Appendix E

Errata

Page 13, Exercise 1.5.6 (6):

Replace "vector space of functions $\mathcal{V}$." with "vector space of functions $\mathcal{V}$ which is closed under taking absolute values."

Page 13, Exercise 1.5.6 (8) should read:

Prove that f is a regulated function on $I = [a, b]$ if and only if both of the limits

$$\lim_{x \to c+} f(x) \quad \text{and} \quad \lim_{x \to d-} f(x)$$

exist for every $c \in [a, b)$ and every $d \in (a, b]$.

Page 68, Proof of Theorem 4.2.1: This proof assumes without justification that $h_n(x) \geq h(x)$ for all x so $h_n(x) - h(x)$ is non-negative. Essentially the same proof treats the case the $h(x) \geq h_n(x)$ so $h(x) - h_n(x)$ is non-negative.

For the general case we define $h_n^+(x) = \max\{h(x), h_n(x)\}$ and and $h_n^-(x) = \min\{h(x), h_n(x)\}$. Observe that $h_n^+(x) \geq h(x)$ and $h(x) \geq h_n^-(x)$ and $\lim h_n^+(x) = h(x) = \lim h_n^+(x)$. So the special cases for which the proof is valid show $\lim \int h_n^+ \, d\mu = \int h \, d\mu = \lim \int h_n^- d\mu$.

Also note that $h_n^+(x) + h_n^-(x) = h_n(x) + h(x)$. Thus the special cases together with part (5) of Exercise 4.1.8 imply

$$\begin{aligned} 2\int h\ d\mu &= \lim_{n\to\infty}\int h_n^+\ d\mu\ + \lim_{n\to\infty}\int h_n^-\ d\mu \\ &= \lim_{n\to\infty}\int h_n^+ + h_n^-\ d\mu \\ &= \lim_{n\to\infty}\int h_n + h\ d\mu \\ &= \lim_{n\to\infty}\int h_n\ d\mu + \int h\ d\mu \end{aligned}$$

and the result follows.

Page 76, Exercise 4.3.9 (2) should read:

Let X be an uncountable set and let $\mathcal{A}$ be the σ-algebra of all subsets of X. Prove there is no finite measure on X which assigns a positive measure to every finite non-empty set.

Page 77, the last line of exercise 4.3.9 (6) delete the phrase: with respect to μ.

Page 87: "togther" should read "together"

Page 88, Exercise 5.1.10 (1) should read: Give an example of a function $f : [a, b] \to \mathbb{R}$ such that f is integrable but f^2 is not.

Page 91, line 5 should read:

... $\leq 4n^2\mu(A) + \delta^2\mu(A^c) \leq 4n^2\delta + (b-a)\delta^2$.

Page 91, line 7 should read:

$\|f_n - g\| \leq \sqrt{4n^2\delta + (b-a)\delta^2} < \frac{\varepsilon}{2}$.

Page 102, line 7 should read:

$s_n = \sum_{i=0}^{n}\langle w, u_i\rangle u_i$. Then since the family is orthogonal, we know

Page 108, line 5 should read:

$$= \|w\|^2 - \sum_{n=0}^{N}|a_n|^2 + \sum_{n=0}^{N}(a_n - c_n)\overline{(a_n - c_n)}$$

Page 109, line -4 should read:

$$\sum_{i=0}^{\infty} |\langle w, u_i \rangle|^2 = \|w\|^2$$

Page 116, lines 13–14 should read:

with $|g(x) - p(x)| < \varepsilon/4$ for all x. So

$$\|g - p\|^2 = \frac{1}{\pi} \int (g-p)^2 \; d\mu \leq \frac{1}{\pi} \int \frac{\varepsilon^2}{16} \; d\mu = \frac{\varepsilon^2}{8}.$$

Hence, $\|f - p\| \leq \|f - g\| + \|g - p\| < \varepsilon/2 + \varepsilon/\sqrt{8} < \varepsilon$.

Page 118, the last line of exercise 6.1.10 (1) should read:

$$\sin\theta_1 \sin\theta_2 = \frac{1}{2}\big(\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)\big).$$

Page 122, the penultimate sentence of the proof of 6.2.5 should read:

Also, $f\bar{g} = (u_1u_2 + v_1v_2) + i(v_1u_2 - u_1v_2)$.

Page 135, the second paragraph of the proof of 7.2.2 should read:

Conversely, if f is a T-invariant measurable function then its real part, $u(x)$, and imaginary part, $v(x)$, are T-invariant. If f is not ν-almost everywhere constant, then at least one of $u(x)$ and $v(x)$ is not ν-almost everywhere constant. Suppose, without loss of generality, that it is $u(x)$. Then there is a $c \in \mathbb{R}$ such that the set $A = u^{-1}([-\infty, c))$ satisfies $\nu(A) > 0$ and $\nu(A^c) > 0$. The set A is T-invariant.

Page 138, line 4 of Corollary 7.3.2 should read:

$A \cap \{T^k(x)\}_{k=0}^{m-1}$, i.e., the number of points in A from the orbit segment

Page 169, line 2 should read:

$$|\langle v, w\rangle|^2 = \|\langle v, w\rangle w\|^2 \leq \|\langle v, w\rangle w\|^2 + \|v - \langle v, w\rangle w\|^2 = 1$$

Page 187, Lemma B.3.9 and its proof should read:

Lemma B.3.9. *Suppose* A_0 *and* B_0 *are disjoint sets in* $\mathcal{M}_0$ *and* X_0 *is an arbitrary subset of* $\mathbb{R}$. *If* $A = A_0 \cap X_0$ *and* $B = B_0 \cap X_0$ *then*

$$\mu^*(A \cup B) = \mu^*(A) + \mu^*(B).$$

The analogous result for a finite union of disjoint sets is also valid.

Proof. Since A_0 and B_0 are disjoint, so are A_0 and B, and therefore $A_0 \cap B = \emptyset$ and $A_0^c \cap B = B$. Hence

$$A_0 \cap (A \cup B) = (A_0 \cap X_0) \cup (A_0 \cap B \cap X_0) = A_0 \cap X_0 = A.$$

Likewise,

$$A_0^c \cap (A \cup B) = (A_0^c \cap A_0 \cap X_0) \cup (A_0^c \cap B) = B.$$

Hence, the fact that A_0 is in $\mathcal{M}_0$ (using $A \cup B$ for X in the definition of $\mathcal{M}_0$) tells us

$$\begin{aligned}\mu^*(A \cup B) &= \mu^*(A_0 \cap (A \cup B)) + \mu^*(A_0^c \cap (A \cup B)) \\ &= \mu^*(A) + \mu^*(B).\end{aligned}$$

The result for a finite collection $A_0 \cap X_0, A_1 \cap X_0, \dots, A_n \cap X_0$, where $A_i \in \mathcal{M}_0$ follows immediately by induction on n. □

Page 188, the first displayed equation should read:

$$B_{n+1} = A_{n+1} \setminus \bigcup_{i=1}^{n} A_i = A_{n+1} \cap \Big(\bigcup_{i=1}^{n} A_i\Big)^c.$$

Page 197, item [C]:

Replace “sumas” with “sums”

Page 197, item [M]:

Replace “Erdodic” with “Ergodic”

Aug. 17, 2020.