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CORRECTIONS AND CLARIFICATIONS OF "INTRODUCTION TO ALGEBRAIC GEOMETRY"

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page 52, lines 1 - 2: Corollary 2.87 should be: Suppose that X is an affine variety. If $p, q \in X$ are distinct points, then $\mathcal{O}_{X,p} \neq \mathcal{O}_{X,q}$.

page 57, line 18: Suppose that U is an open subset of an affine variety X.

page 82, lines 3 -4: Proposition 3.36 should be: Suppose that W is a projective variety. Then distinct points of W have distinct local rings.

page 105, line -7: "$S(X)$" should be "$S(X \times W)$".

page 124, line -5: "$\mathbb{P}^3$" should be "$\mathbb{P}^2$".

page 149, after line 5, insert: To see this, observe that there exist affine neighborhoods V of p and W of q such that $\phi(V) \subset W$ and $V \cap \phi^{-1}(q) = p$. Thus by the Nullstellensatz, the ideal $I_V(V \cap \phi^{-1}(q))$ in $k[V]$ is the maximal ideal $I_V(p)$. This is just the radical of $I_W(q)k[V]$. Localizing at $I_V(p)$, we have that the radical of $m_q\mathcal{O}_{X,p}$ is $m_p\mathcal{O}_{X,p}$.

page 166, line 5: $k(X) \rightarrow k(Z)$

page 188, line 7: Should be $x(p) = x_i(p)$.

page 199, line -13: "Let U be an open subset of the projective variety X" should be "Let X be a projective variety".

page 199, line -7, Insert "for every $q \in V$" after "$s(q) = \frac{a}{f} \in N_{(I(q))}$".

page 204, lines -12 and -11: $\mathcal{F}$ is a coherent sheaf on X and $\mathcal{G}$ is a quasi coherent sheaf on X.

page 210, line 4: "x_n" should be "x_r".

page 210, line 5: "$0 \leq i \leq n$" should be "$0 \leq i \leq r$".

page 210, lines 13 - 16: Replace 'Letting $S =$ .... with localization." with "Letting $S = S(X)$, we have by Lemma 11.44, natural isomorphisms of $S_{(x_i)}$-modules

$$\Gamma(X_{x_i}, \mathcal{F} \otimes_{\mathcal{O}_X} \mathcal{O}_X(n)) = N_{(x_i)} \otimes_{S_{(x_i)}} S(n)_{(x_i)} \cong N(n)_{(x_i)} = \Gamma(X_{x_i}, \widetilde{N(n)}).$$

Now formula (11.20) follows from the sheaf axioms."

page 213, insert after line 8: The S-module structure on M is defined as follows. We have a natural homomorphism of quasi-coherent sheaves

$$\mathcal{F}(n) \oplus \mathcal{O}_X(m) \to \mathcal{F}(n) \otimes_{\mathcal{O}_X} \mathcal{O}_X(m) \cong \mathcal{F}(m+n),$$

giving a homomorphism

$$\Gamma(X, \mathcal{F}(n)) \oplus \Gamma(X, \mathcal{O}_X(m)) \to \Gamma(X, \mathcal{F}(m+n)).$$

We then define the graded S-module structure on M from the natural maps $S_m \to \Gamma(X, \mathcal{O}_X(m))$.

page 214, line 4: $\Gamma(X_{x_j}, \mathcal{F}(d))$.

page 251, line 5: $Z(F_j) = D_j$.

page 251, line 6: $Z(G_j) = E_j$.

page 266, line 8: remove "such that $\mathcal{O}_X(D)$ is invertible"

page 271, line 8: $\Gamma(V, \mathcal{L})$.

page 295: Theorem 15.14 should be: Suppose that X is an abstract variety . Then for $p, q \in X$, the stalks $\mathcal{O}_{X,p} \subset k(X)$ and $\mathcal{O}_{X,q} \subset k(X)$ are equal if and only if $p = q$.

page 295, lines -13 and -12: A quasi projective variety is an abstract variety by Proposition 5.8.

page 348, line 2: $\mathcal{O}_R = \mathcal{O}_X / \mathcal{O}_X(-R)$

page 355: Insert after line 9: Suppose that X_1 and X_2 are elliptic curves and $\sigma : X_1 \to X_2$ is an isomorphism. Let $\phi = \phi_{|2\mathbb{P}_2|} : X_2 \to \mathbb{P}^1$ be ramified over $\{0, 1, \lambda, \infty\}$. Then $\phi\sigma = \phi_{|2\mathbb{P}_1|} : X_1 \to \mathbb{P}^1$ (where $p_1 = \sigma^{-1}(p_2)$) is ramified over $\{0, 1, \lambda, \infty\}$. Thus X_1 and X_2 have the same j-invariant $j(\lambda)$.