

QUARTERLY
OF
APPLIED MATHEMATICS

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BOOK REVIEWS

The development of mathematics. By E. T. Bell. Second edition. McGraw-Hill Book Company, Inc., New York and London, 1945. xiii+637 pp. \$5.00.

The second edition of this well known book contains about fifty pages of new material. Listed in the order in which they occur in the book, the major additions deal with Moslem algebra, the development of symbolism, lattice theory, cubic surfaces, Levi-Civita's parallel displacement, definition of lengths, areas and volumes, the transition from intuitive to unintuitive thinking in modern mathematics, the development of mathematics in times of war, algebra of relations and consistency proofs. The index, too, has been considerably enlarged. Shorter additional paragraphs are concerned with applied mathematics in World War II, Egyptian algebra, number mysticism, Greek methods of computation, the Egyptian method of constructing a right angle, mathematical realism, the Greek treatment of loci, number theory, geometrical constructions, projective differential geometry, theory of quantics, intuitionism, Whitehead and Russell's Principia Mathematica, the Burali-Forti paradox, quantum theory, Skolem's theorem, Laplace's work on probability, R. A. Fisher's work on statistics, and matrix algebra in modern statistics.

S. PRAGER

Tables of associated Legendre functions. Prepared by the Mathematical Tables Project conducted under the sponsorship of the National Bureau of Standards. Official Sponsor: Lyman J. Briggs. Project Director: Arnold N. Lowan. Columbia University Press, New York, 1945. xxv+303 pp. \$5.00.

The main body of the present volume contains tables for $P_n^m(x)$, $dP_n^m(x)/dx$, $i^{-n}P_n^m(ix)$, $i^{-n}dP_n^m(ix)/dx$, $(-1)^m Q_n^m(x)$, $(-1)^{m+1}dQ_n^m(x)/dx$, $-i^{n+2m+1}Q_n^m(ix)$, $i^{n+2m-1}dQ_n^m(ix)/dx$, $P_{n+1/2}^m(x)$, $dP_{n+1/2}^m(x)/dx$, $(-1)^m Q_{n+1/2}^m(x)$ and $(-1)^{m+1}dQ_{n+1/2}^m(x)/dx$ for integral values of m (ranging from 0 to 4) and n (ranging from 0 or 1 to 10 for the functions of integral degree and from -1 to 4 for the functions of half-integral degree). For the greater part, these functions are tabulated to six significant figures for values of the argument which increase from 0 or 1 to 10 by steps of 0.1. The functions $P_n^m(\theta)$ and $dP_n^m(\theta)/dx$ are also tabulated for integral values of m (ranging from 0 or 1 to 4) and n (ranging from 1 to 10), the values of the argument increasing from 0° to 90° by steps of 1° . Auxiliary tables facilitate interpolation.

W. PRAGER

BIBLIOGRAPHICAL LIST

The R.T.P. translations listed below are now available from the Durand Reprinting Committee, in care of California Institute of Technology, Pasadena 4, California.

- R.T.P. Translation No. 2503, The Lorin nozzle for aircraft propulsion. By L. J. Goodlet. 40 pages.
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- R.T.P. Translation No. 2523, New equipment of the Junkers research station. By H. Roos. 7 pages.

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