ERRATA March 8, 2024

A FIRST COURSE IN ENUMERATIVE COMBINATORICS (Carl Wagner)

Chapter 1

Exercise 1.21. In a random permutation of the digits 8, 7, 4, 3, and 2 (with the result interpreted as a 5-digit integer), what is the probability that that integer is divisible by 11?

Solution. Apply the alternating sum test for divisibility by 11. There are 10 two-element subsets of {8,7,4.3,2} from which to choose the second and fourth position (reading from left to right) of the permutation. The only case in which the alternating sum is divisible by 11 is when that sum takes the value zero, which happens just when the two-element set is equal to {4,8}. So the probability is 1/10.

Chapter 6

Revise Project 6.B as follows:

1. Add to the introductory paragraph:

Some authors (e.g., Carlitz, et al. (1972), Asymptotic Properties of Eulerian Numbers, *Z. Wahrscheinlichkeitstheorie verw. Geb.* **23**, 47-54) count the first entry of a permutation as an ascent. We follow Carlitz, et al. in denoting the resulting variant of $A(n,k)$ by $A_{n,k}$. When reading an article about Eulerian numbers it is important to ascertain if the author is referring to $A(n,k)$ or $A_{n,k}$.

2. Revise part (d) as follows:

(d) Prove that the Stirling number of the second kind $S(n,k)=\frac{1}{k!}\sum_{j=0}^{n}A(n,j)\binom{j}{n-k}$

$=\frac{1}{k!}\sum_{j=0}^{k}A_{n,j}\binom{n-j}{k-j}$, for $0\le k\le n.$

(e) When $n\ge 1$, and with $\sigma(n,k) := k!S(n,k)$, the second of the above formulas is equivalent to (*) $\sigma(n,k)=\sum_{j=1}^{k}A_{n,j}\binom{n-j}{k-j}$. Recalling that $\sigma(n,k)$ enumerates the ordered partitions $(B_1,\ldots,B_k)$ of $[n]$ with k blocks, use Theorem 2.5.5 to give a combinatorial proof of (*) by exhibiting a map from $OP_{n,k}$, the family of all such ordered partitions, to the set of all permutations of $[n]$ with k or fewer ascents, such that each permutation with j ascents has $\binom{n-j}{k-j}$ pre-images in $OP_{n,k}$.

(f) Prove *Worpitsky's identity,* $x^n = \sum_{k=0}^{n} A(n,k)\binom{x+k}{n}$, for all $n \geq 0$. Note that when $n \geq 1$, the upper limit in this sum may be replaced by $n-1$, since $A(n,n)=0$ if $n \geq 1$.

Chapter 9

Additional part (c) for Exercise 9.2. Using part (a) and Theorem 9.3.1, show that

$$(i) \qquad \sum_{k=0}^{n} f(k)\Delta g(k) = [f(n+1)g(n+1) - f(0)g(0)] - \sum_{k=0}^{n} (\Delta f(k))g(k+1),$$

and use this result along with part (b) and Theorem 9.3.1 to evaluate $\sum_{k=0}^{n} k2^k$. The summation formula (i) is the discrete analogue of the integration formula

$$(ii) \qquad \int_a^b f(x)g'(x)\,dx = [f(b)g(b) - f(a)g(a)] - \int_a^b f'(x)g(x)\,dx \ .$$

Solution. $\sum_{k=0}^{n} k2^k = [(n+1)2^{n+1} - 0\cdot 2^0] - \sum_{k=0}^{n} 1\cdot 2^{k+1} = (n+1)2^{n+1} - (2^{n+2}-2) = (n-1)2^{n+1} + 2.$