ERRATA
A FIRST COURSE IN STOCHASTIC CALCULUS

This is the errata to the textbook *A First Course in Stochastic Calculus* published by the AMS in November 2021. I will keep the errata updated regularly. If you find any typos/mistakes please email me at lparguin@gmail.com. I will add your name to the acknowledgements.

Acknowledgements
I thank my colleague Prof. Elena Kosygina for pointing out omissions and corrections in the book. I am also extremely grateful to the following people for finding many typos/mistakes

- Prof. Joe Chen (Colgate) and his students Chris Deng and Hieu Do;
- Prof. Prof. Vassilis Papanicolaou (NTU Athens);
- Prof. Raluca Balan (Ottawa).

Chapter 1

- Equation (1.12): $\mathbf{E}[e^{a\lambda X}]$ should be $\mathbf{E}[e^{\lambda X}]$.

Chapter 2

- Equation 2.7: The inner integral should be $\int_{-\infty}^{\sqrt{6}-\frac{1}{\sqrt{3}}z_1}$.
- Example 2.14: Consider a Gaussian random vector (X, Y) (instead of (X_1, X_2))
- Example 2.17: As before Write (X, Y) (instead of (X_1, X_2)) to be consistent with Example 2.14
- Box-Mueller should be spelled Box-Muller throughout the chapter.
- Lemma 2.12 and Exercise 2.11: The Gaussian random variables have mean 0.
- Example 2.20: $\mathbf{E}[Z_2^2]$ should $\mathbf{E}[X_1^2]$ instead of $\mathbf{E}[X_2^2]$ twice.
- Lemma 2.23: The assumption that the processes are independent is missing.
- Example 2.29: The first covariance should be
$$\mathrm{Cov}(Y_s, Y_t) = \frac{e^{-(t-s)}}{2}(1 - e^{-2s}).$$
To be consistent, the second covariance should be
$$\mathrm{Cov}(Y_s, Y_t) = \frac{e^{-(t-s)}}{2}.$$
- Exercise 2.3: Factors 1/2 are missing in e^{-x^2} and e^{-y^2}.
- Exercise 2.7: The index k of the summation should be j.
- Numerical Project 2.3: The covariances of the OU process should be $\mathrm{Cov}(Y_s, Y_t) = \frac{e^{-(t-s)}}{2}(1 - e^{-2s})$ and $\mathrm{Cov}(Y_s, Y_t) = \frac{e^{-(t-s)}}{2}$.

Date: Louis-Pierre Arguin, August 13 2023.

- Exercise 2.17 b): $\mathbf{E}[X_i F(\mathbf{Z})]$ should be $\mathbf{E}[X_i F(\mathbf{X})]$

Chapter 3

- Proof of Lemma 3.14, in the last step, one has to take the intersection over a countable number of δ's (say $\delta = 1/n$).
- Proof of Corollary 3.17: To be precise, the statement $\lim_{n\to\infty} \sum_{j=0}^{2^n-1} |B_{t_{j+1}}(\omega) - B_{t_j}(\omega)| < \infty$ should really be a lim sup but I avoided the use of lim sup as mentioned earlier in the chapter.
- Exercise 3.6 (c): The equation to prove should be $\lim_{t\to\infty} \frac{B_t}{t}$.

Chapter 4

- Equation above Example 4.2: $i \leq 1$ should be $i \geq 1$.
- Above Equation 4.14, $(X_n \geq 1)$ should read $(X_n, n \geq 1)$.
- Example 4.29: As stated, the example is a mean 0 random walk, which is more general than a symmetric random walk.
- Example 4.35: I refrained from using inf instead of min. This is ok if the min is thought of being taken on $[0, \infty]$, which is consistent with the convention that $\tau(\omega)$ if $X(\omega)$ never reaches a.
- Page 87-88: On one hand $\to$ On the one hand
- P.91 Line 5: $\{\max_{t\leq T} \widetilde{B}_T \geq a\}$ should be $\{\max_{t\leq T} \widetilde{B}_t \geq a\}$, where $(\widetilde{B}_t, t \geq 0)$ is $(B_t, t \geq 0)$ reflected at τ_a.
- Exercise 4.4c: It should be $\int_{\mathbb{R}} f(x,y)\mathrm{d}y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$ and $\int_{\mathbb{R}} y f(x,y)\mathrm{d}x = \rho x \frac{e^{-x^2/2}}{\sqrt{2\pi}}$.
- Exercise 4.12 (c): It might be preferable to use the parameter λ instead of a in the MGF.
- Exercise 4.15: The last term in the equation should be $2^{n-1} \cdot (S_n - S_{n-1})$.
- Exercise 4.16: Assume that $\mathcal{F}_t$ is in $\mathcal{F}$ for all $t \geq 0$.

Chapter 5

- P.101 Equation after (5.3): M_{j+1} should be $M_{t_{j+1}}$.
- Lemma 5.11: The proof holds under the assumption that (X_t) is bounded.
- Before Equation (5.26): the function should be $g(y) = Ce^{-2\mu y/\sigma^2} + C'$.
- Example 5.8: The equation for the covariance $\mathbf{E}[I_1 J_1]$ on page 107 should be a summation $\sum_{i,j=0}^{2}$ and not $\sum_{i,j=0}^{3}$.
- Proof of Corollary 5.18: We should have
$$I_t^{(n)} = \sum_{i=0}^{j-1} f(t_i)(B_{t_{i+1}} - B_{t_i}) + f(t_j)(B_t - B_{t_j})$$
- Examples 5.19 and 5.20: It would more appropriate to entitle these examples as "OU process/Brownian bridge *in terms* of Itô integrals" as these processes are not exactly Itô integrals.
- Exercise 5.17 (a): The ODE is for the function g not f.
- Exercise 5.19 (b): You should find the mean and variance of X in terms of X_n.

Chapter 6

- Project 6.2: $X_t = (B_t^{(1)})^2 + (B_t^{(2)})^2 - 2t$ and not $-t$.

Chapter 7

- P. 157 Line 8: $\langle I_t \rangle$ should be $\langle I \rangle_t$.
- Example 7.11 on p. 160: the last line should refer to Theorem 9.11.
- Example 7.12 p.161: In the second equation the derivative before $(\mathrm{d}S_t)^2$ should be $\partial_1^2 f(t, S_t)$. Same in the following equation.
- Theorem 7.18: A $\frac{1}{2}$ is missing in front of $\sum_{j,k}$.
- Equation 7.20 p. 164, second line: the integrand of $\mathrm{d}X_u$ should be $\mu'(X_u)$.
- Exercise 7.8d): As a bonus exercise, one can use the stopped martingale $t \wedge \tau - 2X_{t\wedge\tau}$ to show that $\mathbf{E}[\tau] < \infty$, hence $\mathbf{P}(\tau < \infty) = 1$. (Thank you Elena Kosygina for this one!)
- Exercise 7.13: $(V_t, s \leq T)$ should be $(V_s, s \leq T)$. Also the equation to prove is obviously wrong! It should be

$$\mathbf{E}\Big[\Big(\int_0^{t'} V_s \mathrm{d}B_s - \int_0^t V_s \mathrm{d}B_s\Big)^2 - \int_t^{t'} V_s^2 \mathrm{d}s \big| \mathcal{F}_t\Big] = 0.$$

Chapter 8

- Exercise 8.4: It should be $W_t = B_{t+s} - B_s$.

Chapter 9

- Equation 9.19: there should be a $-$ in front of Θ_s in the Itô integral.

Chapter 10

- Figure 10.1: The figure gives the payoff for a European call AND the five option strategies of Example 10.8.
- Equation 10.28: There is a minus missing in Θ_t.
- Example 10.23: The first derivative should read

$$\frac{\partial C_t}{\partial K} = \Big(S_t N'(d_+) - Ke^{-r(T-t)} N'(d_-)\Big)\frac{\partial d_\pm}{\partial K} - e^{-r(T-t)} N(d_-) = -e^{-r(T-t)} N(d_-).$$

  And the second derivative should be

$$\frac{\partial^2 C_t}{\partial K^2} = -e^{-r(T-t)} N'(d_-) \cdot \frac{\partial d_-}{\partial K} = e^{-r(T-t)} \frac{e^{-(d_-)^2/2}}{K\sigma\sqrt{2\pi(T-t)}} > 0$$

- P. 225 Item (2): the second-to-last sentence *with some probability at time T* should be *with some probability at time τ*.
- Equation 10.7: the term $a_t R_t \mathrm{d}t$ should be $a_t R_t D_t^{-1} \mathrm{d}t$.
- Equations 10.19 and 10.20: the a_t's for the calls and the puts in the following equations should have the factor e^{-rT} and not $e^{-r(T-t)}$.
- Right above Equation 10.32: $\widetilde{\mathbf{P}}$ should be $\mathbf{P}$.
- Example 10.33: The function $r(x)$ is actually just the constant function $r(x) = r$. The equation for $\partial_t f$ should then have r instead of rx on the right.

- Example 10.48: An equal sign is missing.
- Equation 10.52: ∂f^2 should be $\partial^2 f$.
- Exercise 10.2: with the data given, the option is a plain butterfly. To get a skip strike butterfly with puts, change the last strike from 200 to 250.
- Exercise 10.9: This is not an option *per se* as the payoff might be negative...
- Exercise 10.11b): The equation should be

$$\widetilde{S}_t = S_0 \int_0^t e^{-rs} \sigma \mathrm{d}\widetilde{B}_s.$$

- Exercise 10.16a): All B's should be $\widetilde{B}$.
- Exercise 10.18c): There is no square in the last $h(s)$ in the integral for $g(t)$.