dan Gottlieb [5] motivated by a question of Dusa McDuff [12]. Let $f : M \rightarrow N$ be a map of closed oriented connected manifolds of the same dimension n . We suppose that M and N are pointed and that f preserves the base point. We consider the mapping space $\text{Map}(M, N; f)$ of continuous maps homotopic to f , and we denote by $\omega : \text{Map}(M, N; f) \rightarrow N$ the evaluation map at the base point $* \in M$, $\omega(g) = g(*)$. In [5] Gottlieb proves that if $\pi_1(\omega) : \pi_1(\text{Map}(M, N; f)) \rightarrow \pi_1(N)$ is non-zero, then either the degree of f , $\deg f = 0$, or the Euler characteristic $\chi(N) = 0$.

In this paper we consider what happens with the higher order homotopy groups when M and N are simply connected. Recall first that in the case $f = id_N$, the space $\text{Map}(M, N; f)$ is the monoid $\text{aut}_1(N)$ of self-equivalences of N homotopic to the identity. Then denote the evaluation map by $ev : \text{aut}_1 N \rightarrow N$. The image of $\pi_q(ev)$ is called the q^{th} Gottlieb group of N , $G_q(N)$. The groups $G_q(N) \otimes \mathbb{Q}$ and the map ev have been intensively studied in rational homotopy. In particular, for each integer q , $G_{2q}(N) \otimes \mathbb{Q} = 0$ and $\sum_q \dim G_q(N) \otimes \mathbb{Q} < n$ [2]. On the other hand, by a result of G. Lupton and the author, $\tilde{H}_*(ev; \mathbb{Q}) = 0$ if $\chi(N) \neq 0$ [4]. See also [10], [11], [6] for more recent results on Gottlieb groups.

Since the composition with f induces a continuous map $j_N : \text{aut}_1 N \rightarrow \text{Map}(M, N; f)$ satisfying $\omega \circ j_N = ev$, we have the inclusion $G_*(N) \subset \text{Im } \pi_*(\omega)$. Our main theorem is essentially a converse to this result.

Theorem 1. (1) If $\deg f \neq 0$, then $\text{Im } \pi_*(\omega) \otimes \mathbb{Q} = G_*(N) \otimes \mathbb{Q}$.
 (2) If $\deg f \neq 0$ and $\chi(N) \neq 0$, then the composition of $\pi_*(\omega)$ with the Hurewicz map $hur_N : \pi_*(N) \otimes \mathbb{Q} \rightarrow H_*(N; \mathbb{Q})$,

$$\pi_*(\text{Map}(M, N; f)) \otimes \mathbb{Q} \rightarrow H_*(N; \mathbb{Q}),$$

is zero.

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An important step in the proof of Theorem 1 is given by Theorem 2.

Theorem 2. *Suppose $\deg f \neq 0$. Then the injection $j_N : \text{aut}_1(N) \rightarrow \text{Map}(M, N; f)$ induces an injection on the rational homotopy groups,*

$$\pi_*(j_N) \otimes \mathbb{Q} : \pi_*(\text{aut}_1(N)) \otimes \mathbb{Q} \rightarrow \pi_*(\text{Map}(M, N; f)) \otimes \mathbb{Q}.$$

Moreover $\pi_*(j_N) \otimes \mathbb{Q}$ admits a retraction σ satisfying $(\pi_*(\text{ev}) \otimes \mathbb{Q}) \circ \sigma = \pi_*(\omega) \otimes \mathbb{Q}$.

Now recall that a continuous map $g : X \rightarrow Y$ is called a rational Gottlieb map if $\pi_*(g) \otimes \mathbb{Q}$ maps $G_*(X) \otimes \mathbb{Q}$ into $G_*(Y) \otimes \mathbb{Q}$ [13]. For instance, when X is an odd-dimensional sphere S^q , then g is a rational Gottlieb map if and only if $[g]$ belongs to the Gottlieb group $G_*(Y) \otimes \mathbb{Q}$. We prove

Theorem 3. *With the above notation, if $\deg f \neq 0$, then f is a rational Gottlieb map.*

For the proofs, we use the machinery of rational homotopy theory as described for instance in [3]. We use more precisely the Poincaré duality model for manifolds given by Lambrechts and Stanley in [8] and the description of the rational homotopy groups of mapping spaces in terms of derivations given by Lupton and Smith in [9].

1. A CONVENIENT MODEL FOR f

A Poincaré duality model for a simply connected Poincaré duality complex X of dimension n is a commutative differential graded algebra (A, d) quasi-isomorphic to the Sullivan minimal model of X and satisfying Poincaré duality in dimension n . This means that there exists a linear map $\varepsilon : A^n \rightarrow \mathbb{Q}$ such that $\varepsilon(dA^{n-1}) = 0$ and such that the induced bilinear forms

$$A^k \otimes A^{n-k} \rightarrow \mathbb{Q}, \quad a \otimes b \mapsto \varepsilon(ab)$$

are non-degenerate. Such a map ε is then called an orientation.

Theorem 4 (Lambrechts, Stanley [8]). (1) *If (A, d) is a commutative differential graded algebra whose cohomology is a simply connected Poincaré duality algebra, then (A, d) is weakly equivalent to a commutative differential graded algebra (A', d) that is a simply connected Poincaré duality algebra.*
 (2) *Moreover, if A is finite type, $A^0 = \mathbb{Q}$, $A^1 = 0$, $A^2 \subset \text{Ker } d$, and $n \geq 7$, then there is a quasi-isomorphism $\varphi : (A, d) \rightarrow (A', d)$.*

We deduce the following generalization,

Proposition 1. *Let $f : M \rightarrow N$ be a map between simply connected n -dimensional manifolds, $n \geq 7$. Suppose that $H^2(f)$ is injective. Then f admits a Sullivan model of the form*

$$\varphi : (A, d) \rightarrow (B, d),$$

where (A, d) and (B, d) are Poincaré duality algebras of dimension n .

Proof. Let $j : (\bigwedge V, d) \rightarrow (\bigwedge V \otimes \bigwedge W, D)$ be a relative minimal model for f and let $(\bigwedge V, d) \rightarrow (A, d)$ be a quasi-isomorphism with (A, d) a Poincaré duality model for N . By taking the tensor product $(A, d) \otimes_{(\bigwedge V, d)} -$, we get another model of f ,

$$1 \otimes j : (A, d) \rightarrow (A \otimes \bigwedge W, D') := (A, d) \otimes_{(\bigwedge V, d)} (\bigwedge V \otimes \bigwedge W, D).$$

Since $H^2(f)$ is injective, $(A \otimes \bigwedge W)^1 = 0$. By Theorem 4(2) there then exists a quasi-isomorphism $g : (A \otimes \bigwedge W, D') \rightarrow (B, d)$ where (B, d) is a Poincaré duality model of M . We define $\varphi : (A, d) \rightarrow (B, d)$ by $\varphi = g \circ (1 \otimes j)$. $\square$

Our next proposition explains the structure of the model φ when $\deg f \neq 0$.

Proposition 2. *With the above notation, if $\deg f \neq 0$, then*

- (1) *the map φ is injective,*
- (2) *the graded vector space B can be decomposed as $B = \varphi(A) \oplus Z$, where $d(Z) \subset Z$ and $Z \cdot \varphi(A) \subset Z$.*

Proof. Denote by ω and ω' rational fundamental classes of A and B with $\varphi(\omega) = \omega'$, and denote by ε' the associated orientation of B . In particular, $\varepsilon(a) = \varepsilon'(\varphi(a))$ for $a \in A$. If $\varphi(a) = 0$ for some a , then $\varphi(\omega) = 0$ because there is an element a' such that $aa' = \omega$, and so φ is injective.

Denote

$$Z = \{x \in B \mid \varepsilon'(x \cdot \varphi(A)) = 0\}.$$

Choose a homogeneous basis $h_1, \dots, h_m$ of A . Using the Poincaré duality of A , there is another family $\{h_j^*\} \subset A$ such that $\varepsilon(h_j^* \cdot h_i) = \delta_{ij}$, where δ_{ij} is the Kronecker symbol. Then $\varphi(h_1), \dots, \varphi(h_m)$ and $\varphi(h_1^*), \dots, \varphi(h_m^*)$ are homogeneous bases of $\varphi(A)$. If $b \in B$, the element

$$b' = b - \sum_j \varepsilon'(b \cdot \varphi(h_j)) \varphi(h_j^*)$$

is in Z . This shows that $B = \varphi(A) \oplus Z$. Now for an element $z \in Z$,

$$\varepsilon'(dz \cdot \varphi(a)) = \varepsilon'(d(z\varphi(a))) - (-1)^{|z|} \varepsilon'(z \cdot d\varphi(a)) = 0.$$

This shows that $d(Z) \subset Z$. We also have $Z \cdot \varphi(A) \subset Z$ because $\varepsilon'[(z \cdot \varphi(a)) \cdot \varphi(A)] = 0$. $\square$

2. MODEL FOR MAPPING SPACES

In this section we recall the derivation model for the rational homotopy groups of a mapping space $\text{Map}(X, Y; f)$, where X and Y are simply connected CW complexes and $\dim X < \infty$. Recall that under those hypothesis, $\text{Map}(M, N; f)$ is a nilpotent space whose rationalization is obtained by taking the composition with a rationalization of N , $\ell : N \rightarrow N_0$ ([7]).

Let $\ell : (\bigwedge V, d) \rightarrow (A, d)$ be a model for f , with $(\bigwedge V, d)$ a Sullivan (non-necessarily minimal) model for Y and (A, d) a connected model for X . We consider the differential graded vector space $(\text{der}^\ell(\bigwedge V, A), \delta)$ of ℓ -derivations, where $\text{der}^\ell(\bigwedge V, A)_m$ is the vector space of linear maps of degree m , $\theta : (\bigwedge V)^* \rightarrow A^{*-m}$ for which $\theta(xy) = \theta(x)\ell(y) + (-1)^{m|x|}\ell(x)\theta(y)$. The differential δ is defined as usual by

$$\delta\theta = d \circ \theta + (-1)^{m+1}\theta \circ d.$$

From now on, we restrict to $\text{Der}^\ell(\bigwedge V, A)$, the positive ℓ -derivations,

$$\text{Der}^\ell(\bigwedge V, A)_r = \begin{cases} \text{der}^\ell(\bigwedge V, A)_r, & \text{if } r > 1, \\ Z\text{der}^\ell(\bigwedge V, A)_1, & \text{if } r = 1, \end{cases}$$

where Z denotes the space of cycles.

It is well known that if $g : (\bigwedge W, d) \rightarrow (\bigwedge V, d)$ is a quasi-isomorphism, then the induced map $\text{Der}^\ell(\bigwedge V, A) \rightarrow \text{Der}^{\ell \circ g}(\bigwedge W, A)$ is a quasi-isomorphism. In the same way, if $h : (A, d) \rightarrow (A', d)$ is a quasi-isomorphism, then the induced map $\text{Der}^\ell(\bigwedge V, A) \rightarrow \text{Der}^{\ell \circ h}(\bigwedge V, A')$ is also a quasi-isomorphism (see for instance [1],

Theorem 2.8). The following theorem follows then directly from [9], where the theorem is proved when $(\bigwedge V, d)$ is minimal and (A, d) is the minimal model of X .

Theorem 5 ([9], [1], Theorem 3.8). *With the above notation,*

- (1) *we have natural isomorphisms of graded vector spaces*

$$\begin{aligned} H_*(\text{Der}^\ell(\bigwedge V, A)) &\cong \pi_*(\text{Map}_*(X, Y; f)) \otimes \mathbb{Q}, \\ H_*(\text{Der}^\ell(\bigwedge V, A^+)) &\cong \pi_*(\text{Map}_*(X, Y; f)) \otimes \mathbb{Q}, \end{aligned}$$

where $\text{Map}_*(X, Y; f)$ denotes the subspace of $\text{Map}(M, N; f)$ consisting of pointed maps.

- (2) *Denote by $\alpha : A \rightarrow \mathbb{Q}$ the augmentation. Then the composition with α induces a morphism of complexes that is a model for $\pi_*(\omega) \otimes \mathbb{Q}$; i.e., we have a commutative diagram*

$$\begin{array}{ccc} H_*(\text{Der}^\ell(\bigwedge V, A)) & \xrightarrow{\hat{\alpha}} & H_*(\text{Der}^{\alpha \circ \ell}(\bigwedge V, \mathbb{Q})) \cong H_*(\text{Hom}(V, \mathbb{Q})) \\ \cong \uparrow & & \uparrow \cong \\ \pi_*(\text{Map}(X, Y; f)) \otimes \mathbb{Q} & \xrightarrow{\pi_*(\omega)} & \pi_*(N) \otimes \mathbb{Q}. \end{array}$$

3. PROOF OF THEOREM 2

We first consider the case $n \geq 7$ and we use the notation of section 1. In particular, $\varphi : (A, d) \rightarrow (B, d)$ is a model for f , and (A, d) and (B, d) are Poincaré duality models for N and M . Let $\psi : (\bigwedge V, d) \rightarrow (A, d)$ be a surjective Sullivan model for (A, d) , and then let $\rho = \varphi \circ \psi : (\bigwedge V, d) \rightarrow (B, d)$.

Recall that $B = \varphi(A) \oplus Z$ with $dZ \subset Z$ and $Z \cdot \varphi(A) \subset Z$. Then $B = \rho(\bigwedge V) \oplus Z$ with $Z \cdot \rho(\bigwedge V) \subset Z$. Define a linear map

$$\Phi : \text{Der}^\rho(\bigwedge V, B) \rightarrow \text{Der}^\psi(\bigwedge V, A)$$

by $\Phi(\theta) = p \circ \theta$, where $p : B \rightarrow \varphi(A) \cong A$ is the linear projection with kernel Z .

Lemma 1. *The morphism $p \circ \theta$ is a ψ -derivation.*

Proof. Write $\theta(a) = x + z$ with $z \in Z$ and $x = \varphi(p\theta(a))$ and $\theta(b) = x' + z'$ with $z' \in Z$ and $x' = \varphi(p\theta(b))$. Then,

$$\begin{aligned} p\theta(ab) &= p(\theta(a) \cdot \rho(b) + (-1)^{|a||\theta|} \rho(a) \cdot \theta(b)) \\ &= p(x\rho(b) + (-1)^{|\theta| \cdot |a|} \rho(a)x') \\ &= p(\varphi(p\theta(a) \cdot \psi(b)) + (-1)^{|\theta| \cdot |a|} \varphi(\psi(a) \cdot p\theta(b))) \\ &= p\theta(a) \cdot \psi(b) + (-1)^{|a||\psi|} \psi(a) \cdot p\theta(b). \end{aligned} \quad \square$$

Lemma 2. *Φ is a morphism of complexes.*

Proof.

$$\delta(p\theta) = dp\theta - (-1)^{|\theta|} (p\theta)d = p(d\theta) - (-1)^{|\theta|} p(\theta d) = p(\delta\theta). \quad \square$$

Since A is a subalgebra of B , the injection of A into B induces a morphism of complexes

$$\Psi : \text{Der}^\psi(\bigwedge V, A) \rightarrow \text{Der}^\rho(\bigwedge V, B)$$

such that $\Phi \circ \Psi = id_{\text{Der}^\psi(\bigwedge V, A)}$.

Proof of Theorem 2 when $n \geq 7$. When $n \geq 7$, Theorem 5 directly yields that the induced injection $\Psi : \text{Der}^\psi(\bigwedge V, A) \rightarrow \text{Der}^\rho(\bigwedge V, B)$ induces in homology the morphism $\pi_*(j_N) \otimes \mathbb{Q}$. A retraction σ to $\pi_*(j_N) \otimes \mathbb{Q}$ is given by $H_*(\Phi)$.

Now the commutativity of the diagram

$$\begin{array}{ccc} \text{Der}^\rho(\bigwedge V, B) & \xrightarrow{\hat{\alpha}} & \text{Der}^{\alpha \circ \varphi}(\bigwedge V, \mathbb{Q}) \\ \Phi \downarrow & & \parallel \\ \text{Der}^\psi(\bigwedge V, A) & \xrightarrow{\hat{\alpha}} & \text{Der}^{\alpha \circ \psi}(\bigwedge V, \mathbb{Q}) \end{array}$$

implies in homology that $(\pi_*(ev) \otimes \mathbb{Q}) \circ \sigma = \pi_*(\omega) \otimes \mathbb{Q}$. $\square$

Proof of Theorem 2 for $n \leq 6$. Let define $g = f \times id_{S^6} : M \times S^6 \rightarrow N \times S^6$. If $f' \sim f$, then $f' \times id \sim f \times id$. This defines a continuous map

$$e : \text{Map}(M, N; f) \rightarrow \text{Map}(M \times S^6, N \times S^6; f \times id).$$

On the other hand, if $g' : M \times S^6 \rightarrow N \times S^6$ is homotopic to $f \times id$, then we consider the composition

$$M \xrightarrow{h} M \times S^6 \xrightarrow{g'} N \times S^6 \xrightarrow{p} N,$$

where $*$ in S^6 is the base point, $h(m) = (m, *)$ and $p(r, x) = r$. Since $pg'h \sim f$, this gives a continuous map

$$r : \text{Map}(M \times S^6, N \times S^6; f \times id) \rightarrow \text{Map}(M, N; f).$$

Clearly $r \circ e = id$; i.e., $\text{Map}(M, N; f)$ is a retract of $\text{Map}(M \times S^6, N \times S^6; f \times id)$.

Since $\dim(N \times S^6) \geq 7$, we have a map σ making commutative the diagram

$$\begin{array}{ccc} \pi_*(\text{Map}(M, N; f)) \otimes \mathbb{Q} & \xrightarrow{\pi_*(\omega)} & \pi_*(N) \otimes \mathbb{Q} \\ \pi_*(e) \downarrow & & \downarrow \pi_*(h') \\ \pi_*(\text{Map}(M \times S^6, N \times S^6; f \times id)) \otimes \mathbb{Q} & \xrightarrow{\pi_*(\omega)} & \pi_*(N \times S^6) \otimes \mathbb{Q} \\ \sigma \downarrow & & \parallel \\ \pi_*(\text{aut}_1(N \times S^6)) \otimes \mathbb{Q} & \xrightarrow{\pi_*(ev)} & \pi_*(N \times S^6) \otimes \mathbb{Q}, \end{array}$$

where $h'(n) = (n, *)$.

In the same way, $\text{aut}_1(N)$ is a retract of $\text{aut}_1(N \times S^6)$. Denote by $i : \text{aut}_1(N) \rightarrow \text{aut}_1(N \times S^6)$ the natural injection. The retraction τ associates to a map $h : N \times S^6 \rightarrow N \times S^6$ the composition

$$N \xrightarrow{h'} N \times S^6 \xrightarrow{h} N \times S^6 \xrightarrow{p} N.$$

By construction we have $ev \circ \tau = p \circ ev$ and a commutative diagram

$$\begin{array}{ccc} \text{aut}_1(N) & \xrightarrow{i} & \text{aut}_1(N \times S^6) \\ j_N \downarrow & & \downarrow j_{N \times S^6} \\ \text{Map}(M, N; f) & \xrightarrow{e} & \text{Map}(M \times S^6, N \times S^6; f \times id). \end{array}$$

Then $\sigma' = (\pi_*(\tau) \otimes \mathbb{Q}) \circ \sigma \circ (\pi_*(e) \otimes \mathbb{Q})$,

$$\begin{aligned} \pi_*(\text{Map}(M, N; f)) \otimes \mathbb{Q} &\rightarrow \pi_*(\text{Map}(M \times S^6, N \times S^6; f \times id)) \otimes \mathbb{Q} \\ &\rightarrow \pi_*(\text{aut}_1(N \times S^6)) \otimes \mathbb{Q} \rightarrow \pi_*(\text{aut}_1(N)) \otimes \mathbb{Q} \end{aligned}$$

is a retraction of $\pi_*(j_N) \otimes \mathbb{Q}$, and by construction $(\pi_*(ev) \otimes \mathbb{Q}) \circ \sigma' = \pi_*(\omega) \otimes \mathbb{Q}$. $\square$

Proof of Theorem 1. Theorem 2 shows that $\text{Im } \pi_*(\omega) \otimes \mathbb{Q} \subset G_*(N) \otimes \mathbb{Q}$. This proves Theorem 1(1). If $\chi(N) \neq 0$, $\tilde{H}_*(ev; \mathbb{Q}) = 0$, and so by Theorem 2, $hur_N \circ (\pi_*(\omega) \otimes \mathbb{Q}) = hur_N \circ (\pi_*(ev) \otimes \mathbb{Q}) \circ \sigma = \tilde{H}_*(ev; \mathbb{Q}) \circ hur_{\text{aut}_1(N)} \circ \sigma = 0$. $\square$

Proof of Theorem 3. Denote by $i_M : \text{aut}_1 M \rightarrow \text{Map}(M, N; f)$ the map associated to the composition with f . The commutativity of the diagram ($q \geq 1$)

$$\begin{array}{ccccc} \pi_q(\text{aut}_1(M)) \otimes \mathbb{Q} & \xrightarrow{\pi_q(i_M)} & \pi_q(\text{Map}(M, N; f) \otimes \mathbb{Q}) & \xrightarrow{\sigma} & \pi_q(\text{aut}_1(N)) \otimes \mathbb{Q} \\ \downarrow \pi_q(ev) & & \downarrow \pi_q(ev) & & \downarrow \pi_q(ev) \\ \pi_q(M) \otimes \mathbb{Q} & \xrightarrow{\pi_q(f)} & \pi_q(N) \otimes \mathbb{Q} & = & \pi_q(N) \otimes \mathbb{Q} \end{array}$$

shows that f maps $G_*(M) \otimes \mathbb{Q}$ into $G_*(N) \otimes \mathbb{Q}$. $\square$

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