Errata and afterthoughts about the book SURV287 "Basic Modern Theory of Linear Complex Analytic q-difference equations"

April 2, 2025

Generalities

First of all, sorry for the heavy presentation: my aim is to have these complements online as soon as possible.

Although the main text has been reread carefully many times, there are stil (many ?) errors. Some of them (few I believe) are substantial mathematical errors and absolutely require correction. Some are errors of redaction which make reading very difficult. All those I will try to track and correct.

I do that by reading through all the book: on paper I find it easier and more efficient than on screen. I will publish corrections "à la volée". The first series will go up to chapter 3 (I found no significant errors in the Foreword, Preface and Introduction).

I expect that attentive and helpful readers will find other errors and tell me. I will publish them along with date of modification at the end of the chapters.

I will not correct typos and other errors when they have no consequence on the understanding of the text. If someday the AMS decides for a new edition (one may dream), then I shall correct them (I keep note).

There is a more substantial afterthought that deserves to be emphasized right now. As explained at various places in the book, the convention $0 < |q| < 1$ is, in essence, innocuous. However some formulas may differ and I announced in the introduction that there would be enough examples and exercises to teach the reader how to adapt the results. I find that in the end this has not been achieved[1] and therefore I intend to prepare a short guide on the matter. It will also be published on this webpage, hopefully within a few months.

[1] Also I had planned the book for the GSM Series and I would therefore have published hints and/or solutions to the exercises. The AMS wizards decided for the SURV Series and I reckon that this means that the reader has to be more autonomous.

Errata and afterthoughts on Chapter 1

p 12 1.4.5 l -6 and l -1: The continuous spiral is $q_0^{-\mathbf{R}_+}$, not $q^{-\mathbf{R}_+}$.

p 19 1.6.2 l 12: Complete as follows: "${}_2F_1$ (see Subsection 1.2.3)".

p 32 1.11.2: About Barnes-Mellin-Watson, also see Appendix C.

Errata and afterthoughts on Chapter 2

p 49 2.2.2.2 Exercise 2.24 (v): Here $[\omega, \omega']$ denotes the lattice $\mathbf{Z}\omega + \mathbf{Z}\omega'$.

p 51 2.2.2.3 l 3: The reference to the modular group is, at best, confusing. One must distinguish 1) the action of the modular group on $\mathcal{H}$, with quotient shown to be isomorphic to $\mathbf{C}$ (appendix A.3.2 or reference [231]); and 2) the action of a lattice Λ on $\mathbf{C}$, with quotient $\mathbf{E}_\Lambda$ (appendices A.1.4, A.1.5).

p 57 2.3.1.2 Exercise 2.36: Instead of "using" read "applying".
The exercise deserves a hint. Take inspiration from Lang [144] chap. I §1 and apply it to functions such that $f(qx) - f(x)$ is constant, like $x{\theta_q}'/\theta_q$ or xe_c'/e_c or elliptic functions. Note however that one cannot obtain all desired conclusions in that way.

p 58 2.3.1.5 Prop. 2.39: In the proof, the correct expression for $\phi_{c,d}$ is $\dfrac{\theta_q(x)\theta_q(cdx)}{\theta_q(cx)\theta_q(dx)}$.

p 61 2.3.2.2 Lemma 2.46: In (ii) of the proof, read ℓ_q^n and not $\ell_q^{(n)}$ (those are powers of ℓ_q).

p 63 2.3.4: The reference [196] will probably be published in 2025.
Exercise 2.53: At the end of the line, one should read "... or $\mu = 0$ and $c \notin q^{\mathbf{Z}}$.

p 64 2.3.4.2: Just before the corollary, the first reference is Subsection 7.3.3 (not B.5 !) of Chapter 7.
Moreover, the vector bundle mentioned in the corollary is denoted either $\mathscr{F}_{A_\mu}$ (according to the general considerations of Appendix B.5.5) or $\mathscr{F}_{A_\mu}^{(0)}$ (according to the special study of Subsection 7.3.3).

p 69 2.4.2.1 l 1: The reference to 4.3.5 is, more precisely, Subsubsection 4.3.5.3.
l 3: What is computed is really $F_q(1)$, where $F_q(x) := f(x)/x$ according to the notations of Section 1.8 (and, in the notations of Subsubsection 2.4.2.2 further below, $F_q = G_q/H_q$, where $G_q(x) = u(x)/x$ and $H_q(x) = v(x)$).
l 6: At the end of the line, read $\beta = 2/5$.

Errata and afterthoughts on Chapter 3

p 73 first alinea ll 9-10: As pointed out in the introduction ("more substantial afterthought") a short report on $q \leftrightarrow q^{-1}$ will appear on this webpage sometimes this year.

p 76 3.1.1.4 l -8: The correct reference is Subsection 3.3.3, not 3.3.1.

p 79 3.1.3.2 l 7: Read $K^{(\mathbf{Z})}$, not $K^{\mathbf{Z}}$.

p 82 3.1.3.4 Example 3.35: The conclusion is right, but the first intermediate calculation should be $S - b = cS^k(S - a) = cS^{k+1} - c\sigma_q^k(a)S^k$.

p 94 3.3.1 l 2: References (1.2), (1.6) and 4.4 respectively mean Sections 1.2, 1.6 and 4.4.
3.3.1.1 l -9: there is a missing word "$(a;q)_n$ vanishes".

p 98 3.3.1.4 l 11: A left parenthesis is missing in $(-1)^n(\alpha)_n$.

p 100 3.3.2.1 l -2: Replace "to those" by "for those".

p 105 3.3.4.1 l 11: There are missing words, one should read "... then g is given as in Statement (i). Statement (ii) will be used ...".

End of February: complements for chapters 4,5,6,7.

Errata and afterthoughts on Chapter 4

p 108 4.1.1.1 l 1: To be consistent with the immediately following remark, one should read $u^{[p^{-1}]}$ instead of $u^{[p]}$.
4.1.1.2 l -3: the group of classes of line bundles appears only implicitly in Section 4.5, rather see Subsections 9.1.2 and 10.1.2.

p 116 4.2.1.2 l -4: Replace all "sq" by σ_q (three times).

p 119 4.2.2.3 Exercise 4.29: The end of the sentence should rather read "... at most simple poles on $[-c; q]$ (and nowhere else).".

p 119 4.3.1 l -2: Condition $b \neq 0$ should be added (see Subsubsection 3.2.1.1).

p 121 4.3.1.4 Exercise 4.34: Replace "in this number" by "in this Subsubsection".

p 126 4.3.3.2 Exercise 4.42: Actually *almost* telescopic, there remains a factor a that requires further multiplication by a^{-n} (see Example 4.45).

p 128 4.3.3.2 Exercise 4.49: Instead of fl_q read $f\ell_q$.

p 132 4.3.5.3 l 10: The logical equivalence written here is correct; what is not equivalent is the rank 2 system with the (nonlinear) equation $\frac{u}{v} = \cdots$; only the converse implication holds.

p 138 4.4.1.5 l -3: Read "... again in Section 12.3.".

p 139 4.4.2.3 Footnote 8: The other degenerate cases are rather $c \in q^{\mathbf{N}^* \setminus \{1\}}$ (since $c \in q^{-\mathbf{N}}$ has already been excluded).

p 142 4.4.3.1 l 9: Read "meromorphy in y is equivalent to meromorphy in x".
p 142 4.4.3.1 Equation (4.6): The symbols "$o \Longleftrightarrow$" at the beginning should be omitted.

p 149 4.5.1.3 l 2: The stated uniqueness is that of the choice of U_0 among the connected components of $\pi^{-1}(V)$.

Errata and afterthoughts on Chapter 5

p 156 5.1.1.2 l-3: The true "small additive q-de Rham complex" is rather $K' \stackrel{L}{\to} K'$, it is here completed into an exact sequence with its homology; see Subsection 7.1.5 and Appendix E.1.2.

p 158 5.1.3.3: Replace l_q by ℓ_q everywhere.

p 161 5.2.1.3 l 3 of the Proof: replace "minimal" by "maximal".

p 164 5.2.2.2 l -3: Read "links" instead of "lonks".

p 168 5.2.3.2 l 6: read "this common valuation is of course ...".

p 170 5.3.1.1 l 10: "The whole theory" is meant here excluding confluent q-hypergeometric series.
p 170 5.3.1.2 l -3: The associated homogeneous equation is really $\sigma_q f = xf$.

p 171 5.3.2.1 l 4: "when needed", see however Subsection 5.4.3.
p 171 5.3.2.1 l 11: Instead of "L is standard unitary" read "L has the form".
p 171 5.3.2.1 l 14: Instead of "$\sum_{n\geq 0}$ read "$\sum_{j\geq 0}$.

p 172 5.3.2.1 l -2: Read "may have" instead of "have"; indeed, even in the case of nonresonancy there may be no need for q-logarithms, for instance if $A := \mathrm{Diag}(1, q)$. As an exercise, try to find nondiagonal examples.

p 175 5.4.1.2 Lemma 5.40: As stated, the conclusion is incorrect; replace (for instance) $M' = L'\alpha$ by $M' = L'\alpha^{-1}$.

p 177 5.4.2.3 Theorem 5.5: In the first line, read $L \neq 0$·

Errata and afterthoughts on Chapter 6

p 184 6.1.1.2 l 2: Instead of "of the affine space" read "it underlies the affine space".
p 184 6.1.1.4 l -3: Replace "nonzerio" by "nonzero".
p 184 6.1.1.5 l -2: Read "If $c \in q^{\mathbf{N}} \ldots$".
p 184 6.1.1.5 l -1: Read "the equation $(q^n - c)f_n = g_n \ldots$".

p 188 6.1.2.5 l 11: Read $\mathrm{Im}(L, \mathbf{C}\{x\}) = \mathrm{Ker}\ \lambda$.

p 190 6.1.3.2 l 5: Notations rather come from Subsubsection 6.1.2.5.
p 190 6.1.3.2 l 7: Read $\overline{e}(1) = 1$.
p 190 6.1.3.2 l 11: Read $\alpha := \overline{g}(1)$.

p 193 6.2.1.2 l 3: Reference to "the appendix" is Appendix D.2.2.

p 195 6.2.2.3: It should be recalled here that in all cases condition (6.5) from Subsubsection 6.2.2.2 is assumed.

p 196 6.2.2.4 l 7: Reference to "Appendix D" is Corollary D.12 of D.2.1.
p 196 6.2.2.4 l 14: Reference to "Appendix D" is Theorem D.1 of D.2.1.

p 198 6.2.3.1 l 3: Read $\eta_{q,s} := \sum_{n \geq 0} q^{sn(n+1)/2} x^n$ and $\eta^*_{q,s} := \sum_{n \geq 0} q^{-sn(n+1)/2} x^n$.

p 199 6.2.3.2 l 8: Reference to "the appendix" is Appendix D.2.2 and D.2.3.

p 201 6.2.3.3 l 4: Replace $m \leq k$ by $m \leq K$.

p 203 6.2.3.5 Fact (1) after Proposition 6.43: Actually, equality occurs if, and only if, $\mu := -1/s$ is *not* a slope.

p 204 6.2.3.7 Corollary 6.47 l 3: Read "nonexceptional values of s" (not of q).

p 205 6.3.1.1 l 5: In the definition of q-Gevrey-Beurling of order s, the upper bound is $|q|^{-sn(n+1)/2}$ (missing minus sign in the exponent).

p 207 6.3.1.3 Proof of Lemma 6.53: C should be written $\overline{\mathcal{C}_q}$ in accordance with General Notations from 0.4.2.7.

p 215 6.4.2.1 l 3: Instead of $L_1 \cdot L_n$, read $L_1 \cdots L_n$.

p 216 6.4.2.3 Corollary 6.81: Read $\chi(L, \mathbf{C}((x))/\mathbf{C}(\{x\}))$ (missing "/").
p 216 6.4.2.3 l 3: The irregularity is defined twice (see p 197) and the equivalence of the two definitions is not explicitly stated (nor proved). It follows easily from combining Proposition 6.33 with Corollary 6.81; indeed, as shown by Figure 6.3, $v(a_0) - v(L) = -\sum_{\mu_i < 0} r_i \mu_i$.

p 217 6.4.3.1 l 7: Instead of $a'i$, read a'_i.

Errata and afterthoughts on Chapter 7

p 221 7.1.1.2 Exercise 7.5: Statement (i) is generically true but particular values of d/a (*e.g.* $a = d$) may invalidate it. Which ones and how ? Well, this is an Exercise !

p 233 7.1.6.1 l -2: Reference to Appendix D is, more precisely, Appendix D.1.3, Proposition D.5.

p 243 7.4.2.2 l 4: There is no such "corresponding discussion" in the introduction; a more correct reference is Appendix F.4.3.
p 243 7.4.2.2 Proof of Proposition 7.63: There is no such "lemma", use again Appendix F.4.3 along with "easy calculations".

p 245 7.3.1.3 ll -5 and -2: The minimum defining c_q must be taken for $n \geq 1$, not for $n \in \mathbf{Z}$.

p 246 Footnote 8: This remark is rather shortsighted. There are further links of the formal case with vector bundles, see in particular the paper [248] by van der Put and Reversat. This theme will be pursued in [CV] (chapter on arbitrary slopes).

p 246 7.3.2.1 l 8: One should rather mention germs of open sets of the form $\big(\pi^{-1}(V), 0\big)$ since the study here is local at 0.

p 250 Footnote 10: Actually that terminology ("nonresonancy" for the singularities) is not "purely local", it is taken up in the paper [176] by Ohyama, Ramis and Sauloy and in its sequel [196] to appear in 2025.

Beginning of April: complements for chapters 8,9,10.

Errata and afterthoughts on Chapter 8

About reference [37]: this reference occurs at quite a few places in Chapters 8 and 9, and once in each of Appendices D and F. *The corresponding item in the bibliography is wrong.* It mentions Bourbaki, Algèbre, Chapter 9, which is *never* used in the book. Actually, the references do point to Bourbaki, Algèbre (hereafter abbreviated as BAL), but to the first three Chapters (except for the penultimate one on page 598, which points to Chapter 10). I will correct the references and I shall take the opportunity to make them more precise at some points. The Chapters in BAL will be numbered in roman: I, II, III; except from 10 (just a personal habit ...).

p 260 8.1.3.1 On lines -10 to -8 A is A_L.

p 261 8.1.3.3 From line 7 on: $b_i := -a_{n-i}/a_n$ for $i < n$ and $b_n := -1/a_n$ (thus the first line of A_L^{-1}).

p 265 8.1.5.1 Proof of lemma 8.27 l 4, after 3: close parenthesis.
Theorem 8.1 It should have been mentioned that the given proof is due to Di Vizio and Katz, see [58].
Footnote 3, the letter in the first line should be a small ϕ (in accordance with the sequel)
8.1.5.2 Reference [37] here should be BAL III §7.

p 267 8.2.1.1 Example 8.31 l 5 replace $M \simeq M_P$ by $\mathcal{D}_{K,q}v \simeq M_P$.

p 272 8.2.1.4 Last item, 4 lines before the end, the formula has been cut unhappily, it should read:

$$M_i = \mathcal{D}_{K,q}R_{i+1} \cdots R_n/\mathcal{D}_{K,q}P \simeq \mathcal{D}_{K,q}/\mathcal{D}_{K,q}R_1 \cdots R_i.$$

8.2.1.5 l -8 reference [37] should be BAL I §4 no 7.

p 273 8.2.2.1 Exercise 8.43 penultimate line read $\mathrm{Mat}_{n,1}$.

p 276 8.2.3.1 l 3 read "in the context of basic linear algebra".
l 14 read "This is related".

p 280 8.3.1.1 first line or point (2) read "and a commutative diagram".

p 281 8.3.1.3 l 7 reference [37] should be BAL II §1 no 9.

Errata and afterthoughts on Chapter 9

p 297 9.1.1.3 l 4 Remove $\in$ just before $\mapsto$.

p 298 9.1.2 Replace [37] by BAL II §3

p 299 9.1.2.1 Proof of Lemma 9.9, in the computation: on the second line the left hand side is redundant; on the third line, second $\otimes$-factor of left hand side, remove factor $\Phi(e_i)$; and remove equal sign and its right hand side.

p 300 9.1.2.1 Remark 9.11, penultimate line, instead of $\sigma_q^* V \otimes_K W$ rather write $\sigma_q^*(V \otimes_K W)$. There will be more details on that point of view in the chapter of [CV] devoted to arbitrary slopes.
Example 9.13 l 2 There is a redundant sentence beginning with "In a description ...".

p 301 9.1.2.2 Proof of Proposition 9.15 Replace reference [37] by BAL, the remainder of the reference is all right.

p 303 9.1.3 l 5 Replace reference [37] by BAL II §4.

p 306 9.1.4.1 l 7 Read $M \otimes \underline{\mathbf{1}} = M$.

p 311 9.2.4.1 l 5 Add "is clear" before "and".

p 318 9.3.3.1 l -2 The exponent of the second $\otimes$-factor should be p.

p 319 9.3.4 l 5 read "last in 9.3.4.3".
9.3.4.1 l -9: the correct reference for "Exercise ??" is 9.42 p. 314 (at the end of Subsubsection 0.3.1.3).
Proposition 9.51: actually, equality $\Gamma(M) = H^0(\mathbf{E}_q, \mathscr{F}_M^{(0)})$ holds (not merely an inclusion).

Errata and afterthoughts on Chapter 10

p 325 10.1.2.1 Example 10.2 The word "degree" is perhaps confusing, its refers to its use for divisors, not for q-difference modules.

p 327 10.1.3.1 Lemma 10.10 The formula should read $\psi_M(\alpha,\mu) = \psi_{M'}(R_r(\alpha,\mu))$.

p 328 10.1.3.2 Definition 10.13: In [248], a different convention is used for slopes; the slopes of M are absolute and computed with respect to base q, they remain unchanged under ramification. I am not sure which convention is best.

p 330 10.1.4.1 Corollary 10.20 Statement (ii), read "If $S(M_1) \cap S(M_2) = \emptyset$, ...".

p 341 10.3.2.3 Proof of Lemma 10.53, the reference to 11.1.2 is more precisely 11.1.2.5.

p 344 10.3.4.1 three lines above diagram (G_i) is the label (or name) of relation $v_i \circ f_i = u_i$. (So we have relations F_1, F_2, G_1, G_2 and soon afterwards E_1, E_2.)

p 347 10.3.5.1 Example 10.59 l 2 Remove "with" after "Also".
l -3 Read $\hat{F}_{U,V} \in \mathrm{Mat}_{r_1,r_2}(\mathbf{C}(\{x\}))$ (not $\mathrm{Mat}_{r_1,r_2}(\mathbf{C}((x)))$).

p 349 10.3.5.2 last line: this refers to Footnote 3 page 356.

p 350 10.3.5.4 l 15 Rather that "out diagonal" read "off-diagonal".

p 352 10.4.2 There will be more about Harder-Narasimhan filtrations in [CV].