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A First Course in Sobolev Spaces

Giovanni Leoni
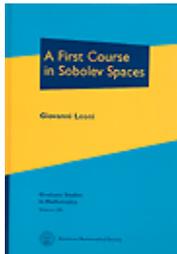
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MAA Review

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[Reviewed by Michael Berg, on 12/03/2009]

I'm loath to admit that despite my having been honorably discharged from graduate school over twenty years ago, my contact with Sobolev spaces has heretofore been nigh on nil. As a number theorist I didn't encounter these beasties except anecdotally or marginally, e.g., in graduate school colloquia in analysis which I attended as an interested (or coerced) outsider — but unquestionably an outsider — or leafing though a journal or a book, again generally far outside of my area of activity. To be sure, until recently, I have not had any need for Sobolev spaces in my research. But things have just changed for me in a rather dramatic manner and I find myself faced with having to learn functional analysis, and in particular certain things about Sobolev spaces, at a deeper level than what I have available as a bequest from my graduate studies. For this reason, admittedly a somewhat selfish one, Giovanni Leoni's brand new *A First Course in Sobolev Spaces* appears on the scene as a welcome surprise. Says Leoni: "To my knowledge, this is one of the first books to follow the… approach… [of looking at Sobolev spaces] as the natural development and unfolding of [the theory of] monotone, absolutely continuous, and

B[ounded] V[ariation] functions of one variable."

And so we find on p. 279 (!) of this big (~ 600 pp.) book the definition that a Sobolev space is a subspace of an L^p-space on some open set in $\mathbf{R}^n$ characterized by the condition that the members of the subspace are functions all of whose distributional first-order partial derivatives live in the given L^p-space. Fair enough, but two questions arise: what's this distributions business, and what's with the 278-page build-up?

Well, there's a single answer to these questions available: Leoni opted to structure his book in accord with accommodations he had to make to a segment of nonplussed beginning graduate students he encountered in his 2006 and 2008 courses titled "Sobolev Spaces." These latter-day youths manifestly knew not that a stiff dose of functional analysis were *de rigeur* for having a go at Sobolev spaces, and so Leoni had to change his selection of pitches if there was to be a game at all. Hence Leoni characterizes the first part of *A First Course in Sobolev Spaces* as amenable to being used for "an advanced undergraduate or beginning graduate course on real analysis or functions of one variable."

The book's second part "begins with [a] chapter on absolutely continuous transformations from a domains of $\mathbf{R}^N$ into $\mathbf{R}^N$," followed by a discussion of Laurent Schwartz's distributions: the stage is set — at last. And this second part of the book is serious analysis indeed: coverage of weak derivatives, BV functions of several variables, Besov spaces, and more. Hard core hard analysis. The game's afoot…

Leoni has worked hard to make *A First Course in Sobolev Spaces* maximally effective pedagogically: solid but accessible prose, great attention to detail, exercises, and a good sense of humor in evidence. This is a book that will see a lot of use. And, to get back to my earlier personal disclosures, I do propose to use it myself when the time is right — soon, I hope.

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