version of the special type of power series $y=a_0x+a_1x^2+a_2x^3+\cdots$ to obtain $x$ as an integral power series in $z\equiv y/a_0$, whose coefficients are given as polynomials in $b_i\equiv -a_i/a_0$, as far as the term involving $z^{13}$. Now the explicit expansion for (1) in terms of the derivatives of $f(z)$ at $z_0$ can be written down immediately, as far as $\nu=13$, from Van Orstrand's expansion on pp. 369–370, merely by

(A) replacing $b_i$ in his formula by $-f^{(i+1)}(z_0)/(i+1)!f'(z_0)$,

(B) replacing *his* $z$ by $-f(z_0)/f'(z_0)$, and

(C) adding the constant term $z_0$.

The truth of the last statement is obvious from the fact that when (1) is applied at the origin it yields Van Orstrand's expansion and from the uniqueness of Van Orstrand's expansion.

# A NOTE ON THE CORRECTION OF GEIGER MÜLLER COUNTER DATA*

By H. B. MANN (*Ohio State University*)

The correction of Geiger Müller Counter data has been considered in a previous paper by J. D. Kurbatov and the author.[1] According to the model described there the following result was proved: If the density of radiation is a constant $a$ and if $\tau$ denotes the resolving time, $B(T)$ the number of discharges during the time $T$; then

$$B(T) = \frac{aT}{1 + a\tau} + \eta, \tag{1}$$

where $\eta$ is given by

$$\eta = -a\int_0^T \epsilon(t)dt \tag{2}$$

and $\epsilon(t)$ satisfies the conditions

$$\left.\begin{aligned} \epsilon(t) &= -a\int_{t-\tau}^{t} \epsilon(x)dx \qquad \text{for} \quad t \geqq \tau, \\ \epsilon(t) &= 1 - e^{-at} - \frac{a\tau}{1 + a\tau} \quad \text{for} \quad 0 \leqq t \leqq \tau. \end{aligned}\right\} \tag{3}$$

It was further shown that for $a\tau<1$.

$$|\eta| \leqq \frac{(a\tau)^2}{1-(a\tau)^2}\left[1 - (a\tau)^{s+1}\right],$$

where $s$ is the largest integer not larger than $T/\tau$. In this paper an upper bound for $|\eta|$ will be derived without the restriction $a\tau<1$. We shall prove the following inequality:

* Received May 29, 1946.

[1] J. D. Kurbatov and H. B. Mann, *A correction for Geiger Müller counter data*, Phys. Rev. **68**, 40–43 (1945).

Let $[x]$ denote the largest integer not larger than $x$; then

$$|\eta(T)| \leqq \frac{a\tau}{1+a\tau}(2e^{a\tau}-1) + \frac{a^2\tau}{1+a\tau}\left(T - \left[\frac{T}{\tau}\right]\tau\right)(1-e^{-a\tau})^{[T/2\tau]}. \tag{4}$$

*Proof of the inequality* (4). From (3) we see that $\epsilon(t)$ is a continuous function of $t$. Applying the mean value theorem to (3) we have

$$\epsilon(t) = -a\tau\epsilon(t^*), \qquad t-\tau \leqq t^* \leqq t.$$

Hence $\epsilon(t)$ changes its sign at least once in every open interval of length $\tau$ and will therefore be 0 at least once in every such interval. Hence we have

PROPOSITION 1. *In every open interval of length $\tau$ there is at least one point for which $\epsilon(t)=0$.*

Differentiating (3) with respect to $t$ we obtain

$$\epsilon'(t) = a\epsilon(t-\tau) - a\epsilon(t). \tag{5}$$

In the interval $\bar{t} \leqq t \leqq \bar{t}+\tau$ Eq. (5) may be considered as a differential equation for $\epsilon(t)$ with the initial condition that its solution be equal to $\epsilon(\bar{t})$ at the point $\bar{t}$. Solving (5) with this initial condition we have, for $\bar{t} \leqq t \leqq \bar{t}+\tau$,

$$\epsilon(t) = \epsilon(\bar{t})e^{-a(t-\bar{t})} + ae^{-at}\int_{\bar{t}}^{t} e^{ax}\epsilon(x-\tau)dx. \tag{6}$$

Let $M(\bar{t})$ be the maximum of the absolute value of $\epsilon(t)$ in the interval $[\bar{t}-\tau, \bar{t}]$, then

$$|\epsilon(t)| \leqq M(\bar{t})e^{-a(t-\bar{t})} + e^{-at}M(\bar{t})(e^{at} - e^{a\bar{t}}) = M(\bar{t}) \quad \text{for} \quad \bar{t} \leqq t \leqq \bar{t}+\tau. \tag{7}$$

From (7) it follows that $|\epsilon(t)| \leqq M$ for $t \geqq \bar{t}$. Hence we have

PROPOSITION 2. *If $|\epsilon(t)| \leqq M$ for $\bar{t}-\tau \leqq t \leqq \bar{t}$, then $|\epsilon(t)| \leqq M$ for $t \geqq \bar{t}-\tau$.*

If $\epsilon(\bar{t})=0$ then we obtain from (6)

$$|\epsilon(t)| \leqq M(\bar{t})(1-e^{-a(t-\bar{t})}) \leqq M(\bar{t})(1-e^{-a\tau}) \quad \text{for} \quad \bar{t} \leqq t \leqq \bar{t}+\tau. \tag{8}$$

From (8) and Proposition 2 follows

PROPOSITION 3. *If $\epsilon(\bar{t})=0$ and $|\epsilon(t)| \leqq M$ for $\bar{t}-\tau \leqq t \leqq \bar{t}$, then $|\epsilon(t)| \leqq M(1-e^{-a\tau})$ for $t \geqq \bar{t}$.*

According to proposition (1) we have in the interval $\alpha\tau \leqq t \leqq (\alpha+1)\tau$ at least one point $t_\alpha$ for which $\epsilon(t_\alpha)=0$.

Consider the points $t_1, t_3, \cdots, t_{2n+1}$. If $M$ is the maximum of $|\epsilon(t)|$ in $0 \leqq t \leqq \tau$ we must have, according to Propositions 1, 2, and 3,

$$\begin{aligned}
|\epsilon(t)| &\leqq M \quad \text{for} \quad 0 \leqq t \leqq t_1, \\
|\epsilon(t)| &\leqq M(1-e^{-a\tau}) \quad \text{for} \quad t_1 \leqq t \leqq t_3, \\
&\cdot \\
&\cdot \\
&\cdot \\
|\epsilon(t)| &\leqq M(1-e^{-a\tau})^k \quad \text{for} \quad t_{2k-1} \leqq t \leqq t_{2k+1}.
\end{aligned}$$

Hence

$$|\eta| = \left| -a\int_0^T \epsilon(x)dx \right| = \left| \sum_{\alpha=1}^{\alpha=[T/\tau]} \epsilon(\alpha\tau) - a\int_{[T/\tau]\tau}^{T} \epsilon(x)dx \right|$$

$$\leqq M + 2M(1 - e^{-a\tau}) + 2M(1 - e^{-a\tau})^2 + \cdots + aM\left(T - \left[\frac{T}{\tau}\right]\tau\right)(1 - e^{-a\tau})^{[T/2\tau]}$$

$$\leqq M(2e^{a\tau} - 1) + aM\left(T - \left[\frac{T}{\tau}\right]\tau\right)(1 - e^{-a\tau})^{[T/2\tau]}.$$

From (3) it can be seen that $M = a\tau/(1+a\tau)$ and (4) follows.

The inequality (4) is very satisfactory and shows that even for large values of $a\tau$ the quantity $\eta$ will be very small compared to $aT/(1+a\tau)$ even if $T$ is only a few minutes.

CORRECTIONS TO OUR PAPER

# STABILITY OF COLUMNS AND STRINGS UNDER PERIODICALLY VARYING FORCES*

QUARTERLY OF APPLIED MATHEMATICS, **3**, 215–236 (1943)

BY S. LUBKIN AND J. J. STOKER (*New York University*)

The following errors were found in the tables printed on pp. 232–235.

| $\beta$ \ $\alpha(C_0)$ | for | read |
|---|---|---|
| 1.6 | −0.77898 | −0.77897 |
| 1.8 | −0.92281 | −0.92282 |
| 7.6 | −5.71537 | −5.71538 |
| 9.2 | −7.11974 | −7.11975 |

| $\beta$ \ $\alpha(C_1)$ | for | read |
|---|---|---|
| 3.6 | −2.32402 | −2.32401 |

| $\beta$ \ $\alpha(S_1)$ | for | read |
|---|---|---|
| 0.8 | 0.55906 | 0.55406 |
| 1.4 | 0.63015 | 0.63016 |
| 4.4 | −0.29781 | −0.29780 |
| 7.6 | −2.08644 | −2.08648 |
| 11.0 | −4.29436 | −4.29437 |

| $\beta$ \ $\alpha(S_2)$ | for | read |
|---|---|---|
| 3.8 | −0.00468 | −0.00464 |
| 6.8 | −1.60383 | −1.60379 |
| 8.4 | −2.58478 | −2.58477 |

| $\beta$ \ $\alpha(C_2)$ | for | read |
|---|---|---|
| 0.6 | 1.12806 | 1.12810 |
| 3.4 | 2.01478 | 2.01477 |
| 20.0 | −5.05198 | −5.05199 |

| $\beta$ \ $\alpha(C_3)$ | for | read |
|---|---|---|
| 0.6 | 2.26622 | 2.26621 |
| 1.0 | 2.28515 | 2.28516 |
| 2.2 | 2.31495 | 2.31493 |
| 2.8 | 2.29660 | 2.29661 |
| 5.6 | 1.85589 | 1.85591 |

* Received Aug. 16, 1946.