# MATHEMATICS ACROSS THE IRON CURTAIN: A HISTORY OF THE ALGEBRAIC THEORY OF SEMIGROUPS ERRATA AND ADDITIONAL REFERENCES

#### CHRISTOPHER HOLLINGS

### Chapter 1. Algebra at the Beginning of the Twentieth Century

Section 1.3. An overview of the development of semigroup theory

The following two brief articles on the history of algebraic semigroup theory may be added to the list at the bottom of p. 10:

M. Paula Marques Smith, Semigroups also have history, in *Research seminar on history and epistemology of mathematics* (eds. Assis Azevedo, M. Elfrida Ralha and Lisa Santos), Centro de Matemática de Universidade do Minho, 2010, pp. 157–172.

M. Lurdes Teixeira, Finite semigroup theory, an historical perspective, in *Research seminar on history and epistemology of mathematics* (eds. Assis Azevedo, M. Elfrida Ralha and Lisa Santos), Centro de Matemática de Universidade do Minho, 2010, pp. 173–184.

### Chapter 2. Communication between East and West

An expanded version of this chapter, in which a broader perspective is taken (i.e., one that goes beyond mathematics to science more generally) may be found in:

Christopher D. Hollings, Scientific communication across the Iron Curtain, Springer-Briefs in History of Science and Technology, Springer, 2016.

#### Chapter 3. Anton Kazimirovich Sushkevich

Section 3.1. Biography

A few comments on Sushkevich's work on the history of algebra (as mentioned at the top of p. 54) may be found in:

Christopher Hollings, Investigating a claim for Russian priority in the abstract definition of a ring, BSHM Bulletin: Journal of the British Society for the History of Mathematics **29**(2) (2014), 111–119.

Date: April 1, 2016.

### Chapter 4. Unique Factorisation in Semigroups

Section 4.1. Postulational analysis

The E. T. Bell quotation on lines 11–12 of p. 83 should in fact read: "the decadent vice of playing with barren postulates".

Section 4.2. E. T. Bell and the arithmetisation of algebra

Further commentary on E. T. Bell's attitude towards abstract algebra, and his presentation of it to a general readership, may be found in:

Christopher Hollings, A tale of mathematical myth-making: E. T. Bell and the 'arithmetization of algebra', BSHM Bulletin: Journal of the British Society for the History of Mathematics 31(1) (2016), 69–80.

### Section 4.5. Arithmetic of ova

The statement on p. 102 that the material on factorisation in semigroups found in Nathan Jacobson's Lectures on abstract algebra (vol. 1, D. Van Nostrand, Princeton, NJ, 1951) was removed from subsequent editions is incorrect. In fact, much the same material may be found in Section 2.14 of Jacobson's later Basic algebra (W. H. Freeman, San Fransisco, CA, 1974), although it is rather less prominent than in the earlier book — the reason suggested on p. 102 for the supposed absence of this material from later editions may be applied instead to its comparatively minor position.

### Chapter 5. Embedding Semigroups in Groups

An extended version of this chapter, which expands upon the comments made about the work of Pták, Lambek, Jackson and Bush on p. 133, may be found in:

Christopher Hollings, Embedding semigroups in groups: not as simple as it might seem, Archive for History of Exact Sciences **68**(5) (2014), 641–692.

### Chapter 6. The Rees Theorem

Section 6.6. Unions of groups and semigroups

Contrary to the statement on p. 156 that the name 'completely regular semigroup' was introduced by Mario Petrich in his *Introduction to semigroups* (Merrill, Columbus, OH, 1973), the Russian equivalent of this term ('вполне регулярная полугруппа') can in fact be found on p. 104 of E. S. Lyapin's *Semigroups* (Gos. Izdat. Fiz.-Mat. Lit., Moscow, 1960).

## Chapter 8. The Expansion of the Theory in the 1940s and 1950s

Section 8.4. The Japanese school

The figure 12,418,001,077,381,302,684 may be added to Table 8.1 on p. 203 as the number of nonequivalent semigroups of order 10; this is taken from:

Andreas Distler, Chris Jefferson, Tom Kelsey and Lars Kotthoff, The semigroups of order 10, in *Principles and practice of constraint programming: 18th International Conference, CP 2012, Québec City, QC, Canada, October 8–12, 2012, Proceedings* (ed. Michela Milano), Springer, 2012, pp. 883–899.

The following articles may be added to the list of references in note 38 on p. 358 (concerning the enumeration of finite semigroups):

Petr Gajdoš and Martin Kuřil, Ordered semigroups of size at most 7 and linearly ordered semigroups of size at most 10, Semigroup Forum 89(3) (2014), 639–663.

Martin E. Malandro, Enumeration of finite inverse semigroups, arXiv:1312.7192v1, 2013.

#### Section 8.6. British authors

An additional biographical source for John M. Howie (to be added to those listed in note 57 on p. 359) is:

Gracinda M. S. Gomes and Nik Ruškuc, John Macintosh Howie: work and legacy, *Semi-group Forum* **89**(1) (2014), 2–19.

### Chapter 9. The Post-Sushkevich Soviet School

A fuller account of the survey volumes discussed on pp. 217–218, and the picture that they present of the development of abstract algebra in the USSR, may be found in:

Christopher Hollings, The acceptance of abstract algebra in the USSR, as viewed through periodic surveys of the progress of Soviet mathematical science, *Historia Mathematica* **42**(2) (2015), 193–222.

# Section 9.2. Lyapin's mathematical work

The statement that the notion of a densely embedded ideal has never been adopted into Western semigroup theory is not quite right. In fact, it has appeared from time to time; see, for example: Mario Petrich, The translational hull in semigroups and rings, *Semigroup Forum* 1 (1970), 283–360.

### Chapter 10. The Development of Inverse Semigroups

A technical addendum to this chapter, outlining the three main approaches to the subsequent study of inverse semigroups (namely, those via inductive groupoids, fundamental inverse semigroups and Munn representations, and E-unitary/proper inverse semigroups and P-semigroups), may be found in:

Christopher Hollings, Three approaches to inverse semigroups, European Journal of Pure and Applied Mathematics 8(3) (2015), 294–323.

Amongst other things, this latter article notes the brief appearance of the minimum group congruence  $\sigma$  in V. V. Wagner's 1953 paper 'Theory of generalised heaps and generalised groups' (*Mat. sb.* 32 (1953), 545–632), a fact not mentioned in the relevant part of Chapter 10.

### Section 10.6. Preston and inverse semigroups

An exposition of the work of David Rees on partial one-one transformations, and his use of these in connection with the embedding problems discussed in **Chapter 5** may be found in:

J. Madhusudana Rao and A. V. Ramakrishna, On embedding a semigroup in a group, Resonance 19(8) (2014), 740–752.

### Chapter 12. Books, Seminars, Conferences, and Journals

Section 12.3. Czechoslovakia, 1968, and Semigroup Forum

The following conferences may be added to the list given in note 31 on pp. 370–371:

- (52) Workshop on Computational Algebra, Lisbon, Portugal, 21–23 July 2014.
- (53) Semigroups, Languages and Algebras, Akita University, Japan, 7–9 August 2014.
- (54) Workshop on Groups and Semigroups on the occasion of the 60th birthday of Mikhail Volkov, Porto, Portugal, 9 June 2015.
- (55) International Conference on Semigroup Algebras and Applications, Cochin University of Science and Technology, Cochin, Kerala, India, 17–19 September 2015.
- (56) International Conference on Semigroups and Automata celebrating the 60th birth-day of Jorge Almeida and Gracinda Gomes, Lisbon, Portugal, 20–24 June 2016.

## **Bibliography**

Updated details are available for the following item:

Mark V. Lawson, Liam O'Carroll and Sarah Rees, David Rees 1918–2013, Semigroup Forum 88(3) (2014), 513–522.

A further obituary of David Rees is:

Rodney Y. Sharp, David Rees, FRS 1918–2013, Bulletin of the London Mathematical Society, 2016.

MATHEMATICAL INSTITUTE, UNIVERSITY OF OXFORD, ANDREW WILES BUILDING, RADCLIFFE OBSERVATORY QUARTER, WOODSTOCK ROAD, OXFORD, OX2 6GG, UNITED KINGDOM

THE QUEEN'S COLLEGE, OXFORD, OX1 4AW, UNITED KINGDOM *E-mail address*: christopher.hollings@maths.ox.ac.uk