Karen E. Smith Interview

Conducted by Laure Flapan

Communicated by Alexander Diaz-Lopez

Karen E. Smith is Keeler Professor of Mathematics at the University of Michigan. Recipient of the 2001 AMS Ruth Lyttle Satter Prize, Smith is interested in commutative algebra and algebraic geometry. She is a co-author of *An Invitation to Algebraic Geometry*. Her e-mail address is kesmith@umich.edu.

EDITOR'S NOTE. Karen E. Smith's article "Noether's Legacy: Rings in Geometry" appeared in *Notices*, January 2016, www.ams.org/notices/201601/rnoti-p7.pdf.

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Flapan: *When and how did you know you wanted to be a mathematician?*

Smith: That is an interesting question. I definitely enjoyed math from a young age. As early as middle school I played with different mathematical ideas on my own and read math books. For example, in seventh grade I devoured a book about the Fibonacci numbers, which I reread multiple times over the years (I wish I could remember the name!). However, I was perhaps in my fourth year of graduate school before I really began to seriously consider "mathematician" as a career option.

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You see, I never really knew that mathematician was a career at all! My parents encouraged me to study engineering in college and were disappointed when I switched my major to math, fearing I would be unemployable. I switched because I loved my freshman calculus class out of Spivak's *Calculus* book, especially solving the problems for the final take-home exam. I guess I was stubborn and impractical enough to ignore their advice.

Growing up, I didn't know anyone with a PhD. Mentoring undergraduates was not something most Princeton professors did, at least not mentoring me. So, upon graduation, I landed a job as a high school math teacher. Continuing my education never occurred to me. For starters, I was eager to pay off my student loans, and I had no idea that "graduate student" was sort of a job itself. First-year high school teaching is extremely difficult, and I was in a miserable school district with little support. I soon resolved to do something else. By a stroke of luck, I bumped into one of my peers from college, who mentioned that he was getting paid to be a student in a math PhD program! This was an amazing revelation for me! I applied immediately.

Karen E. Smith (second from right) with current and former students Robert Walker (Ford Foundation Fellow), Sarah Mayes (assistant professor at the University of Toronto), and Will Traves (chair of the US Naval Academy math department).

I was thrilled to be a TA at Michigan, where I earned more to teach one class and study math than I earned teaching five high school classes! For me, grad school was a terrific job. I never envisioned myself actually writing a PhD; I simply enjoyed studying and teaching college students. It wasn't until I started getting attention for proving new results that I began to dream about being a professional mathematician.

Flapan: *Who encouraged or inspired you?*

Smith: My seventh-grade math teacher, Mr. Eckert, taught me modular arithmetic and gave me challenging problems to explore on my own. I think this is my earliest memory of "doing math" in the sense of playing, experimenting, conjecturing. Likewise, my twelfth-grade calculus teacher, Mr. Driscoll, offered an extra math class on number theory using Underwood Dudley's book. I loved that book as well! His take-home exam gave me my first taste of the satisfaction of solving a hard problem after suffering over it a week. These two teachers really nurtured my love for math.

In college, no one particularly encouraged me, and a few actively discouraged me. One exception was Charlie Fefferman, my freshman calculus teacher, who expressed surprise that I was an engineering student despite being "so good at math." This remark, in a brief but obviously influential conversation, was enough to make me switch majors. Other professors inspired me, especially Nick Katz. His homework sets were terrific, with a coherent set of exercises leading us to discover and develop nice chunks of mathematics on our own. I still remember one on p-adic numbers. I emulate Katz's teaching today in my own classes; ask my students about my "worksheets."

In graduate school, on the other hand, I did feel actively encouraged. Carolyn Dean, an assistant professor and the only female faculty member, took me out to lunch to chat several times my first year. Small things, like the fact that our department chair occasionally asked me how my classes were going, had a big impact. I was lucky to stumble into Mel Hochster's commutative algebra course. Mel was enthusiastic and encouraging and eventually became my thesis advisor. He has been a fantastic advisor and mentor ever since.

As a postdoc, I learned a tremendous amount of algebraic geometry from young mathematicians in Boston/Cambridge, especially my friend Sándor Kovács. He introduced me to János Kollár, who asked me on the spot, at the Santa Cruz algebraic geometry conference in 1995, if I could give a talk the next day. The conversations that followed that talk were critical to my research development.

As a faculty member, I have always been grateful to Bill Fulton, who still often encourages me by having seemingly tremendous faith in me.

But by far the most important encouragement came from my late husband, Juha Heinonen. His unflagging but breezy belief in me made a huge difference. Although he died ten years ago, I can still hear him say, "If not you, then who?"

Flapan: *How would you describe your research to a graduate student?*

Smith: I study commutative algebra, mostly motivated by problems in algebraic geometry. One of my favorite tools is the Frobenius or pth power map, which can be iterated, for example, to show that certain cohomology classes must be trivial on certain varieties. These methods can be used to prove theorems about varieties defined over the complex numbers by reduction to prime characteristic.

My favorite kinds of problems are those rich in interesting examples that connect different types of mathematics. For example, I have recently played with cluster algebras[1] (highly combinatorial objects defined by Sergey Fomin and Andrei Zelevinsky) by reducing to characteristic p to prove theorems about the structure of the varieties they define. This is joint work with my postdocs Angelica Benito, Greg Muller, and Jenna Rajchgot.

Flapan: *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

Smith: I am not sure I have a clear favorite among my theorems. I was excited to prove that the test ideal (a notion from characteristic p commutative algebra of importance in Hochster and Huneke's theory of tight closure) and the multiplier ideal (a notion from complex birational algebraic geometry) are essentially the same—that is, the multiplier ideal "reduces modulo p" to the test ideal for every sufficiently large prime p. This type of result spawned a great deal of research into the connections between these two fields, which continues today. I can't keep up with it all myself!

Another project that was especially fruitful was with Rob Lazarsfeld and Lawrence Ein: we found some applications of multiplier ideals—or more accurately, an asymptotic version of them—to some problems in commutative algebra, including a surprising comparison theorem for symbolic and ordinary powers of ideals in a regular ring. The third paper in that series, on an application to understanding valuation ideals, required resolution of singularities for an Abhyankar valuation.

Perhaps the theorem I am most pleased with at this moment is one with Michel Van den Bergh on the structure of the ring of differential operators on rings of invariants

[1] *See* "What is a cluster algebra?" *by Andrei Zelevinsky,* Notices, *December 2007,* www.ams.org/notices/200711/tx071101494p.pdf.

in characteristic p: we found some conditions on a ring in prime characteristic that guarantee its ring of differential operators is simple. The idea of "finite F-representation type," which we introduced in that paper, is arising again in a project on "noncommutative resolutions of singularities" right now with my postdocs Eleonore Faber and Greg Muller.

Flapan: *Do you think your approach to or style of mathematics has changed with time? If so, how?*

Smith: Perhaps now I am less interested in solving every aspect of a problem and more interested in solving the big main cases while leaving some harder technical generalizations for others. I would rather find some new interesting phenomenon or surprising new connection, say, in polynomial rings and then move on to something completely different rather than spend a long time generalizing the result to arbitrary regular rings, say.

I have also come in and out of different interests—for a while I wanted nothing to do with prime characteristic, but I have since come back.

For sure, I am now a much better expositor of mathematics. Sometimes I cringe to read my earlier papers.

Flapan: *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

Smith: I take a break—the length depends on the magnitude of the discouragement—and eventually just get right back up. Life and math go on. It is important to make a habit of putting in the time. Eventually all those calculations—all the blood, sweat, and tears—pay off. I know that younger mathematicians often have a harder time believing that the payoff is coming, but after many disappointments I can assure you, it is. Rarely does anything understood deeply turn out to be useless, even though it may not serve the purpose you had hoped.

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Flapan: *What is something people might be surprised to learn about you?*

Smith: I had many jobs before mathematician, including lifeguard, hotel maid, deli meat slicer, computer parts recycling factory worker, pizza delivery person, SAT prep course instructor, and high school teacher. Of course, I had a few more typical jobs as well, such as babysitter, math tutor, and camp counselor. I went to a fancy college, but I have more modest working class roots. I know how to hustle to make a buck!

Flapan: *What advice do you have for current graduate students in math?*

Smith: Start where you are at, and don't compare yourself to others. Work hard, get help, and stay on the path. Sometimes you will fail. That's OK. Enjoy what you are doing now, and don't forget to play, mathematically and otherwise. Do lots of calculations and examples, be curious, be solid on the basics.

Also, remember to take care of yourself. Take one day a week off work. Sleep well and exercise. Have a social life.

Find advice and mentoring from many different people at different places in their careers and even in different careers. Take it all in carefully, but much will be contradictory, so sort out what feels right and best for you.

You can soothe a lot of anxiety by helping others. So instead of looking around your graduate program and worrying about how many students are "better" than you, why not look around for someone you can help pull up?

Flapan: *If you could recommend one book or lecture to graduate students, what would it be?*

Smith: Shafarevich's *Basic Notions in Algebra* is one of my favorites. I also thought that Manjul Bhargava gave the best colloquium talk I've ever seen at the Seoul ICM in 2014. You can watch it online.[2]

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Laure Flapan

ABOUT THE INTERVIEWER

Laure Flapan just completed her PhD in algebraic geometry from the University of California, Los Angeles and is headed to a postdoc at Northeastern University in the fall of 2017. Her work is in algebraic geometry, particularly Hodge theory.

[2] https://www.youtube.com/watch?v=Vx-4MUKCMPg.