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ERRATUM TO "OPEN SETS OF AXIOM A FLOWS WITH EXPONENTIALLY MIXING ATTRACTORS"

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In "Open sets of Axiom A flows with exponentially mixing attractors" there was an oversight concerning the regularity of the Markov partition for higher dimensional flows. Previously [1, p. 2978] we claimed that any element of a Markov partition of a non-trivial attractor for an Axiom A vector field, after quotienting out the stable leaves, is a C^2 disk, hence a "John domain" [2, Definition 2.1]. The argument for exponential mixing presented in the paper relies on the application of results [2, Theorem 2.7] where the "John domain" condition is required.

Bowen [3] showed that, for higher dimensional systems, the boundaries of the Markov partition elements cannot be smooth (here smooth means piecewise $\mathcal{C}^1$), in particular the objects of interest cannot be expected to be C^2 disks as previously claimed. In general there is no reason to expect that the elements of the Markov partition are John domains. When the unstable bundle is higher dimensional and the expansion is not isotropic, there seems no hope that the sets are John domains (for evidence of this consult the estimates and comments in [4, §A.2]).

Here we show that the originally claimed result remains valid. First observe that, as previously described [1, §4], for any $d \geq 3$, it is possible to construct examples of vector fields with Axiom A attractors which have 1D unstable bundle and which satisfy the requirements of the rest of the construction. If the unstable bundle is 1D then the relevant element of the Markov partition is a John domain. This is because it is constructed [1, §3] as the connected subset of a local unstable manifold. Such a connected 1D set is automatically a John domain. The above observation gives immediately the main results as follows.

Theorem A. *Given any Riemannian manifold M of dimension $d \geq 3$ there exists a $\mathcal{C}^1$-open subset of $\mathcal{C}^3$-vector fields $\mathcal{U} \subset \mathfrak{X}^3(M)$ such that for each $X \in \mathcal{U}$ the associated flow is Axiom A and exhibits a non-trivial attractor Λ which mixes exponentially with respect to the unique SRB measure of Λ.*

Theorem B. *Suppose that $X^t : M \to M$ is a $\mathcal{C}^2$ Axiom A flow, Λ is an attractor (in particular closed and topologically transitive), that the stable foliation is $\mathcal{C}^2$ and that the unstable bundle is one-dimensional. If the stable and unstable foliations are*

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not jointly integrable, then the flow mixes exponentially with respect to the unique SRB of Λ*.*

The first theorem stands exactly as previously stated [1, Theorem A]. The second remains as previously stated [1, Theorem B] except for the addition of the assumption that the unstable bundle is one-dimensional.

The restriction that each of the flows constructed must have unstable bundle which is 1D could be removed using further improvements of the methods. Studying exponentially mixing Anosov flows Butterley & War [4] showed that elements of the Markov partition of the system obtained by quotienting along local stable manifolds can be identified with a finite number of connected subsets of $\mathbb{R}^d$ which satisfy some weak geometric properties [4, Appendix A]. Moreover these properties suffice for showing exponential mixing of the relevant suspension semiflow (a condition inspired by John domains, but weaker is used). It appears that the resultant theorem [4, Theorem 1] holds true (with the same proof) in the case of Axiom A attractors and not only for Anosov flows (i.e., Theorem B without requiring the 1D unstable bundle). Since the argument involves reducing the problem to an expanding semiflow by quotienting, the key is to show that the quotient system satisfies the assumptions of the theorem [4, Theorem 3] concerning exponential mixing of expanding semiflows. All the hyperbolicity properties are as before, the new details are the geometric properties (as proven in [4, Appendix A]). The key observation is that the systems of interest are attractors and not just Axiom A which means the system after quotienting along local stable manifolds sufficiently resembles the one obtained if the original system were Anosov. This is because, for a hyperbolic attractor, the local unstable manifold of each point of the attractor is contained within the attractor.

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