THE BOHNENBLUST-HILLE INEQUALITY COMBINED WITH AN INEQUALITY OF HELSON

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ABSTRACT. We give a variant of the Bohnenblust-Hille inequality which, for certain families of polynomials, leads to constants with polynomial growth in the degree.

1. INTRODUCTION

Hardy and Littlewood showed in [7] that there exists a constant $K > 0$ such that for every $f \in H^1$ we have

$$\left(\int_{\mathbb{D}} |f(z)|^2 dm(z) \right)^{1/2} \leq K \int_{\mathbb{T}} |f(w)| d\sigma(w),$$

where dm and $d\sigma$ denote respectively the normalised Lebesgue measures on the complex unit disc $\mathbb{D}$ and the torus (or unit circle) $\mathbb{T}$. Equivalently, this means that the Hardy space $H_1(\mathbb{T})$ is contained in the Bergman space $B_2(\mathbb{D})$. Shapiro [12, pp. 117-118] showed that the inequality holds with $K = \pi$, and Mateljević [10] (see also [11, 13]) showed that actually the constant could be taken to be $K = 1$. A simple reformulation of the Bergman norm then gives that if $\sum_{n=0}^{\infty} a_n z^n$ is the Fourier series expansion of $f \in H^1(\mathbb{D})$, we have

$$\left(\sum_{n=0}^{\infty} \frac{|a_n|^2}{n+1} \right)^{1/2} \leq \int_{\mathbb{T}} |f(w)| d\sigma(w).$$

A few years later Helson in [9] generalised this inequality to functions in N variables. For $n \in \mathbb{N}$ denote by $d(n)$ the number of divisors and by $p^\alpha = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$ the prime decomposition of n . Then we have that for every $f \in H^1(\mathbb{T}^N)$ with Fourier series expansion $\sum_{\alpha \in \mathbb{N}_0^N} c_\alpha z^\alpha$

$$(1.1) \quad \left(\sum_{\alpha \in \mathbb{N}_0^N} \frac{|c_\alpha|^2}{d(p^\alpha)} \right)^{1/2} \leq \int_{\mathbb{T}^N} |f(w)| d\sigma(w).$$

Given a multiindex α , we write $\alpha + 1 = (\alpha_1 + 1) \cdots (\alpha_k + 1)$. Note that, with this notation, we have $d(p^\alpha) = \alpha + 1$.

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On the other hand, by the Bohnenblust-Hille inequality [4] as presented in [5] there is a constant $C > 0$ such that for every m -homogeneous polynomial in N variables $P(z) = \sum_{|\alpha|=m} c_\alpha z^\alpha$ with $z \in \mathbb{C}^N$ we have

$$(1.2) \quad \left(\sum_{|\alpha|=m} |c_\alpha|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq C^m \sup_{z \in \mathbb{D}^N} |P(z)|.$$

The proof of this inequality given in [5] consists basically of two steps: first to decompose the sum in (1.2) as the product of certain mixed sums and second to bound each one of these sums by a term including $\|P\|$, the supremum of $|P|$ on $\mathbb{D}^N$. For this second step usually the following result of Bayart [1] is used: for every m -homogeneous polynomial in N variables we have

$$(1.3) \quad \left(\sum_{|\alpha|=m} |c_\alpha|^2 \right)^{1/2} \leq 2^{m/2} \int_{\mathbb{T}^N} \left| \sum_{|\alpha|=m} c_\alpha w^\alpha \right| d\sigma(w).$$

Very recently, it was proved in [2, Corollary 5.3] that for every $\varepsilon > 0$ there exists $\kappa > 0$ such that we can take $\kappa(1+\varepsilon)^m$ as the constant in (1.2). Our aim in this note is to get a variant of (1.2) by using (1.1) instead of (1.3). With this variant, we see that for polynomials P , each of whose monomials involve a uniformly bounded number of variables, the obtained constants have polynomial growth in m .

2. MAIN RESULT AND SOME REMARKS

The following is our main result.

Theorem 2.1. *Let $\Lambda \subseteq \{\alpha \in \mathbb{N}_0^N : |\alpha| = m\}$ be an indexing set. Then for every family $(c_\alpha)_{\alpha \in \Lambda}$ we have*

$$\left(\sum_{\alpha \in \Lambda} \left(\frac{|c_\alpha|}{\sqrt{\alpha+1}} \right)^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq m^{\frac{m-1}{2m}} \left(1 - \frac{1}{m-1} \right)^{m-1} \sup_{z \in \mathbb{D}^N} \left| \sum_{\alpha \in \Lambda} c_\alpha z^\alpha \right|.$$

We give several remarks before we present the proof.

Remarks 2.2.

- (1) It is easy to see that $\sqrt{\alpha+1} \leq \sqrt{2}^m$. Hence the preceding inequality includes the hypercontractive version of the Bohnenblust-Hille inequality from (1.2) as a special case.
- (2) Thanks to the term $\sqrt{\alpha+1}$, the constants in the previous inequality grow much more slowly than the constants in (1.2). Actually, we have

$$m^{\frac{m-1}{2m}} \left(1 - \frac{1}{m-1} \right)^{m-1} = \frac{\sqrt{m}}{e} (1 + o(m)).$$

- (3) Let $\text{vars}(\alpha)$ denote the number of different variables involved in the monomial z^α . In other words, $\text{vars}(\alpha) = \text{card} \{j : \alpha_j \neq 0\}$. Given M we consider the set

$$\Lambda_{N,M} = \{\alpha \in \mathbb{N}_0^N : |\alpha| = m \text{ and } \text{vars}(\alpha) \leq M\}$$

(note that if $M \geq N$, then $\Lambda_{N,M} = \Lambda_{N,N}$). An application of Lagrange multipliers gives that for any $\alpha \in \Lambda_{N,M}$ we have for every N and M ,

$$\alpha + 1 = (\alpha_1 + 1) \cdots (\alpha_k + 1) \cdots \leq \left(\frac{m}{M} + 1 \right)^M.$$

Combining this with Theorem 2.1 we obtain for every m, N, M ,

$$\begin{aligned} \left(\sum_{\alpha \in \Lambda_{N,M}} |c_\alpha|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} &\leq \left(\frac{m}{M} + 1 \right)^{M/2} \left(\sum_{\alpha \in \Lambda_{N,M}} \left(\frac{|c_\alpha|}{\sqrt{\alpha+1}} \right)^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \\ &\leq \left(\frac{m}{M} + 1 \right)^{M/2} m^{\frac{m-1}{2m}} \left(1 - \frac{1}{m-1} \right)^{m-1} \sup_{z \in \mathbb{D}^N} \left| \sum_{\alpha \in \Lambda_{N,M}} c_\alpha z^\alpha \right|; \end{aligned}$$

hence

$$(2.1) \quad \left(\sum_{\alpha \in \Lambda_{N,M}} |c_\alpha|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq 2^{\frac{M}{2}} m^{\frac{M+1}{2}} \sup_{z \in \mathbb{D}^N} \left| \sum_{\alpha \in \Lambda_{N,M}} c_\alpha z^\alpha \right|.$$

This means that for polynomials whose monomials have a uniformly bounded number M of different variables, we get a Bohnenblust-Hille type inequality with a constant of polynomial growth in m . We remark that the dimension N plays no role in this inequality; the only important point here is the number of different variables in each monomial. As a consequence, an analogue of (2.1) holds for m -homogeneous polynomials on c_0 : let $P: c_0 \rightarrow \mathbb{C}$ be an m -homogeneous polynomial and

$$\Lambda_M = \{\alpha \in \mathbb{N}_0^{(\mathbb{N})} : |\alpha| = m \text{ and } \text{vars}(\alpha) \leq M\}.$$

Then for every M and m

$$\left(\sum_{\alpha \in \Lambda_M} |c_\alpha(P)|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq 2^{\frac{M}{2}} m^{\frac{M+1}{2}} \|P\|,$$

where the $c_\alpha(P)$ are the coefficients of P and $\|P\|$ is the supremum of $|P|$ on the unit ball of c_0 .

- (4) In [6, Theorem 5.3] a very general version of the Bohnenblust-Hille inequality is given, involving operators with values on a Banach lattice. A straightforward combination of the proof of Theorem 2.1 (see the final section) and the arguments presented in [6, Theorem 5.3] easily gives a version of Theorem 2.1 in that setting.

3. THE PROOF

Let us fix some notation before we prove our main result. We are going to use the following indexing sets:

$$\begin{aligned} \mathcal{M}(m, N) &= \{\mathbf{i} = (i_1, \dots, i_m) : 1 \leq i_j \leq N, j = 1, \dots, m\}, \\ \mathcal{J}(m, N) &= \{\mathbf{i} \in \mathcal{M}(m, N) : 1 \leq i_1 \leq \dots \leq i_m \leq N\}. \end{aligned}$$

In $\mathcal{M}(m, N)$ we define an equivalence relation by $\mathbf{i} \sim \mathbf{j}$ if there is a permutation σ of $\{1, \dots, m\}$ such that $j_k = i_{\sigma(k)}$ for every k . With this, if matrix $(a_{i_1, \dots, i_m})$ is symmetric, then we have

$$\sum_{\mathbf{i} \in \mathcal{M}(m, N)} a_{\mathbf{i}} = \sum_{\mathbf{i} \in \mathcal{J}(m, N)} \sum_{\mathbf{j} \in [\mathbf{i}]} a_{\mathbf{j}} = \sum_{\mathbf{i} \in \mathcal{J}(m, N)} \text{card}[\mathbf{i}] a_{\mathbf{i}}.$$

Also, given $\mathbf{i} \in \mathcal{M}(m-1, N)$ and $j \in \{1, \dots, N\}$, for $1 \leq k \leq m-1$ we define $(\mathbf{i}, k, j) = (i_1, \dots, i_{k-1}, j, i_k, \dots, i_{m-1}) \in \mathcal{M}(m, N)$ (that is, we put j in the k -th position, shifting the rest to the right).

There is a one-to-one correspondence between $\mathcal{J}(m, N)$ and $\{\alpha \in \mathbb{N}_0^N : |\alpha| = m\}$ defined as follows. For each $\mathbf{i}$ we define $\alpha = (\alpha_1, \dots, \alpha_N)$ by $\alpha_r = \text{card}\{j : i_j = r\}$ (i.e. α_r counts how many times r appears in $\mathbf{i}$); on the other hand, given α we define $\mathbf{i} = (1, \overset{\alpha_1}{\cdot}, 1, \dots, N, \overset{\alpha_N}{\cdot}, N) \in \mathcal{J}(m, N)$.

Each m -homogeneous polynomial on N variables has a unique symmetric m -linear form $L : \mathbb{C}^N \times \dots \times \mathbb{C}^N \rightarrow \mathbb{C}$ such that $P(z) = L(z, \dots, z)$ for every z . If (c_α) are the coefficients of the polynomial and $a_{i_1, \dots, i_m} = L(e_{i_1}, \dots, e_{i_m})$ is the matrix of L , we have $c_\alpha = \text{card}[\mathbf{i}]a_{\mathbf{i}}$, where α and $\mathbf{i}$ are related to each other.

Finally, if α and $\mathbf{i}$ are related and $p_1 < p_2 < \dots$ denotes the sequence of prime numbers, we will write $p^\alpha = p_1^{\alpha_1} \dots p_N^{\alpha_N} = p_{i_1} \dots p_{i_m} = p_{\mathbf{i}}$.

Proof of Theorem 2.1. We follow essentially the guidelines of the proof of the Bohnenblust-Hille inequality as presented in [5]. First of all let us assume that $c_\alpha = 0$ for every $\alpha \notin \Lambda$; then we have

$$\begin{aligned} \left(\sum_{\alpha \in \Lambda} \left(\frac{|c_\alpha|}{\sqrt{\alpha+1}} \right)^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} &= \left(\sum_{\mathbf{i} \in \mathcal{J}(m, N)} \left| \text{card}[\mathbf{i}] \frac{a_{\mathbf{i}}}{\sqrt{d(p_{\mathbf{i}})}} \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \\ &= \left(\sum_{\mathbf{i} \in \mathcal{M}(m, N)} \frac{1}{\text{card}[\mathbf{i}]} \left| \text{card}[\mathbf{i}] \frac{a_{\mathbf{i}}}{\sqrt{d(p_{\mathbf{i}})}} \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \\ &= \left(\sum_{\mathbf{i} \in \mathcal{M}(m, N)} \left| \text{card}[\mathbf{i}]^{1-\frac{m+1}{2m}} \frac{a_{\mathbf{i}}}{\sqrt{d(p_{\mathbf{i}})}} \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}}. \end{aligned}$$

We now use an inequality due to Blei [3, Lemma 5.3] (see also [5, Lemma 1]): for any family of complex numbers $(b_{\mathbf{i}})_{\mathbf{i} \in \mathcal{M}(m, N)}$ we have

$$(3.1) \quad \sum_{\mathbf{i} \in \mathcal{M}(m, N)} |b_{\mathbf{i}}|^{\frac{2m}{m+1}} \leq \prod_{k=1}^m \left(\sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} |b_{(\mathbf{i}, k, j)}|^2 \right)^{1/2} \right)^{\frac{2}{m-1}}.$$

Using this and the fact that $\text{card}[(\mathbf{i}, k, j)] \leq m \text{card}[\mathbf{i}]$ we get

$$\begin{aligned} &\left(\sum_{\alpha \in \Lambda} \left(\frac{|c_\alpha|}{\sqrt{\alpha+1}} \right)^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \\ &\leq \prod_{k=1}^m \left(\sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} \left| \text{card}[(\mathbf{i}, k, j)]^{\frac{m-1}{2m}} \frac{a_{(\mathbf{i}, k, j)}}{\sqrt{d(p_{(\mathbf{i}, k, j)})}} \right|^2 \right)^{1/2} \right)^{\frac{1}{m}} \\ &\leq \prod_{k=1}^m \left(\sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} \left| \text{card}[\mathbf{i}]^{\frac{m-1}{2m}} m^{\frac{m-1}{2m}} \frac{a_{(\mathbf{i}, k, j)}}{\sqrt{d(p_{(\mathbf{i}, k, j)})}} \right|^2 \right)^{1/2} \right)^{\frac{1}{m}} \\ &= m^{\frac{m-1}{2m}} \prod_{k=1}^m \left(\sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} \text{card}[\mathbf{i}] \left| \frac{a_{(\mathbf{i}, k, j)}}{\sqrt{d(p_{(\mathbf{i}, k, j)})}} \right|^2 \right)^{1/2} \right)^{\frac{1}{m}}. \end{aligned}$$

We now bound each one of the sums in the product. We use the fact that the coefficients a_j are symmetric. Also, if q divides $p_{i_1} \dots p_{i_m} = p_{\mathbf{i}}$, then it also divides $p_{i_1} \dots p_{i_m} p_j = p_{(\mathbf{i}, k, j)}$; hence $d(p_{\mathbf{i}}) \leq d(p_{(\mathbf{i}, k, j)})$ for every $\mathbf{i}$ and every j . This

altogether gives

$$\begin{aligned} \sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} \text{card}[\mathbf{i}] \left| \frac{a(\mathbf{i}, k, j)}{\sqrt{d(p(\mathbf{i}, k, j))}} \right|^2 \right)^{1/2} \\ = \sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{J}(m-1, N)} \text{card}[\mathbf{i}]^2 \frac{|a(\mathbf{i}, k, j)|^2}{d(p(\mathbf{i}, k, j))} \right)^{1/2} \\ \leq \sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{J}(m-1, N)} \frac{|\text{card}[\mathbf{i}] a(\mathbf{i}, k, j)|^2}{d(p_{\mathbf{i}})} \right)^{1/2}. \end{aligned}$$

Let us note that what we have here are the coefficients of an $(m-1)$ -homogeneous polynomial in N variables. We now use (1.1) to conclude our argument:

$$\begin{aligned} \sum_{j=1}^N \left(\sum_{\mathbf{i} \in \mathcal{J}(m-1, N)} \frac{|\text{card}[\mathbf{i}] a(\mathbf{i}, k, j)|^2}{d(p_{\mathbf{i}})} \right)^{1/2} \\ \leq \sum_{j=1}^N \int_{\mathbb{T}^N} \left| \sum_{\mathbf{i} \in \mathcal{J}(m-1, N)} \text{card}[\mathbf{i}] a(\mathbf{i}, k, j) w_{i_1} \cdots w_{i_{m-1}} \right| d\sigma(w) \\ \leq \int_{\mathbb{T}^N} \sum_{j=1}^N \left| \sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} a(\mathbf{i}, k, j) w_{i_1} \cdots w_{i_{m-1}} \right| d\sigma(w) \\ \leq \sup_{z \in \mathbb{D}^N} \sum_{j=1}^N \left| \sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} a(\mathbf{i}, k, j) z_{i_1} \cdots z_{i_{m-1}} \right| \\ = \sup_{z \in \mathbb{D}^N} \sup_{y \in \mathbb{D}^N} \left| \sum_{j=1}^N \sum_{\mathbf{i} \in \mathcal{M}(m-1, N)} a(\mathbf{i}, k, j) z_{i_1} \cdots z_{i_{m-1}} y_j \right| \\ \leq \left(1 - \frac{1}{m-1} \right)^{m-1} \sup_{z \in \mathbb{D}^N} \left| \sum_{\alpha \in \Lambda} c_{\alpha} z^{\alpha} \right|, \end{aligned}$$

where the last inequality follows from a result of Harris [8, Theorem 1] (see also [5, (13)]). This completes the proof. $\square$

As we have already mentioned, very recently [2, Corollary 5.3] has shown that for every $\varepsilon > 0$ there exists $\kappa > 0$ such that (1.2) holds with $\kappa(1 + \varepsilon)^m$. The main idea for the proof is to replace (3.1) by a similar inequality in which we have mixed sums with k and $m-k$ indices (instead of 1 and $m-1$, as we have here). This allows us to use instead of (1.3) the following inequality:

$$\left(\sum_{|\alpha|=m} |c_{\alpha}|^2 \right)^{1/2} \leq c_p^m \left(\int_{\mathbb{T}^N} \left| \sum_{|\alpha|=m} c_{\alpha} w^{\alpha} \right|^p d\sigma(w) \right)^{\frac{1}{p}} \quad \text{for } 1 \leq p \leq 2.$$

A good control on the constants c_p (that tend to 1 as p goes to 2) gives an improvement on the constant in (1.2) presented in [2]. In our setting, by dividing by $\alpha + 1$, we are using (1.1), which already has constant 1. Hence this new approach does not improve the constants in our setting.

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