Corrections, Updates, and Addenda
to
Invariant Theory of Finite Groups
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Thanks for bringing typos, errors, and corrections to my attention goes to David Pengelley and Frank Williams.

1. Corrections

I will not list punctuation mistakes or font errors unless they are too distracting.

• **Page** 37, **Line** -2
Replace $\mathbb{F}[V]_i$ *by* A_i.

• **Page** 93, **Line** -11
The second half of the proof of Theorem 4.2.4 does not prove the desired statement. Starting at line -11 the proof should read as follows.

By hypothesis $p \not| d = e_0 \cdots e_{m-1}$ so $p \not| |\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}|$ and hence the restriction of the Noether map

$$\eta : \mathbb{F}[\Omega]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}} \longrightarrow \mathbb{F}[V]^G$$

is an epimorphism. We need to show that

$$\mathbb{F}[\Omega]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}}$$

is generated by multipolarized elementary symmetric polynomials.

This is done with an iterated application of the first fundamental theorem of invariant theory and induction on m.

For $m = 1$ this is a consequence of the first fundamental theorem for the symmetric group Σ_{e_0}, cf. page 87. So suppose $m > 1$ and we have shown that for the defining representation Ω of $\left(\cdots\left(\Sigma_{e_{m-2}} \wr \Sigma_{e_{m-3}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}$ and any $k \in \mathbb{N}$ the algebra $\mathbb{F}[\bigsqcup_k \Omega]^{\left(\cdots\left(\Sigma_{e_{m-2}} \wr \Sigma_{e_{m-3}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}}$ is generated by multipolarized elementary symmetric polynomials. Note that

$$\mathbb{F}[\Omega]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_1}\right) \wr \Sigma_{e_0}} = \left(\mathbb{F}[\Omega]^{\left(\times_{e_0}\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}\right)}\right)^{\Sigma_{e_0}}.$$

Let $c = d/e_0$. If Γ is the natural permutation representation for Σ_c, then

$$\Omega = \underleftrightarrow{\Gamma \sqcup \cdots \sqcup \Gamma}_{e_0}$$

as Σ_c-set. Therefore

$$\mathbb{F}[\Omega]^{\left(\times_{e_0}\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}\right)} = \bigotimes_{e_0} \mathbb{F}[\bigsqcup_k \Gamma]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}}.$$

Γ regarded as a $\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}$-set is the defining representation so $\bigsqcup_k \Gamma$ is a $\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}$-set to which our induction hypothesis can be applied. We therefore conclude that

$$\mathbb{F}[\bigsqcup_k \Gamma]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}}$$

is generated by multipolarized elementary symmetric polynomials. Set

$$A = \mathbb{F}[\bigsqcup_k \Gamma]^{\left(\cdots\left(\Sigma_{e_{m-1}} \wr \Sigma_{e_{m-2}}\right)\cdots \wr \Sigma_{e_2}\right) \wr \Sigma_{e_1}}.$$

Then

$$\mathbb{F}[\Omega]^{\left(\times_{e_0}(\cdots(\Sigma_{e_{m-1}}\wr\Sigma_{e_{m-2}})\cdots\wr\Sigma_{e_2})\wr\Sigma_{e_1}\right)} = \bigotimes_{e_0} \mathbb{F}[\coprod_k \Gamma]^{(\cdots(\Sigma_{e_{m-1}}\wr\Sigma_{e_{m-2}})\cdots\wr\Sigma_{e_2})\wr\Sigma_{e_1}} = \bigotimes_{e_0} A$$

and the action of Σ_{e_0} on $\otimes_{e_0} A$ is by permutation of the factors. Therefore by Theorem 4.1.2 and the inductive hypothesis we obtain

$$\begin{aligned}\mathbb{F}[\Omega]^{(\cdots(\Sigma_{e_{m-1}}\wr\Sigma_{e_{m-2}})\cdots\wr\Sigma_{e_1})\wr\Sigma_{e_0}} &= \left(\mathbb{F}[\Omega]^{\left(\times_{e_0}(\cdots(\Sigma_{e_{m-1}}\wr\Sigma_{e_{m-2}})\cdots\wr\Sigma_{e_2})\wr\Sigma_{e_1})\right)}\right)^{\Sigma_{e_0}} \\ &= \left(\otimes_{e_0} A\right)^{\Sigma_{e_0}}\end{aligned}$$

is generated by multipolarized elementary symmetric polynomials. Hence $\mathbb{F}[V]^G$ is generated by their images under the Noether map η, i.e., by fine orbit Chern classes. □

- **Page** 146, **Line** -3

stabilizer

- **Page** 149, **Line** -14

The invariant $q = x_1x_3 + x_2x_5$ *is missing from the list of generators of* $\mathbb{F}[x_1, \ldots, x_5]^{\mathbb{Z}/2}$.

- **Page** 193, **Line** -4

Replace Vp' *by* $V_{p'}$.

- **Page** 195, **Line** 6

The second part of the proof of Theorem 7.2.2 is wrong. It can be corrected as follows.

Theorem 7.2.2 : *Let* $\rho : G \hookrightarrow \mathsf{GL}(n, \mathbb{F})$ *be a representation of a finite group* G *over the field* $\mathbb{F}$. *Assume that* $\mathbb{F}[V]^G = \mathbb{F}[f_1, \ldots, f_n]$*is a polynomial algebra. Then the fundamental class,* $[\mathbb{F}[V]_G]$, *of the ring of coinvariants is a* $\det^{-1}$*-relative invariant.*

Proof

If G contains no transvections, then the given proof goes through.

Denote the fundamental class of $\mathbb{F}[V]_G$ by $[F]$, and assume that G contains a transvection t. Then t generates a cyclic group $< t >$ of order p. Moreover, by the characterization of the fundamental class in Section 5.4 we have that

$$t[F] = \lambda_t[F]$$

for some $\lambda_t \in \mathbb{F}^\times$. Hence

$$\phi :< t > \longrightarrow \mathbb{F}^\times,\ t \mapsto \lambda_t$$

is a group homorphism. Since $< t >$ has order p we find that λ_t is a p-th root of unity and hence 1. □

- **Page** 197, **Line** 2

The matrix must look like

$$\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ & & \ddots & & \\ 0 & & \ldots & 0 & 1 \end{bmatrix}$$

- **Page** 197, **Line** 7

$j = 1, \ldots, n-1$

- **Page** 200, **Line** 10

Replace $\mathbb{H}$ *by* H.

• **Page** 200, **Line** 11
Replace S set *by* Set.
• **Page** 200, **Line** 11
Add the assumption that $a_H = a_{H'}$ *whenever* $s(H) = H'$.
• **Page** 200, **Line** 10
Add the conclusion $s(L) = L$.
• **Page** 200, **Line** -7/6
$L_G = \prod_{H \in \mathcal{H}(G)} \ell_{s_H}^{|s_H|-1}$
• **Page** 200, **Line** -4
Note that Lemma 7.2.5 is applicable because if $s(H_{s'}) = H_{s''}$ *then* $s'' = ss's^{-1}$ *and hence the two pseudoreflections* s' *and* s'' *have the same order.*
• **Page** 224, **Line** 6
$\phi_3 = x^3y - xy^3 = -\frac{1}{4}(c_{top}(x+2y) - (x^2+y^2)^2 - 4x^2y^2)$
• **Page** 231, **Line** -9
The reference [4] *is incorrect. The correct one is indeed missing from the reference list. So here it is:*

J. Adem, The Relations on Steenrod Powers of Cohomology Classes, in: *Algebraic Geometry and Topology*, a symposium in honor of S. Lefschetz, pp 191-238, Princeton University Press, Princeton NJ 1957.
• **Page** 249, **Line** -2
$\mathfrak{p}_W^{n+m}$
• **Page** 253, **Line** 2
$\mathrm{hom}-\mathrm{codim}_{\mathbb{F}_q[V]}(M) \geq r$
• **Page** 258, **Line** -4
The correct formula in the statement of Corollary 8.6.2 is

$$\mathcal{P}^{q^k}(d_{n,i}) = \begin{cases} d_{n,i-1} & \text{for } k = i-1 \geq 0, \\ -d_{n,i}d_{n,n-1} & \text{for } k = n-1 \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

• **Page** 265, **Line** 6
The given proof of Corollary 9.1.7 relies on Lemma 9.1.6, and hence works only for Noetherian algebras. However, direct computation shows the desired statement for all unstable algebras:

Let $f \in \sqrt{I}$. We need to show that $\mathcal{P}^i(f) \in \sqrt{I}$ for all $i \in \mathbb{N}_0$. Since $f \in \sqrt{I}$ there exists an $s \in \mathbb{N}_0$ such that $f^{q^s} \in I$. Therefore the Cartan formulae show that

$$\left(\mathcal{P}^i(f)\right)^{q^s} = \mathcal{P}^{iq^s}(f^{q^s}) \in I,$$

because I is $\mathcal{P}^*$-invariant. Hence $\mathcal{P}^i(f) \in \sqrt{I}$. □
• **Page** 283, **Line** 14
Zarati
• **Page** 288, **Line** -8
relevance
• **Page** 292, **Line** -13
Replace D *by* $D(n)^{q^t}$ *in the proof of Corollary 10.2.3 (four times).*
• **Page** 293, **Line** 3
$n = \dim(\mathbf{T_U}(\mathbf{D}(\mathbf{n})))$
• **Page** 293, **Line** -3
$\phi : H'' \longrightarrow H'$

- **Page** 295, **Line** 4

$\mathbf{T_U}(\mathbf{H}') =$

- **Page** 295, **Line** 14

$H'' \xrightarrow{\phi} H'$

- **Page** 295, **Line** -11

The subscript) should be erased.

- **Page** 297, **Line** -18

finite dimensional

- **Page** 298ff, **Line** -4

In Section 10.4 sub- and superscripts of Tor *have to be switched, so that the subscript carries the ring and the superscript the degree. This is just to make it consistent with the notation elsewhere in the book.*

- **Page** 302, **Line** -2

$\mathbb{F}_q[z_1, \ldots, z_n]$

- **Page** 305, **Line** 8

annihilates

- **Page** 306, **Line** 12

Lannes's

- **Page** 306, **Line** 23

Erase line 23.

- **Page** 307, **Line** -14

the Auslander-Buchsbaum equality

- **Page** 310, **Line** -8

$\mathbf{T}_{\mathbf{U},\alpha}(\mathrm{T}\mathbf{r}^{\mathbf{G}}) :$

- **Page** 339, **Reference** 161

Die Frage der...

- **Page** 340, **Reference** 181

S. Iyengar and L. Smith,

- **Page** 343, **Reference** 225

P. S. Landweber, L. Smith, and R. E. Stong,

- **Page** 344, **Reference** 246

Lannes's **T**-Functor and Noetherian Finiteness

- **Page** 352, **Reference** 413

L. Smith and R. E. Stong

- **Page** 353, **Reference** 437

R. M. W. Wood

- **Page** 353, **Reference** 438

R. M. W. Wood

- **Page** 359, **Column** 1

Erase lines 13 - 17.

- **Page** 360, **Column** 1

Erase lines 11 - 12.

- **Page** 360, **Column** 1, **Line** 13

pseudoreflections

- **Page** 364, **Column** 1, **Line** -8

composite functor theorem

- **Page** 364, **Column** 2, **Line** -15

degree theorem

• **Page** 364, **Column** 2, **Line** -6
derivation lemma

• **Page** 366, **Column** 1, **Line** -14
Noetherian

• **Page** 366, **Column** 1, **Line** -10
first fundamental theorem

• **Page** 366, **Column** 1, **Line** -9
first main theorem of invariant theory

• **Page** 368, **Column** 2, **Line** 12
Noether normalization theorem

• **Page** 368, **Column** 2, **Line** 20
Noetherian finiteness

• **Page** 370, **Column** 1, **Line** -10
formula

• **Page** 370, **Column** 1, **Line** -9
subgroups

• **Page** 371, **Column** 1, **Line** -1
$W(\mathbf{F_4})$

• **Page** 371, **Column** 2, **Line** 1
Erase line 1.

2. Updates on the Publication Status of Preprints in the References

• **Reference** 115
M. Feshbach, The Mod 2 Cohomology Rings of the Symmetric Groups and Invariants, *Topology* 41 (2002), 57-84.

• **Reference** 198
G. Kemper, The Depth of Invariant Rings and Cohomology, with an appendix by K. Magaard, *J. of Algebra* 245 (2001), 463-531.

• **Reference** 295
M. D. Neusel The Transfer in the Invariant Theory of Modular Permutation Representations II,*Canadian Math. Bulletin* 45 (2002), 272-283.

• **Reference** 296
M.D.Neusel, The Lasker-Noether Theorem in the Category $\mathcal{U}(\mathrm{H}^*)$, *Journal of Pure and Applied Algebra* 163 (2001), 221-233.

• **Reference** 384
L. Smith, Invariants and Coinvariants of Pseudoreflection Groups, Jacobian Determinants and Steenrod Operations, *Proc. Edinb. Math. Soc.* 44 (2001), 597-611.

• **Reference** 385
L. Smith, Invariants of 2×2-matrices over finite fields. *Finite Fields Appl.* 8 (2002) 504–510.

• **Reference** 440
G. Walker, and R.M.W. Wood, Linking first occurrence polynomials over $\mathbb{F}_2$ by Steenrod operations, *J. Algebra* 246 (2001) 739–760.

MDN, Lubbock March 2008.