

2017 Leonard Eisenbud Prize for Mathematics and Physics



László Erdős



Horng-Tzer Yau

LÁSZLÓ ERDŐS and HORNG-TZER YAU were awarded the 2017 Leonard Eisenbud Prize for Mathematics and Physics at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017.

Citation

The 2017 Leonard Eisenbud Prize is awarded to László Erdős and Horng-Tzer Yau for proving the universality of eigenvalue statistics of Wigner random matrices. In the 1950s, Eugene Wigner, motivated by the study of the complex spectra of highly excited nuclei, initiated an investigation of random matrices of a seemingly simple form: $N \times N$ symmetric, Hermitian, or quaternion self-dual random matrices with independent, identically distributed entries. Lacking a truly microscopic theory, Wigner proposed randomly selecting a quantum Hamiltonian. Since then, modeling through random matrices has been surprisingly useful: in determining the distribution of the zeros of the Riemann ζ -function; in characterizing the spectra of quantum Hamiltonians whose classical limit generates a chaotic dynamics; in developing lattice gauge theories; and in carrying out statistical analysis on large data sets. By browsing the *Oxford Handbook of Random Matrix Theory*, one gains an appreciation of how much the subject has flourished.

A quantity of prime interest is the gap probability, i.e., the distribution of the distance between neighboring eigenvalues. For matrices whose entries have a Gaussian distribution, F. Dyson computed the gap probability in the limit of large N . In fact, he determined the entire local spectrum of eigenvalues and found a structure now known

as Pfaffian, or determinantal, point processes with a specific defining kernel. At the time, pioneering numerical simulations suggested that the eigenvalue statistics are universal, in the sense that for large N there is no longer a dependence on the particular distribution of the matrix entries. In applications, this kind of robustness is a crucial assumption. The true Hamiltonian will not resemble a particular Wigner matrix. But for the purpose of predicting universal features, such a model may suffice.

A proof of the universality conjecture remained as an unsolved, challenging problem for many years. K. Johansson established the desired result for complex Hermitian matrices under the assumption that the distribution of the entries has a Gaussian component. László Erdős and Horng-Tzer Yau, jointly with collaborators, introduced and perfected the technique of using Dyson's Brownian motion as an interpolating scheme. The dynamics start with the eigenvalues of A and reach the eigenvalues of a Gaussian random matrix at time $t \rightarrow \infty$.

To summarize their amazing result: Let $\{A_{ij}\}_{1 \leq i, j \leq N}$ be a collection of either real or complex random variables, independent up to symmetry, such that $\mathbb{E}(A_{ij}) = 0$, $\mathbb{E}(|A_{ij}|^2) = N^{-1}$, and $\mathbb{E}(|\sqrt{N}A_{ij}|^{4+\epsilon}) < C$ for some constants C and $\epsilon > 0$. Then the typical distance between eigenvalues is of the order N^{-1} . We focus at a point $E \in \mathbb{R}$ at which the average density of eigenvalues is strictly positive and consider the eigenvalues lying in the interval $[E - \ell N^{-1}, E + \ell N^{-1}]$ with arbitrary $\ell > 0$. Shift these eigenvalues by E and rescale by N . Then we arrive at the collection of eigenvalues $\{\lambda_j^{(N)}\}_{j=1, \dots, n}$, with random n , located in the interval $[-\ell, \ell]$. The assertion of the theorem is that in the limit $N \rightarrow \infty$, the point process $\{\lambda_j^{(N)}\}_{j=1, \dots, n}$ converges to the limiting point process for the eigenvalues of the Gaussian Wigner random matrix $\{\lambda_j^{(G)}\}_{j=1, \dots, n}$.

A complete formulation can be found in Erdős and Yau's review "Universality of local spectral statistics of random matrices," published in the *Bulletin of the AMS* in 2012, where the notion used for the convergence of point processes is specified.

The initial breakthrough was published in 2010 in "Bulk universality for Wigner matrices," *Communications on Pure and Applied Mathematics* (with S. Péché, J. Ramírez, and B. Schlein), with important subsequent improvements in "Universality of random matrices and local relaxation flow," *Inventiones Mathematicae* (2011, with B. Schlein), "Bulk universality for generalized Wigner matrices," *Probability Theory and Related Fields* (2012, with J. Yin), "Gap

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universality of generalized Wigner and β -ensembles,” *Journal of European Mathematical Society* (2015), and “Fixed energy universality for generalized Wigner matrices,” *Communications on Pure and Applied Mathematics* (2016, with P. Bourgade and J. Yin).

Biographical Sketch: László Erdős

László Erdős was born in Budapest in 1966 and completed his university education in mathematics at the Lorand Eötvös University in 1990. He received his PhD at Princeton University in 1994 under the supervision of Elliott H. Lieb. After postdoc positions in Zürich and New York he first became a faculty member at Georgia Tech in Atlanta, then obtained a chair professorship at the Ludwig-Maximilian University in Munich, Germany. Since 2013 he has been professor at the Institute of Science and Technology Austria, near Vienna. He was an invited speaker at ICM 2014. He is a corresponding member of the Austrian Academy of Sciences, an external member of the Hungarian Academy of Sciences, and member of the Academia Europaea. Erdős’s research focuses on mathematical physics, in particular many-body quantum mechanics, disordered quantum systems, and random matrices.

Response from László Erdős

It is a great pleasure and honor to be selected as a co-recipient of the 2017 Leonard Eisenbud Prize. I am grateful to the committee for this recognition of our work.

I am very fortunate to have learned the importance of combining physical motivations with sharp analysis from the very beginning of my career, starting in the Budapest dynamical system school led by Doma Szász and continuing at Princeton under the guidance of Elliott Lieb, whose infallible scientific taste and mathematical mastery have shaped my research ever since. Finally, I owe most of my research aptitude to my former postdoctoral advisor and long-term collaborator, Horng-Tzer Yau, with whom sharing this prize is a great distinction.

A very special acknowledgment goes to our younger collaborators with whom we shared many parts of this long journey toward the solution of the Wigner-Dyson-Mehta conjecture. The results would not have been possible without the multitude of ideas and indefatigable engagement by Paul Bourgade, Antti Knowles, Benjamin Schlein, and Jun Yin, together with shorter but essential collaborations with Jose Ramírez and Sandrine Péché. I thank all of them.

Biographical Sketch: Horng-Tzer Yau

Horng-Tzer Yau received his BSc from National Taiwan University in 1981 and his PhD degree from Princeton University in 1987, under the supervision of Elliott Lieb. He has held faculty positions at the Courant Institute and Stanford University, and since 2005 he has been a professor of mathematics at Harvard University. Yau received the Henri Poincaré Prize in 2000 and was the Distinguished Visiting Professor at the Institute for Advanced Study from 2013 to 2014. He has received fellowships from the Sloan Foundation, Packard Foundation, and MacArthur Foundation and has been a member of the American Academy

of Arts and Sciences since 2001 and the National Academy of Sciences since 2013. Currently, Yau is a Simons Investigator and the Editor-in-Chief of *Communications in Mathematical Physics*. His work focuses on quantum many-body systems, interacting particle systems, and random matrix theory.

Response from Horng-Tzer Yau

It is a great pleasure and honor to receive this prize. As a student, I saw E. Wigner many times in the colloquium at Princeton. During those years, it never occurred to me that one day I would work on a problem in his area of interest. My coworker, László Erdős, and I came to the universality problem accidentally after many years of working on random Schrödinger equations, which were believed to exhibit random matrix statistics. At the time, study of the universality of random matrices was under the reign of integrable methods. It was fortunate for us that the Green’s function method and probabilistic tools were mature enough by then to be applied to this problem. These tools allowed us to understand the universality problem through analytic methods and to make the connection with Dyson’s work.

I would like to take this opportunity to thank my thesis adviser, E. Lieb, who taught me to believe in the simplicity of mathematics and physics. I also would like to thank Raghu Varadhan, from whom I learned probability theory during my postdoctoral time at the Courant Institute. In addition to Erdős, I am also indebted to my other coworkers. Among them, Paul Bourgade, Benjamin Schlein, and Jun Yin collaborated with Erdős and me on several papers and generated many key ideas in this project. I also would like to thank the committee for selecting this work for the Leonard Eisenbud Prize. Finally, I would like to thank my wife, Chuan-Chuan, for her patience and care through my career.

About the Prize

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of “English to Erdős and Erdős to English.” He was one of the founders of the Physics Department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the AMS during 2003–2004. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US\$5,000 prize will be awarded every three years for a work published in the preceding six years.

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The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2017 prize, the members of the selection committee were

- Ezra Getzler
- Alice Guionnet
- Herbert Spohn (Chair)

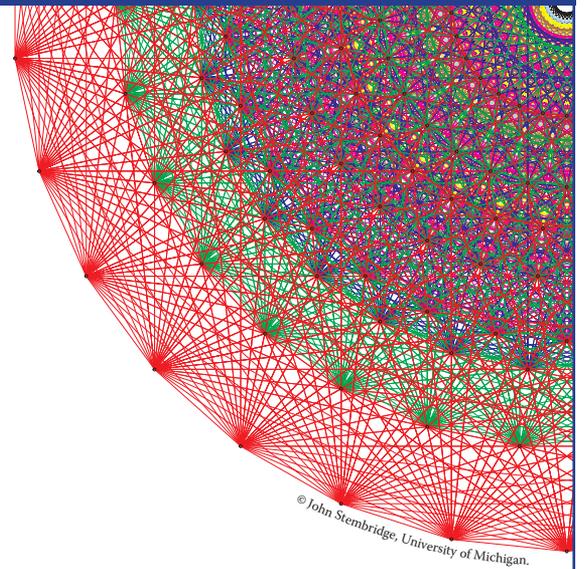
The complete list of recipients of the Leonard Eisenbud Prize for Mathematics and Physics follows:

- Hiroshi Ooguri, Andrew Strominger, Cumrun Vafa (2008)
- Herbert Spohn (2011)
- Gregory W. Moore (2014)
- László Erdős, Horng-Tzer Yau (2017)

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