CORRIGENDUM ON “ALMOST AUTOMORPHIC SOLUTIONS OF SEMILINEAR EVOLUTION EQUATIONS”

J. A. GOLDSTEIN AND G. M. N'GUÉRÉKATA

(Communicated by Ken Ono)

ABSTRACT. In this paper we correct an error contained in our paper *Almost automorphic solutions of semilinear evolution equations*, Proceedings of the American Mathematical Society 133 (2005), 2401-2408.

We are grateful to En-Wei Zheng and Qin He for pointing out a small error in the proof of Proposition 3.3 in [2]. It is easy to fix the proof, provided we modify the statement of Proposition 3.3 by replacing $AA(X)$ by the slightly smaller space $AA_u(X)$, $(X, \|\cdot\|)$ being a Banach space. This restriction seems not to restrict the existing or potential applications in partial differential equations. Here are the definitions. A function $f \in BC(\mathbb{R}; X)$ is said to belong to $AA(X)$ [resp., $AA_u(X)$] if for every sequence of real numbers (s'_n) there exists a subsequence (s_n) such that

$$\lim_{n \rightarrow \infty} f(t + s_n) = g(t), \quad \lim_{n \rightarrow \infty} g(t - s_n) = f(t)$$

pointwise on $\mathbb{R}$ [resp., uniformly on compacta of $\mathbb{R}$].

In [2] we asserted that a particular sequence in the space $BC^\delta(\mathbb{R}, Y)$ converged uniformly on $\mathbb{R}$, but our proof (based on the Arzelà-Ascoli theorem) only established uniform convergence on compacta. We treated $BC^\delta(\mathbb{R}, Y)$ as a Banach space under the norm $|\cdot|_{\delta, Y}$ for $0 \leq \delta \leq 1$. But it can also be viewed as a locally convex Fréchet space under the invariant metric

$$\rho(f, g) = \sum_{n=1}^{\infty} 2^{-n} \frac{\rho_n(f, g)}{1 + \rho_n(f, g)},$$

where, for $h = f - g$,

$$\rho_n(f, g) = \rho_n(h, 0) = \|h\|_{C[-n, n]} + \delta \sup \left\{ \frac{|h(t) - h(s)|}{|t - s|^\delta} : t, s \in [-n, n], t \neq s \right\}.$$

Let $Y_{(\delta)}$ be the Fréchet space $(BC^\delta(\mathbb{R}, Y), \rho)$. This gives $Y_{(\delta)}$ the compact-open topology, in which convergence of a sequence means uniform convergence on compacta of $\mathbb{R}$ of the functions and their δ -Hölder difference quotients. Then Proposition 3.3 [2] is valid if

(1) $BC^\delta(\mathbb{R}, Y)$, $BC(\mathbb{R}, X)$ are replaced by $Y_{(\delta)}$, $Y_{(0)}$,

Received by the editors November 11, 2010 and, in revised form, December 8, 2010.

2010 *Mathematics Subject Classification*. Primary 34A05, 34K05, 34G20.

Key words and phrases. Almost automorphic function, evolution equation, mild solution.

respectively. Since the Schauder fixed point theorem holds in locally convex spaces, Proposition 3.4 [2] holds in $AA_u(X)$, again assuming (1). The main result now follows, because $AA_u(X)$ is complete in the $Y_{(s)}$ topology. This follows from ([1], Remark p. 16).

Thus in summary we state our main results in [2] as follows.

Consider the semilinear differential equation

$$(2) \quad x'(t) = Ax(t) + F(t, x(t)), \quad t \in \mathbb{R},$$

where the linear operator $A : D(A) \subset X \rightarrow X$ generates an exponentially stable C_0 -semigroup $(T(t))_{t \geq 0}$ such that

$$F(t, x) = P(t)Q(x), \quad t \in \mathbb{R}, \quad x \in X,$$

where $P(t) \in AA_u(\mathbb{Z})$ for each $t \in \mathbb{R}$ with $\mathbb{Z} = B(X, Y)$; P is continuous from $\mathbb{R}$ to $AA_u(\mathbb{Z})$, and $Q : BC(\mathbb{R}, X) \rightarrow BC(\mathbb{R}, X)$ is continuous and satisfies the estimate

$$\|Q\varphi\|_\infty \leq \mathcal{M}(\|\varphi\|_\infty)$$

and $\mathcal{M} \in C(\mathbb{R}^+, \mathbb{R}^+)$ satisfies

$$\lim_{r \rightarrow \infty} \frac{\mathcal{M}(r)}{r} = 0.$$

Then we have

Proposition. $BC^\delta(\mathbb{R}, Y)$ is compactly contained in $BC(\mathbb{R}, X)$; in other words, the canonical injection $id : BC^\delta(\mathbb{R}, Y) \rightarrow BC^\delta(\mathbb{R}, X)$ is compact, which implies that

$$id : BC^\delta(\mathbb{R}, Y) \cap AA_u(X) \rightarrow AA_u(X)$$

is compact too.

Theorem. Under the above assumptions, (2) has a mild solution in $AA_u(X)$.

REFERENCES

1. B. Basit and A. J. Pryde, *Asymptotic behavior of orbits of C_0 -semigroups and solutions of linear and semilinear abstract differential equations*, Russian Journal of Mathematical Physics **13** (2006), 13-30. MR2262809 (2007i:34087)
2. Jerome A. Goldstein and G. M. N'Guérékata, *Almost automorphic solutions of semilinear evolution equations*, Proc. Amer. Math. Soc. **133** (2005), 2401-2408. MR2138883 (2006a:34175)

DEPARTMENT OF MATHEMATICAL SCIENCES, UNIVERSITY OF MEMPHIS, MEMPHIS, TENNESSEE 38152-3240

E-mail address: jgoldste@memphis.edu

DEPARTMENT OF MATHEMATICS, MORGAN STATE UNIVERSITY, 1700 E. COLD SPRING LANE, BALTIMORE, MARYLAND 21251

E-mail address: Gaston.N'Guerekata@morgan.edu