Jordan Ellenberg Interview

Photo by Mats Rudels.

Jordan Ellenberg is the John D. MacArthur Professor of Mathematics and the Vilas Distinguished Achievement Professor of Mathematics at the University of Wisconsin.

Diaz-Lopez: *When did you know you wanted to be a mathematician?*

Ellenberg: I knew I was interested in mathematics very early, in fact, as early as my memories go. I loved mental computation and loved thinking about numbers and how they fit together. One of my parents told me about the Pythagorean theorem, and I remember that I worked out a way of generating an infinite family of Pythagorean triples. I think this must be the first time I felt the excitement of putting together things I knew into a new piece of math (new to me, at any rate!).

I'm not sure I knew at that time I wanted to be a mathematician or even what a mathematician was!

Diaz-Lopez: *Who encouraged or inspired you?*

Ellenberg: My parents, of course, first. They're statisticians, and my mom, before that, taught math in high school, so our house was full of mathematical stuff. Then Eric Walstein, a teacher in Montgomery County who made a habit of working with younger kids who were ahead of grade level in math. And, when I was in high school, Larry Washington at the University of Maryland, the first real mathematician I spent a lot of time with. He taught me number theory. I remember learning about Dirichlet series from him, and it was like the top of my head getting ripped off (in a good way).

Diaz-Lopez: *How would you describe your research to a graduate student?*

Ellenberg: I work in arithmetic geometry, especially Diophantine geometry, which means I study the solutions to equations in whole numbers—what number theorists have been doing for more than two thousand years! Nowadays, this topic has gotten intermingled with all kinds of other areas.

For instance, I think a lot about things like the Cohen-Lenstra conjectures, which, loosely speaking, ask, "What does the class group of a random number field look like?" It turns out that the version of Cohen-Lenstra having to do with random extensions of the field of rational functions $k(t)$ ends up being questions about the topology of certain moduli spaces of curves, called Hurwitz spaces. I never expected to be thinking about the complex of curves and stable cohomology theorems in algebraic topology in order to do number theory!

Another example I like: thanks largely to a beautiful result of Zeev Dvir, there is now a whole industry of using algebraic geometry to solve problems in extremal geometric combinatorics; a great example is Larry Guth and Nets Katz's solution of the Erdős Distance Conjecture, that N points in the plane determine at least $N/\log N$ distinct

DOI: http://dx.doi.org/10.1090/noti1395

Read a lot. Learn a lot. Do not decide "I like this kind of math but not that kind or that other kind."

distances. One thing I've been doing with coauthors like Marton Hablicsek and Daniel Erman is exploring the ways one can get new results in combinatorics from rather deep notions in algebraic geometry: for instance, the idea of degenerating a bunch of points in the plane to a "fat point" that is somehow many points all located in one spot (technically, a 0-dimensional punctual scheme) or the phenomenon of curves on which every point is a flex point (something that can only happen in characteristic p).

I also spend a lot of time thinking about general structures on moduli spaces. On the one hand, these are the things you study when you want to do something concrete, like counting solutions to equations. On the other hand, they often carry unexpectedly rich structures, actions of interesting categories that my collaborators and students and I are busy investigating. So representation theory, too, comes into the arithmetic geometry picture!

Diaz-Lopez: *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

Ellenberg: I'm really proud of a theorem I proved with Akshay Venkatesh and Craig Westerland about the geometry of a family of spaces called Hurwitz spaces. This is not the place to explain exactly what that means, but I'll explain the aspect I was proud of. We had been studying one of the Cohen-Lenstra conjectures that concerns the variation in the structure of ideal class groups as you move around a large population of quadratic fields. We had formulated a kind of geometric analogue of this conjecture, which predicted a very unexpected regularity in the topology (specifically, in the cohomology) of these Hurwitz spaces. That seemed crazy! It made us doubt that the Cohen-Lenstra conjectures were true in the first place. But then, the more we thought about this topological question, the more plausible it seemed that the regularity we were looking for was actually there. And then we proved it! It was incredibly satisfying to first perceive, without any ideas about proof at all, that a certain theorem "should" be there and then, bit by bit, to bring the proof into view.

Diaz-Lopez: *What advice do you have for graduate students?*

Ellenberg: Read a lot. Learn a lot. Do not decide "I like this kind of math but not that kind or that other kind." If you like math, you like math. And you'd be surprised what you wind up needing.

Diaz-Lopez: *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

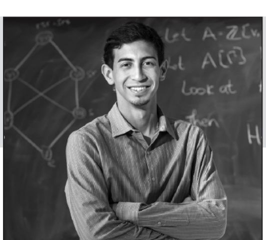
Ellenberg: I think you should always have at least three projects going at once: one you know you can do, one you think you can do, one you're pretty sure you can't do. That helps keep some part of your life free from discouragement. Also, I try to remember that never being discouraged in math means you're working on problems that are too easy.

Diaz-Lopez: *If you were not a mathematician, what would you be?*

Ellenberg: Probably a writer. I spent a whole year after college doing a creative writing degree. I thought seriously about whether I wanted to do that for a living. It's a very hard life, so it's lucky for me there's such a thing as math.

Diaz-Lopez: *If you could recommend one lecture to graduate students, what would it be?*

Ellenberg: All young number theorists should read Serre's *A Course in Arithmetic*; it somehow captures an immense amount of the spirit of the subject in a tiny amount of space.



Alexander Diaz-Lopez is a PhD student at the University of Notre Dame. Diaz-Lopez is the first graduate student member of the *Notices* Editorial Board.