Errata for the book

"Functional Analysis — An Elementary Introduction"
(GSM 156, AMS, Providence 2014)

by Markus Haase

In the following I collect all those misprints or mistakes that have been identified by now (7 September 2021). Most of them are not very serious, in the sense that they can easily be spotted and corrected. Only two are real grave: **Exercises 14.6 and 14.16 are simply wrong and should be deleted, as well the references to them in the main text (pages 250 and 252).** (I am quite ashamed of the fact that I haven't realized that from the very start and included them in the text.)

The list below also contains typographical corrections and passages to include in a future second edition.

I am very grateful to Marko Budisic (Clarkson University) and his students for initiating this errata pages and contributing substantially to it.

Markus Haase

Preface

p. xvi, "Chapter 10" : should be $\mathrm{H}^1(a,b)$ instead of $\mathrm{H}^1[(a,b)$.

p. xvii, "Chapter 16": should be $\mathrm{L}^1(a,b)$ and $\mathrm{L}^p(a,b)$ instead of $\mathrm{L}^1[a,b]$ and $\mathrm{L}^p[a,b]$, respectively. (Just for the sake of notational consistency.)

Chapter 1, Inner Product Spaces

p. 12, Exercise 1.14: "orthnormal" → "orthonormal"

Chapter 2, Normed Spaces

Chapter 3, Distance and Approximation

p.52. Exercise 3.14, last line: this might look awkward, but is actually correct.

p.53, Exercise 3.20: should read $\ell^p \subseteq \ell^q$.

p.53, Exercise 3.22.b, display: should be "$-\mathrm{e}^{-nt}f(t)$" instead of "$+\mathrm{e}^{-nt}f(t)$".

Chapter 4, Continuity and Compactness

p.70, Lemma 32: "Let F be *a* closed linear ..."

p.75: Exercise 4.18, display: should read $d((x_1,x_2),(y_1,y_2)) := d_1(x_1,y_1)+d_2(x_2,y_2)$

Chapter 5, Banach Spaces

Chapter 6, The Contraction Principle

Chapter 7, The Lebesgue Spaces

Chapter 8, Hilbert Spaces Fundamentals

Chapter 9, Approximation Theory and Fourier Analysis

p. 148, Theorem 9.2.b: "Let E, F be normed space*s*"

p.151, proof of Lemma 9.6: "uniform for $-1 \leq 1 - s^2 \leq 1 \ldots$ "

p.153, line -10: leave out additional space before "In order" [Latex fix]

p.159, proof of Theorem 9.14: "We use the characterization (ii) from ..."

p.160, first greyshaded box: delete space in "$2\pi \mathrm{i}\, t$" [Latex fix]

p.168, Section 9.7, second line: "Lemma 9.3.b)"$\rightarrow$ "Exercise 9.3.b)"

p.168, Theorem 9.28, display: should be $\|T\|_{E\to F} = \|T_0\|_{E_0\to F}$.

p.173, Exercise 9.9, display: should be $G := \{f \in E \mid \text{is a Cauchy sequence in } F\}$.

p.174, Exercise 9.15, display: should be $p_{n+1}(s) := p_n(s) + \ldots \quad (n \geq 0,\ s \in [0,1])$.

Chapter 10, Sobolev Spaces and the Poisson Problem

p.184, 5th display: should be $(J^2 f)(t) = \displaystyle\int_a^t (t-s) f(s)\,\mathrm{d}s$

p.190, Exercise 10.14.(iii): should read "for all $v \in \ldots$ "

p.190, Exercise 10.15, definition of u_0: should be $u_0(s) = g(a) + \dfrac{g(b)-g(a)}{b-a}(s-a)$

Chapter 11, Operator Theory I

p.197, line 6: put some space between $\mathrm{d}s$ and $\mathrm{d}t$ [Latex fix]

p.197, Theorem 11.6: should read just "associated integral operator" (the term "Hilbert–Schmidt integral operator is defined only afterwards).

p.197, proof of Theorem 11.6: replace [a,b] by X

p.208, Exercise 11.23, second display: should be $(\mathcal{H}f)(x)$

Chapter 12, Operator Theory II

p.211, last display: should be $\mathcal{C}_0(E;F)$ instead of $\mathcal{C}_0(H;K)$

p.215, proof of Theorem 12.7: should be $\|A_n - A\|$ instead of $\|A_n \to A\|$

p.217, Theorem 12.9, first display: should receive a tag

p.217, proof of Theorem 12.9: "By (12.2)" should be replaced by a reference to the *first* display in the formulation of Theorem 12.9.

p.217, proof of Theorem 12.9: replace "whence" by "and hence"

Chapter 13, Spectral Theory of Compact Self-Adjoint Operators

p.234, Theorem 13.8: should read "on a Hilbert space H"

p.238, proof of Corollary 13.2: in the beginning it should read "a) and b)" instead of just "a)"; furthermore, replace " by a)" by "by what has already been shown",

p.241, after Theorem 13.14: "in the framework *of*" and "see ... *or* ..."

p.243, definition of the resolvent: should be $R(\lambda, A) := (\lambda \mathrm{I} - A)^{-1}$

p.243, Exercise 13.5: should read $R(\lambda, A) = \sum_{n=0}^{\infty} (\mu - \lambda)^n R(\mu, A)^{n+1}$

p.245, Exercise 13.21.e): should be $A(F^{\perp}) \subseteq F^{\perp}$

Chapter 14, Applications of the Spectral Theorem

p.247, line -7: should read "Since g is symmetric"

p.249, first display: should be $(0, 1)$ instead of (a, b) (twice)

p.249, Lemma 14.2: should read "either $u = 0$ or $\lambda > 0$"

p.249, proof of Lemma 14.2, display: should be $\langle qu, u \rangle$ instead of $\langle pu, u \rangle$

p.249, def'n of the Sturm–Liouville problem: should be $(0, 1)$ instead of (a, b) (twice)

p.250, last two lines: close bracket after "analogously" and delete the reference to Exercise 14.6 completely, as that exercise is erroneous (see below).

p.252, after the greyshaded box: delete the whole paragraph, as Exercise 14.16 is erroneous (see below).

p.254, last display: should be $\|k\|_2^2$ instead of $\|k\|_{\mathrm{HS}}^2$

p.255, proof of Lemma 14.5: refer to Corollary 12.10 instead of to Lemma 12.12

p.255, after proof of Lemma 14.5: replace "By the above lemma" by "Hence"

p.258, Exercise 14.6: This exercise is erroneous and should be deleted!

p.259, Exercise 14.16: This exercise is erroneous and should be deleted!

Chapter 15, Baire's Theorem and Its Conseqences

p.276, after Exercise 15.18: put the following exercise: "Let E, F be Banach spaces and $T \in \mathcal{L}(E; F)$ a surjective bounded linear operator. Show that if $S \in \mathcal{L}(E; F)$ with $\|T - S\| < 1$, then S is surjective as well."

Chapter 16, Duality and the Hahn–Banach Theorem

p.279, display (16.2): should read "for all $f \in E$"

p.283, proof of Theorem 16.4, 1st line: should read "ordered pairs (E_1, φ_1)"

p.289, proof of Theorem 16.4.a): should read "specializing $T : F \to E$ the inclusion mapping"

p.289, after the proof of Theorem 16.4: replace "to find solutions" by "for finding solutions"

p.294, Reamrk 16.20: rename "Milman's theorem" by "the Milman–Pettis theorem"; amend the reference to Lax as [Lax02, Chap.8, Thm.11]

Historical Remarks

p.306, 2nd par: "research" → "research"

p.308, 1st par: put "Fredholm[17]," instead of "Fredholm,[17]"

Appendix A, Background

p.311, greyshaded box: replace $(x_n)_{n\in\mathbb{N}} \subseteq \mathbb{N}$ by $(x_n)_{n\in\mathbb{N}} \subseteq X$

p.312, before beginning of Section A.2: should be $\pi(k)$ instead of $n_{\pi(k)}$

p.325, line 13: should read "If $T : E \to E$ is linear"

p.327, Lemma A.7, last line: should be $\mathrm{Lin}(E)$ instead of $\mathrm{Lin}(E; F)$

p.328, Sesquilinear Forms: it should be pointed out that for each sesquilinear form a one has the following two identities:

$$\begin{aligned} &q_a(f+g) + q_q(f-g) = 2q_a(f) + 2q_a(g) \qquad \text{(parallelogram)} \\ &q_a(f+g) - q_a(f-g) = 2a(f,g) + 2a(g,f). \end{aligned}$$

If $\mathbb{K} = \mathbb{C}$ then one may replace g by $\mathrm{i}g$ here in the second identity and obtains

$$q_a(f+\mathrm{i}g) - q_a(f-\mathrm{i}g) = 2\mathrm{i}\big(-a(f,g) + a(g,f)\big).$$

This leads to the general polarization identity

$$a(f,g) = \frac{1}{4}\Big(q_a(f+g) - q_a(f-g) + \mathrm{i}\big(q_a(f+\mathrm{i}g) - q_a(f-\mathrm{i}g)\big)\Big).$$

It follows that in the case $\mathbb{K} = \mathbb{C}$, each sesquilinear form is determined by the associated quadratic form. Moreover, it follows that such a sesquilinear form a is hermitian if and only if q_a is real-valued.

p.330, last display: leave out the comma

p.330, last line: should read "$f(x) = f(y)$ implies that $x = y$"

Appendix B, The Completion of a Metric Space

Appendix C, Bernstein's Proof of Weierstrass' Theorem

Appendix D, Smooth Cutoff Functions

Appendix E, Some Topics from Fourier Analysis

Appendix F, General Orthonormal Systems

Bibliography

Symbol Index, Subject Index, Author Index