It's not true what I write.
It's true what I mean.

ERRATA LIST and COMMENTS for **Jordan Structures in Lie Algebras**

To the reader. *Information about misprints and possible mistakes found in the text, as well as comments on it, will be most appreciated by the author.*

page ix: The correct title of Section 14.6 is: Lie algebras generated by ad-nilpotent elements.

page 6, line -7: There are two commas after the word *derivation*. Of course, one is enough!

line -4: I meant *induction* argument not *induced* argument.

page 7, Proof of Lemma 1.2: Use the Leibniz rule in the equalities $d^k(xy) = 0$, for $m \leq k \leq 2m-2$ and $x, y \in L$, to eliminate the superfluous summands. Then the proof follows...

page 11, line 6: $(I)_\Phi$ must be replaced by $\mathrm{span}_\Phi(I)$ twice.

page 14, line -11: S instead of X.

page 16, line 15: The word *law* was incorrectly written as *las*.

page 18, Exercise 1.37: a *finite-dimensional* semiprime algebra.

page 19, line 17: of *a* locally finite...

page 23, line -1: $\mathrm{ad}_{A^n x}$.

page 25, line 3 of Example 2.19: less *than* or equal to...

page 34, line -17: $I_1 = I$.

page 40, line 16: I meant *call* not *called*.

page 41, Corollary 2.69: The statement must be replaced by the following: If L is *simple*, then any *nontrivial* finite $\mathbb{Z}$-pregrading of L is actually a grading. This is just what is used in the proof of Lemma 11.33(i).

page 49, Exercise 2.92(3): $\mathrm{span}\{x, xa, \ldots, xa^{n-1}\}$ (curly brackets instead of parenthesis).

line 2 of Exercise 2.98: The restriction $n \geq 1$ is unnecessary.

page 51, line 5: divisors**.** On (the period is missing).

page 61, line -8: for any $y \in L$.

line -6: for $y \in L$.

page 62, line 12: I meant *would imply* not *would be imply*.

page 64, line 1: part of *his* investigation.

page 65, line 14: i.e. *a prime* associative.

page 67, Proof of Proposition 3.39: Reference to Lemma 3.38 is necessary to justify that $\mathrm{ad}_t^2 k = 0$ implies $(kt - tk)Rt^2 = 0$ for every $k \in K$. Hence, *either* $t \in Z(K)$ *or* $t^2 = 0$.

page 69, line 3: I meant *giving* not *given*.

page 72, Proposition 4.13: Cite [**FLGGL09**] is not that listed in Bibliography.

page 77. In equation (4.13): the required condition is $i + j \geq \mathbf{8}$.

page 80, line -1 of Proof of Theorem 4.29: applied to u (not to v).

page 81, lines 14-16: The sentence beginning with *prove a necessary...* must be ignored. The connection between minimal abelian inner ideasl and minimal one-sided ideals is studied in Chapter 13.

page 82, line -5: Proposition **4.35** not 4.34.

page 85, Theorem 4.47(ii): condition $a^2Ka^2 = 0$ is superfluous.

page 86, Exercise 4.56: Give **an** easier.

page 90, Theorem 5.11(iv): $\frac{1}{2}\mathrm{ad}_e^2 : L_{-2}^h \to L_2^h$, $\frac{1}{2}\mathrm{ad}_f^2 : L_2^h \to L_{-2}^h$.

page 92, line -8: c is symmetric or skew-symmetric, then d can be chosen symmetric or skew-symmetric.

page 93, line 1: c is skew-symmetric (the case that c symmetric is similar).

page 97, line 17: Cite [**GGL12**] is not that listed in Bibliography.
line 5 of Definition 5.31: and $\mathcal{F}_n = L$. **I**t is clear

page 104, Remark 6.14: $\mathrm{char}(\mathbb{F}) \geq 5$,

page 110, line -1: $[[x, e, y, e], [x, e, z, e]]$

page 118, line -6: $\mathrm{id}_L(x') \cap \mathrm{id}_L(y') = 0$.

page 119: (we do not know *whether simplicity* is inherited).

page 120, line -12: I meant *Cayley algebra* not *Cayley Lie algebra*

page 121, Macdonald Principle: [**McC04**, II.5.1.1].
Shirshov-Cohn Principle: [**McC04**, II.5.1.4].

page 122, Lemma 8.9: [**McC04**, II.10.2].
line -4: a positive integer **n**.

page 124, line 2 of Proposition 8.19: U_aJ **is** a strict..
lines 10,12: The statement is not correct, what I tried to say is the following: *An algebraic unital Jordan algebra* J *over a field* $\mathbb{F}$ *of characteristic not* 2 *is a division*

algebra iff the minumum polynomial $m_a(x)$ *of every nonzero element* $a \in J$ *is irreducible:* $\langle a\rangle_J \cong \mathbb{F}[x]/(m_a(x))$.

line 21: [**McC04**, II.18.1.4].

Proof of Prop. 8.20, line 2: J is a nondegenerate *Jordan* algebra.

page 125, line 15: $\{x, y, z\} + \{y, x, z\} = 4(x \bullet y) \bullet z$,

page 126, lines -14 and -13: $\mathrm{Mc}_1(J)$.

page 127, line -3: has a nonzero linear trace tr: $\tilde{J} \to \mathbb{F}$.

page 129, line 12: proved **in**.

Proof of Theorem 8.41: J is an I-algebra...

page 133, lines 3 and 4 of the proof of (3): of L such that $\mathrm{ad}_a^2 B \neq 0$. Since by Corollary 4.20 $\mathrm{ad}_a^2 B$ is a minimal abelian inner ideal of L, we have by Lemma 8.50(ii) that $\overline{B}$ is a minimal inner ideal of L_a.

page 135, Remark 8.58: The reference to **Bah76** is incorrect. The correct one is Bahturin's book *Identical relations in Lie algebras.* Translated from the Russian by Bahturin. VNU Science Press, b.v., Utrecht, 1987.

page 137, line -20: b is (delete **as**) Jordan element.

line -3: a nonzero *Jordan* element.

page 138, line 3: algebra**s** are understood.

Proposition 8.70: *If* R *is primitive, then* $\overline{R}$ *is primitive whenever it contains a nonzero Jordan element. Conversely,...*

Corollary 8.71: *If* R *is primitive and contains nonzero nilpotent elements, then* $\overline{R}$ *is primitive.*

Proposition 8.72(ii): less than *or equal to* 16.

page 141, line 6: By Exercise 2.99(3), for any $c \in [x, H^\perp]$, $\mathrm{ad}_c^2 \mathfrak{o}(X, \langle\,,\rangle) \subset [x, H^\perp]$, so it remains to show that for every $v \in H^\perp$ there exists...

line 7 of Remark 8.82: we get $\gamma = 0$ and hence

$$[x, xa] = [x, \beta x + w] = [x, w] \in [x, H^\perp].$$

Proposition 8.83(3): DELETE: so Rc^2 is a minimal left ideal of R.

Proposition 8.83(5): R is a subring of $\mathcal{L}_X(X)$...

page 142, line 5: (6) As seen in the proof of (5), c^2R is a minimal right ideal of $\mathcal{F}_X(X)$, so $\mathrm{rank}(c^2) = 1$.

line -5: is no*t* complete.

page 145. line -5: (Example 8.4). *Moreover,* this Jordan...

page 154. line 3: 3-torsion free is enough.

page 156: The proof of Theorem 9.12 actually proves that L *is nondegenerate if and only if it is a strongly nondegenerate.* Nevertheless, this is just what is used later.

page 157, line 5: $0 < |k| \leq n$.

page 160, 9.4, line 2: of its associated Lie *algebra.*

page 165, line 13: with **a** result.

page 168, lines -19, -18: Replace *overline*$\mathbb{F}$ by $\overline{\overline{\mathbb{F}}}$.
lines -16 and -14: Replace $\mathbb{F}$ by $\overline{\overline{\mathbb{F}}}$.

page 169, line -13: a**n** $\mathbb{F}$-subalgebra.

page 170, line 6: $g(t)$ **is nonzero**.

page 171, line 14: action **of** an element...
line 18: In Section 11.5 we revisited...
line 7: for $g(t)$ with nonzero...

page 172, line 1: the **analogues** of...

page 173, line 9: $[D(u,v), D(x,y)] = ...$

page 174, line -13: a**n** m-sequence.

page 175, Proposition 11.6: $x \bullet y = \frac{1}{2}\{x, v, y\}$

page 176, line 1 of the proof of Proposition 11.10: well defined, *linear*, and one-to-one.

page 178, line 6: is defined.

page 178, line -8: on **pairs**.

page 180, Remark 11.20(a)l lines 2-4: To avoid confusion, replace V by U in the following formulas: $I = W^*V, \;\; V \leq X_2.$ ... and V have...
line -1: called **the** Tits-Kantor-Koecher algebra

page 181, line 11: $\delta(x^+, y^-) - \delta(y^+, x^-))\oplus$

page 183, lines 10, 17 and 20: $\mathcal{F}_{X\oplus Y}(X \oplus Y)$ instead of $\mathcal{F}(X \oplus Y)$.
line 19: where $K = \mathrm{Skew}(\mathcal{F}_{X\oplus Y}(X \oplus Y), h)$
lines 25 and 26: where $K = \mathrm{Skew}(\mathcal{F}_M(M), *)$ and Z is the center of $\mathcal{F}_M(M)$,

page 184, 11.3, line 7: $[a_n, \cdots, a_2, a_1] = \mathrm{ad}_{a_n} \cdots \mathrm{ad}_{a_2} a_1$
Lemma 11.30: Proof of assertion (i) needs to be clarified.

(i) *For* $m \geq 1$ *and any nonempty subset* $X \subset L_n$,

$$(\mathrm{ad}(L)^m X) \cap L_n \subset \sum_{i=1}^{r} \mathrm{ad}(L_0)^i X,$$

for some $r \geq 1$.

Proof. By the grading properties,

$$(\mathrm{ad}(L)^m X) \cap L_n = \sum_{(i_1,\ldots,i_m)} \prod_{j=1}^{m} \mathrm{ad}(L_{i_j})X$$

where $-n \le i_j \le n$ for all $1 \le j \le m$, and for each $1 \le k \le m$, $\sum_{j=k}^m i_j \le 0$ (since $\sum_{j=k}^m i_j > 0$ would imply $\prod_{j=k}^m \mathrm{ad}(L_{i_j})X = 0$). Suppose then that for some $1 < k \le m$ we have $\sum_{j=k}^m i_j \le 0$ and $i_{k-1} > 0$. By transposition of adjacent factors, we can move $\mathrm{ad}(L_{i_{k-1}})$ forward enough to reach a position where the product vanishes, yielding a sum of factors of length less than m (see Exercise 11.77 for examples). An induction process completes the proof.

page 188, 11.37, line 4: Φ-submodule.

page 193, Proposition 11.56: Cite [**FLGGL09**] is not that listed in Bibliography.

page 194, line -4: a nondegenerate **Lie** algebra.

page 201, line 4: over a field $\mathbb{F}$

page 203, line -9: $[x, [x, \mathrm{Ker}_L[e]]] \subset [(e), [(e), \mathrm{Ker}_L(e)]] = 0$.

page 208, line 3: Proposition 11.47(v) and...

page 213, line -16: algebras **or** Jordan pairs

page 214, line -14: *argument is the fact*

lines -8 and -7: The expression "$\mathrm{ad}_x \mathrm{ad}_y = 0$ if Lie, $Q_{x,y} = 0$ if the Jordan" was involuntarily repeated.

page 217, Lemma 3.1. *Let U be a Φ-submodule of L*

page 221, line -7: $\mathrm{Inn}(U, f)$.

page 230, line 20: applying (**3.4**).

page 231, lines 3 and 4: I meant *Clifford.*

page 234, line 5 of the proof of Theorem 13.63: nonzero, then V_0 would contain...

line -12: (see also [**Loo75**, 12.5]).

page 238, line -10: of $\overline{\mathrm{K}'}$ **it**

page 240, Lemma 13.78(iii): a $*$-orthogonal *idempotent.*

line -6: hence, by Theorem 13.32,

page 242, line -8: modul**o** the center of...

page 244, line -4: what implies...

page 247, line 24: be **the** root...

page 248, line 16: classical ones, A_n, B_n, C_n and D_n, are particular cases **of** those...

line 25: The sentence *the root decomposition of L relative to H* is repeated.

page 249, line 3 of the central paragraph: I meant *up to.*

page 255, line 13 of Introduction: **ad**-nilpotent elements.

page 259, line 19: DJA(L)

page 260, line 10: By Theorem 11.32 **and Lemma 11.33** ... is **central** simple.

line 16: The reference to Lemma 10.1 must be replaced by [**Bath 76, Theorem 2**], since we are not assuming that the field is algebraically closed.

page 263, line 8: locally finite **Lie** algebras

line 11: over an algebraically.

line 12: be **a** nonzero subalgebra.

page 264, line 2 of Lemma 14.18: subalgebra L of R^- is...

lines 18-19: so $a = 0$ (Remark 2.84).

line -6: will denote.

page 265, Theorem 14.25: For a better statement of this theorem, move *locally finite* from the initial assumption to items (ii), (iii), (iv), (v).

page 266, Theorem 14.26: The qualifier *of diagonal type* was unintentionally included in the statement.

page 272, lines -12, -11: if D is a derivation of a normed algebra A, then Sep(D) is an ideal of A.

page 284, Exercise 15.44(2): I meant *it does.*

page 288, [FLGGL09]: The reference given here is not correct. The correct one is: Antonio Fernández López, Esther García, and Miguel Gómez Lozano. Inner ideal structure of nearly Artinian Lie algebras. *Proc. Amer. Math. Soc.*, 137(1): 1-9 (2009).

page 289, [GGL12]: The reference given here is not correct. The correct one is; Esther García and Miguel Gómez Lozano. Principal filtrations of Lie algebras. *Comm. Algebra*, 40(10): 3622-3628 (2012).

COMMENTS

• The definitions of product of ideals, semipriminess and primeness given in Chapter 1 are not suitable when we deal with Jordan algebras. Given I and K ideals of a Jordan algebra J it is shown, using Macdonal Principle, that $U_I K$ spans an inner ideal. With respect to semiprimeness and primeness, a Jordan algebra J is called *semiprime* (*prime*) if $U_I I = 0$ implies $I = 0$ ($U_I K = 0$ implies $I = 0$ or $K = 0$). However, under this notion of semiprimeness, the following chain of equivalences still holds, for I, K ideals of J:

$$U_I K = 0 \Leftrightarrow U_K I = 0 \Leftrightarrow K \cap I = 0 \Leftrightarrow I \bullet K = 0.$$

In general, $I \cap K = 0 \Rightarrow I \bullet K = 0 \Rightarrow U_I K = 0$. Conversely, $I \bullet K = 0$ implies $U_{I\cap K} I \cap K = 0 \Rightarrow I \cap K = 0$ by semiprimeness.

• Notice that the continuity of every derivation D of a strongly prime Banach Lie algebra L with nonzero socle M (a particular case of Corollary 15.4) can be derived from the fact that the restriction of D to M is the adjoint operator of a continuous linear map (Proposition 14.13 together with Theorem 15.25) and therefore continuous.

• In [**BF16**, Example 6.17], the authors provide an example of an isotropic inner ideal (**Definition 13.2**) of a Lie algebra R^-, where R is an associative algebra, and therefore an inner ideal of the Jordan algebra R^+, which is not an inner ideal of R (**Definition 13.19**).

• In the proof of Proposition 11.41(ii) (page 189) we use the following result: *Let J be a nondegenerate* linear *Jordan algebra, and let $z \in J$ be such that $U_x z = 0$ for all $x \in J$. Then $z = 0$.* Using nondegeneracy twice, it suffices to show that $U_{U_z x} y = 0$ for all $x, y \in J$: $U_{U_z x} y = U_z U_x U_z y = 0$ in virtue of the identity

$$2U_x U_z y = V(x, z)^2 y - V(U_x z, z)y$$

and the fact that $V(x, z)y = U_{x+y} z - U_x z - U_y z = 0$.