Corrections and improvements to the book: 'One-dimensional turbulence and the stochastic Burgers equation'.

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July 15, 2022

- P. 17. In the first line below (1.2.9), replace "decreasing" by "non-increasing".
- P. 17. In the second line above (1.2.10), replace "decreasing" by "non-increasing".
- P. 22. After Lemma 1.3.3, add the paragraph, containing the sentence: "The assertion of the lemma remains true if $h(x)$ is a function with a non-zero mean, and $|\cdot|_{m,p}$ and $|\cdot|_q$ stand for the norms in spaces of functions on S^1 of any mean-value."
- P. 33. In the first displayed formula, in the second sum replace "$(2\pi j)$" with "$|2\pi j|$".
- P. 67. In the first displayed formula in item ii), replace "$(2\pi s)$" with "$|2\pi s|$".
- P.91. After (6.0.8) add: "More specifically, the theory claims that if the flow $u(t,x)$ is homogeneous and isotropic, then the l.h.s. of (6.0.8) equals $-(4/5)\epsilon r$, where ϵ is the rate of dissipation of energy (see [114] and Section 6.2 of [68]). In this form the relation is called *Kolmogorov's* 4/5-*law*."
- P. 108. After Problem 7.2.4, add the following paragraph:

  Signed moments of increments of velocity fields.

  The structure function $S_{p,l}(u)$ is made from the unsigned (or absolute) moments of increments $u(t,x+l)-u(t,x)$. But the results of previous chapters also allow to treat their signed moments. In particular, we can establish for burgulence a 1d version of the weak form (6.0.8) of Kolmogorov's 4/5-law. Namely, we claim that under the same assumptions on l and ν as in Theorem 7.2.3 (up to a possible change of constants C_1, c_2 and c_*), there are constants $c',\ C'>0$ such that any solution $u=u(t,x;u_0)$ with $u_0\in H^1$ satisfies

  $$-C'\ell^3 \le \langle\langle \int_{S^1} (u(t,x+\ell)-u(t,x))^3\,dx\rangle\rangle \le -c'\ell^3.$$

  Indeed, the lower bound clearly follows from (7.2.5) with $p=3$. Since $u=-|u|+2u^+$, then in view of (7.2.5) to establish the upper one it suffices to prove that

  $$A := \langle\langle \int_{S^1} ((u(t,x+\ell)-u(t,x))^+)^3\ dx\rangle\rangle \le S_{\ell,3}/3.$$

  But as $u(x+\ell)-u(x)\le \int_x^{x+\ell} u_x^+(y)\,dy$, then

  $$\begin{aligned} A &\le \langle\langle \int_{S^1} \Big(\int_x^{x+\ell} u_x^+(t,y)dy\Big)^3\,dx\rangle\rangle \\ &\le \langle\langle \int_{S^1} (\sup_x(u_x^+(t,x))\cdot\ell)^3\ dx\rangle\rangle = \ell^3\langle\langle(\sup_x(u_x^+(t,x))^3\rangle\rangle \le K\ell^3, \end{aligned}$$

  for some constant $K>0$, where the last inequality follows from Corollary 2.1.5. Consequently, $A\le S_{3,\ell}/3$ under the same assumptions on ℓ,ν as in Theorem 7.2.3 (possibly for modified constants C_1, c_2 and c_*).

- P. 121. In (8.2.1) replace "$u^+_{0x}(0,\cdot)$" with "u^+_{0x}"
- P. 121. In (8.2.2) twice replace "$_+$" with "$^+$".
- P. 157. In the assertion of Theorem 11.1.3 replace "monotone increasing" with "non-decreasing".
- P. 157. The assertion of Theorem 11.1.4 remains true with the same proof for $0 < \alpha < \infty$.
- P. 158. In the first displayed formula, replace "$(2\pi s)$" with "$|2\pi s|$".
- P. 158. In l.4 replace "Lebesgue dominated convergence theorem" with "Fatou's lemma".
- P. 158. The assertions of Corollary 11.1.5 remain true with the same proof for $0 < p < \infty$.
- P. 158. The assertions of Theorem 11.1.6 remain true with the same proof for $0 < \alpha < \infty$.
- P. 158. In the proof of Theorem 11.1.6 replace "the dominated convergence theorem" with "Fatou's lemma". And replace "using the monotone convergence theorem" with "using once again Fatou's lemma".
- P. 158. The assertions of Corollary 11.1.7 remain true with the same proof for $0 < \alpha < \infty$.
- P. 159. In the assertion of Theorem 11.1.12 replace "monotonically increasing" with "non-decreasing", and replace "monotonically decreasing" with "non-increasing".
- P. 159. In the last displayed formula, in the integral add $d\mu$.
- P. 160. In l.6 it should be "with $0 \leq m \leq 1/2$
- P. 171. In formulas (11.8.1) and (11.8.3) remove the signs "$\sum_{j=1}^n$"
- P. 186. In the third line of reference [64] replace "ref. [17]" with "ref. [18]"