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The Quarterly prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts submitted for publication in the Quarterly of Applied Mathematics should be sent to the Managing Editor, Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. Seventy-five reprints without covers will be furnished free; additional reprints and covers will be supplied at cost.

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CONTENTS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; if authors wish to add material, they may do so at their own expense.

Titles: The titles should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: Only very simple symbols and formulas should be typewritten. All others should be carefully written by hand in ink. Ample space for marking should be allowed above and below all equations. Greek letters used in formulas should be designated by name in the margin.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

All subscripts and exponents should be clearly marked, and dots, bars, tildes, etc. over letters should be avoided.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that reoccurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t\,(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be given as footnotes. Only in longer expository articles may references be grouped together in a bibliography at the end of the manuscript.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) **36**, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: Since the printed text will not break up into pages in the same manner as the manuscript, footnotes should be numbered continuously.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

with similar expressions for the other stress components. If, therefore, in Eq. (16) we replace z by γz and then divide the right side by γ^2, the resulting expression can be used as the stress function. Hence the change to elliptic coordinates, which is usually done by replacing z by $\gamma \sinh (u + iv)$, can be accomplished in Eq. (16) by substituting $\sinh (u + iv)$ for z and then dividing the right side by γ^2. Thus we find the following expression for the stress function in elliptic coordinates:

$$F = p \frac{\sin v_0}{v_0 + \sin v_0 \cos v_0} [yv + (\sin^2 v_0)e^{-u} \cos v] \tag{19}$$

where the hyperbola $y^2/a^2 - x^2/b^2 = 1$ is given by the particular value, v_0, of the coordinate v. Equation (19) is the stress function for the corresponding problem involving the isotropic plate.[6]

[6]See paper by H. Neuber, Zeitsch. f. Angew. Math. u. Mech. 13, 439 (1933).

BOOK REVIEWS

Eigenfunction expansions associated with second-order differential equations. By E. C. Titchmarsh. Oxford, at the Clarendon Press, 1946. 175 pp. $7.00.

The subject matter of this book is composed of the analysis required to develop and justify the expansion of an arbitrary function $f(x)$ in the eigenfunctions of the differential equation $y'' + [\lambda - q(x)]y = 0$. The classical theory is disposed of in Chapter 1 and the remainder of the book is largely devoted to those situations where $q(x)$ is singular at an end-point of the region and/or those where $f(x)$ is defined over an infinite or semi-infinite domain. In Chapter 4 one finds as examples the expansion formulae for Hermite, Bessel, Legendre, Sonine, Laguerre, and hypergeometric functions. The latter chapters deal with the distribution of the eigenvalues, the nature of the spectrum, and with questions of convergence and summability.

The book is intended primarily for the mathematician, but its interest and value to the physicist and engineer should be great.

G. F. Carrier

Modern operational calculus with applications in technical mathematics. By N. W. McLachlan. MacMillan and Co., Ltd., London, 1948. xiv + 218 pp. $5.00.

This introduction to the theory of the Laplace Transform is intended as a guide to those interested in the application of this transform to engineering problems. However, the questions of rigor are treated in more detail than is customary in texts with this purpose. In fact, nearly one third of the book consists of appendices dealing with such items as convergence questions. In Chapters 1 and 2 the transform is defined and the fundamental theorems and identities are deduced. In Chapters 3 and 4, the solution by transform methods of ordinary and partial linear differential equations is discussed. These and the subsequent chapters which, for the most part, are concerned with integral evaluation contain many examples drawn largely from electric and acoustic problems. It is regrettable that no reference has been made to the role of this transform in solving stability problems.

The book should be especially useful to engineers whose background is essentially non-mathematical since the points of rigor discussed appear to have been selected for just such an audience.

G. F. Carrier