2017 AWM LECTURE SAMPLER

Photo from the banquet at the 2015 AWM Research Symposium. Back row (l-r): 2015 AWM Presidential Award winner Sylvia Bozeman; Past AWM President Kristin Lauter; former AWM President Jill Pipher. Front row (l-r): AWM Executive Committee member Talitha Washington; 2015 Keynote Speaker Shirley Malcom; and former AWM President Ruth Charney.

The AWM Research Symposium 2017 (April 8–9, 2017 at UCLA) will showcase the research of women in the mathematical professions. Some of the plenary speakers and members of the committee have offered to share a sneak-peek of their presentations with *Notices*.

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Kristin Lauter and Ami Radunskaya

AWM Research Symposium April 2017: Introduction

As the recent book and movie *Hidden Figures* bring to light, women's contributions to mathematics have often been overlooked and minimized. The 2017 AWM Research Symposium celebrates the mathematical contributions of many women mathematicians at all stages in their careers.

On the fortieth anniversary of AWM in 2011, AWM launched a new tradition of biennial AWM Research Symposia. The first one was held at Brown University in 2011, then Santa Clara University in 2013, University of Maryland in 2015, and now UCLA in 2017. These are large weekend research meetings on the model of AMS sectional conferences, bringing together more than three hundred female research mathematicians from around the world and featuring more than one hundred fifty research talks by female mathematicians.

The conference at UCLA on April 8–9 will feature eighteen special sessions, including eight sessions for Research Networks for Women supported by the AWM ADVANCE grant, four plenary talks, a poster session for graduate students, a reception for student chapters, a jobs panel, exhibit booths for sponsors, an awards banquet, and an AWM Presidential Award!

The AWM Research Symposia were originally inspired by the annual meetings of the Korean Women in Mathematical Sciences society. The plenary speakers at AWM Symposia have included a Fields Medalist, a president of the IMU, and an NSF math institute director. This year's plenary speakers are no less eminent: AWM Past President Ruth Charney, the 2016 Blackwell–Tapia Prize winner Mariel Vazquez, AWM Sadosky Prize winner Svitlana Mayboroda, and the first AWM Sonya Kovalevsky Lecturer Linda Petzold.

Photo Credits

Photo of Kristin Lauter is courtesy of Michael Svoboda.
Photo of Ami Radunskaya is courtesy of Pomona College.

ABOUT THE AUTHORS

In 1999 **Kristin Lauter** received an AWM Mentoring Grant to study with Jean-Pierre Serre in Paris at the College de France (sketch by Enrico Bombieri) and wrote two papers with appendices by Serre. She enjoys soccer, sailing, bike trips with her husband, and doing art with her twin daughters.

Kristin Lauter

Ami Radunskaya, known as "Dr. Rad" by her students, enjoys transdisciplinary research, using mathematics to answer questions in medicine, power systems, and economics. The second photo shows her working with her father, Roy Radner, on a recently published PNAS article, "Dynamic pricing of network goods." She enjoys hiking, making music, cooking, and eating with family and friends.

Ami Radunskaya

Ruth Charney

Searching for Hyperbolicity

Geometry has come a long way since Euclid. In the nineteenth century, hyperbolic geometry came into its own with the work of Felix Klein and to this day has continued to play a central role in differential geometry and low-dimensional topology. In the late 1980s, Mikhail Gromov introduced a notion of hyperbolicity that applies to a very general class of metric spaces (not necessarily manifolds) and in the process launched the field of geometric group theory.

In geometric group theory, one studies groups acting by isometries on a metric space and explores connections between algebraic properties of the group and geometric properties of the space. Gromov's hyperbolicity condi-

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tion is a particularly powerful property that has been deeply explored and applied to understand groups that act by isometries on such spaces. In recent years, there has been much interest in extending these techniques to broader classes of groups and spaces. Many interesting spaces do not fully satisfy Gromov's hyperbolicity condition and yet display some aspects of hyperbolic behavior. How can we identify such behavior and put it to good use?

Gromov's definition of hyperbolicity is based on a thin triangle condition. Let X be a geodesic metric space, that is, a space in which the distance between two points is equal to the minimal length of a path connecting them. We say that X is hyperbolic if there exists a constant δ such that for any geodesic triangle Δ in X, each side of Δ is contained in the δ-neighborhood of the other two sides. A classic example of this is when X is a tree and all triangles are tripods. In this case, we can take $\delta=0$; that is, each side of a triangle is contained in the union of the other two sides. At the other extreme, it is easy to see that the Euclidean plane fails the thin triangle condition for every choice of δ.

How can we identify such behavior and put it to good use?

Now let's combine the behavior of the tree and the plane. Take Y to be the universal cover of $T^2 \vee S^1$, the wedge of a torus and a circle. Envision Y as a collection of horizontal planes with vertical edges emanating from every lattice point, creating a tree of flats as in Figure 1. If we stay in a horizontal plane, our surroundings do not look at all hyperbolic and triangles can get arbitrarily wide, but if we travel mostly in an upward direction, not spending too much time in any horizontal plane, then our behavior begins to look hyperbolic!

In joint work with H. Sultan, along with subsequent work of M. Cordes and D. Murray, we consider geodesic rays in a metric space X that have hyperbolic-like behavior and encode this information in a new type of boundary for X, called the Morse boundary. We show that the Morse boundary has properties analogous to boundaries of Gromov hyperbolic spaces and (providing the Morse boundary is nonempty) it provides an effective tool to study groups acting isometrically on X.

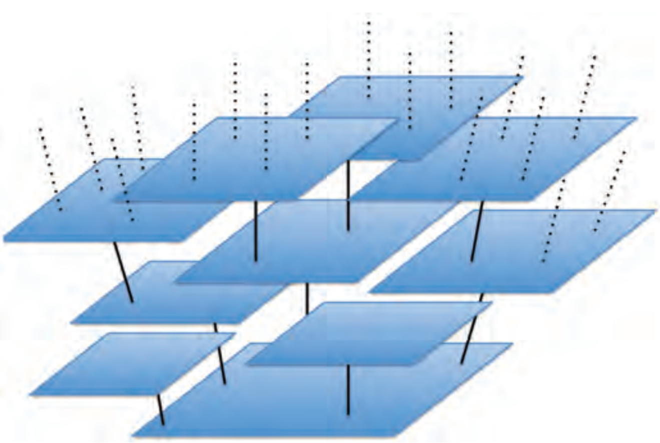

Figure 1. A tree of flats looks hyperbolic vertically, though not horizontally.

Photo Credit

Photo of Ruth Charney courtesy of Michael Lovett.

ABOUT THE AUTHOR

Ruth Charney

Ruth Charney has always been intrigued by the interaction between algebra and topology. She was drawn into the field of geometric group theory at its inception in the late 1980s and has been happily ensconced there ever since. In addition to mathematics, she loves traveling, hiking, dancing, and exploring restaurants

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Svitlana Mayboroda

Localization of Eigenfunctions

At the forefront of modern technology, matter structured at the nanometer and atomic levels increasingly reveals its wavelike nature. This provides new remarkable opportunities and challenges in analysis and partial differential equations. In recent years, pushing the limits of fundamental physics, cutting-edge experiments with ultracold atoms have created new states of matter and offer the possibility of controlling quantum entanglement, with major potential applications to cryptography and quantum computing. In engineering, electronic waves confined in quantum wells have yielded high efficiency LEDs, which are about to revolutionize the energetics of lighting, as recognized by the 2014 Nobel Prize. At these scales, even the slightest disorder or irregularity can trigger one of the most puzzling and ill-understood phenomena—wave localization.

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What is wave localization? It is an astonishing ability of physical systems to maintain vibrations in small portions of their original domains of activity while preventing extended propagation. One should not, in this context, think solely in terms of mechanical vibrations. Light is a particular example of an electromagnetic wave, wifi is delivered by waves, sound is a pressure wave, and, from the vantage point of quantum physics, even matter can be perceived as a type of wave. In mathematical terms, localized eigenfunctions φ of a self-adjoint elliptic operator $L=-\mathrm{div}\, A\nabla+V$ in a domain Ω satisfy $L\varphi=\lambda\varphi$, where φ is extremely close to zero outside some small subset of Ω. This phenomenon can be triggered by irregularities of the coefficients of A, disorder in the potential V, special features of the shape of Ω, or an intricate mixture of all of the above. Over the past century this has been a source of persistent interest in condensed matter physics, engineering, and mathematics. However, it has remained a mystery whether it is possible to directly translate knowledge of A, V and Ω into the specific information on the location and frequencies of localized eigenfunctions or, better yet, to design systems with desired localization patterns.

In 2012, together with M. Filoche, we introduced a new concept of the "localization landscape." This has turned out to have some remarkable and powerful features. Indeed, in a joint work with D. Arnold, G. David, M. Filoche, and D. Jerison, we show that the landscape function, which is defined as a solution to $Lu=1$ on Ω, reveals a clear disjoint partition of Ω into independent regions, and this partition predicts localization domains with exponential accuracy, with the rate of decay governed by the so-called Agmon metric associated to $1/u$ [A+2016]. That is, given any system, one needs only to solve a simple equation $Lu=1$, and determine the corresponding "valley lines" to obtain a map of subregions which host the localized eigenfunctions (see Figure 1). Furthermore, modulo this exponentially small error, the part of the spectrum of L on Ω not exceeding the maximum of $1/u$ can be fully diagonalized, that is, bijectively mapped on the combined spectrum of the subregions determined by the landscape. In particular, an eigenfunction can be massive simultaneously in two landscape subregions if and only if the eigenvalues of these subregions are exponentially close (a fairly unlikely event). In this sense, one could view $1/u$ as an *effective* quantum potential which exhibits clear structure even when V is highly disordered or when is absent and the localization is caused by the geometry and/or by coefficients of A. In other words, $1/u$ suitably quantifies the *uncertainty principle*.

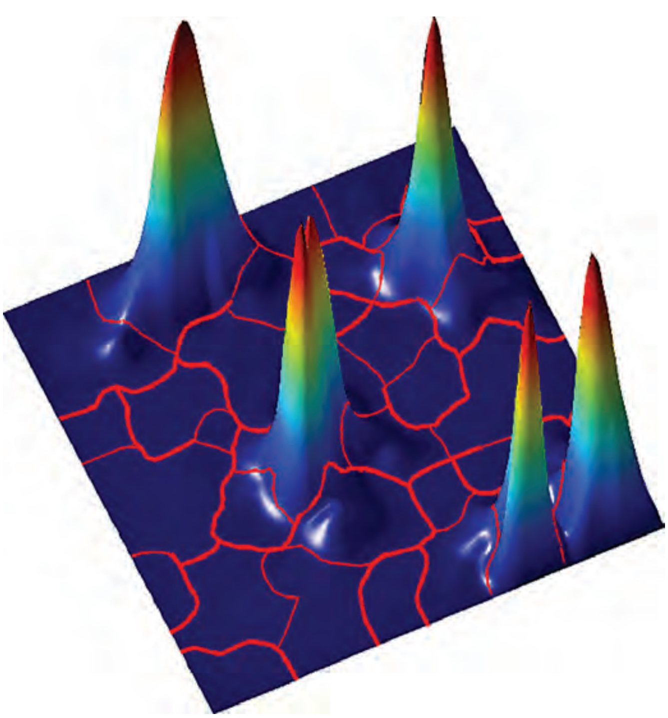

Figure 1. A network of valleys of a localization landscape (in red) and the first five localized eigenfunctions for the Schrödinger operator with disordered potential.

Going further, the localization landscape furnishes a new version of the Weyl law. It appears to give the first universal estimate on the counting function and on the density of states for small eigenvalues in the range where the classical Weyl law notoriously fails. In fact, numerical experiments show that already minima of $1/u$ give a very good approximation for the bottom of the spectrum, but these observations for now remain mathematically inaccessible.

Finally, in the intervening years, these results have found immediate applications in theoretical and experimental physics and in energy engineering to predict the vibration of plates, the spectrum of the bilaplacian with Dirichlet data, the efficiency and quantum droop of GaN LEDs governed by the Poisson–Schrödinger self-consistent system, and the spectral properties of the Schrödinger operator with Anderson or Anderson–Bernoulli potentials in bounded domains.

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Credits

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Figure 1 is courtesy of M. Filoche, S. Mayboroda, "Universal mechanism for Anderson and weak localization," *Proceedings of the National Academy of Sciences* **109**, (2012), (37), 14761–14766, DOI:10.1073/pnas.1120432109.

Svitlana Mayboroda

ABOUT THE AUTHOR

Svitlana Mayboroda works at the interface of harmonic analysis, geometric measure theory, and partial differential equations, with applications to condensed matter physics and engineering.

Linda Petzold

Inference of the Functional Network Controlling Circadian Rhythm

Petzold's lecture focuses on the use of computing and mathematics to better understand circadian rhythm, the process by which living organisms manage to follow a 24-hour cycle. Difficulties in following this cycle are experienced, for example, in jet lag and shift work. Circadian rhythm disorders are known risk factors for heart disease, obesity, and diabetes, as well as numerous psychiatric and neurodegenerative diseases. In mammals, the Suprachiasmatic Nucleus (SCN), a brain region of about 20,000 neurons, serves as the master circadian clock, coordinating timing throughout the body and entraining the body to daily light cycles. The extent to which cells in the SCN can synchronize and entrain to external signals depends both on the properties of the individual oscillators (neurons) and on the communication network between individual cell oscillators. Petzold's lecture explores both the development of mathematical models for the oscillators and inference of the structure of the network that connects the neurons.

Linda Petzold

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Linda Petzold is a computational and applied mathematician working on modeling, analysis, simulation, and software applied to networked systems in biology and medicine.

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