

QUARTERLY  
OF  
APPLIED MATHEMATICS

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VOLUME V

JANUARY • 1948

NUMBER 4

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# QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly under the sponsorship of Brown University, Providence, R.I. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following industrial companies are gratefully acknowledged:

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Entered as second class matter March 14, 1944, at the post office at Providence, Rhode Island, under the act of March 3, 1879. Additional entry at Menasha, Wisconsin.

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Printed by the  
GEORGE BANTA PUBLISHING COMPANY  
Menasha, Wisconsin

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In this seventh edition the tables of the natural logarithm, exponential and hyperbolic functions have been extended to six places, the argument being given to four, three, and three places, respectively. For numbers less than unity  $\ln x + 10$  is given instead of  $-\ln x$  as in the previous edition.

S. PRAGER

*Tables of integrals and other mathematical data.* By H. B. Dwight. Macmillan Company, New York, 2nd ed., 1947. viii+250 pp. \$2.50.

In this revised edition additional material has been included on various progressions, powers of Taylor series, roots of quadratic equations, square roots of complex quantities, inverse trigonometric functions, probability integrals, hyperbolic and Bessel functions. New indefinite integrals involving  $a+bx$ ,  $(ax^2+bx+c)^{1/2}$ ,  $\sin x$ ,  $\cos x$ , and Bessel functions have also been added, together with some definite integrals. Further additions include a table of the normal probability integral and an extension of the bibliography.

S. PRAGER

*Tables of supersonic flow around cones.* By the staff of the Computing Section, Center of Analysis, Massachusetts Institute of Technology, Department of Electrical Engineering. Under the direction of Zdenek Kopal. Cambridge, Massachusetts, 1947. xviii+555 pp. \$5.00.

The introduction to this volume is a concise review of the derivation of the ordinary non-linear differential equation defining the supersonic flow of a compressible fluid past a circular cone and a statement of the relations among the state variables on either side of the shock implied by such a flow. The remainder of the book is a tabulation of the numerically obtained solutions of this problem.

For air ( $\gamma=1.405$ ), the velocity components and velocity of sound are tabulated against the flow direction parameter  $\theta$  for cone angles of  $5^\circ$ ,  $7.5^\circ$ ,  $\dots$ ,  $22.5^\circ$ ,  $25^\circ$ ,  $30^\circ$ ,  $35^\circ$ ,  $40^\circ$ ,  $45^\circ$ , and  $50^\circ$ . For each cone a complete range of wave angles is considered. The increments in  $\theta$  vary from  $1/4^\circ$  to  $1^\circ$  for various solutions and regions. In general there is five digit accuracy. A less extensive but similar table is presented for  $\gamma=4/3$ . Several cross tabulations are also included.

G. F. CARRIER

*Gas dynamics tables for air.* By Howard W. Emmons. Dover Publications, Inc., New York, 1947. 46 pp. \$1.75.

Dr. Emmons has compiled tables of the most commonly used functions occurring in gas dynamics. The quantities tabulated include pressure, velocity, density, stream-tube area, dynamic pressure, temperature, velocity of sound and Prandtl-Meyer angles, as functions of Mach number over the complete range. Similar quantities following a normal shock wave are also tabulated.

A sample of five hundred of the numbers in these tables was taken to check the computing accuracy, and it was found that there were four errors in the fifth published figure and two errors by one unit in the fourth figure. It appears then that, in general, the first four figures are accurate. Unpublished tables similar to these have been in use by the reviewer for several years and have been found to be of great practical utility in performing computations involving the flow of room-temperature air. These tables are accordingly highly recommended for the use of aerodynamicists.

ARTHUR KANTROWITZ