

QUARTERLY

OF

APPLIED MATHEMATICS

EDITED BY

H. W. BODE
J. M. LESSELS

H. L. DRYDEN
W. PRAGER
J. L. SYNGE

TH. v. KÁRMÁN
I. S. SOKOLNIKOFF

WITH THE COLLABORATION OF

M. A. BIOT
J. P. DEN HARTOG
C. FERRARI
J. N. GOODIER
F. D. MURNAGHAN
S. A. SCHELKUNOFF
H. U. SVERDRUP
H. S. TSIEN

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BOOK REVIEWS

Integraltafel. Erster Teil: Unbestimmte integrale. By Wolfgang Gröbner and Nikolaus Hofreiter. Springer-Verlag, Vienna, 1949. viii + 166 pp. \$5.40.

This table of indefinite integrals appears to be more complete than others generally available. There are three sections of the book entitled Rational Integrands (21 pages), Algebraic Irrational Integrands (85 pages), and Transcendental Integrands (60 pages).

The section on Algebraic Irrational Integrands contains integrals having integrands as follows: Rational functions of x and $(ax + b)^{1/n}$; x and $(ax + b)^{1/2}$; x and $[(ax + b)/(cx + d)]^{1/n}$; x , $(ax + b)^{1/2}$, and $(cx + d)^{1/2}$; x and $(ax^2 + 2bx + c)^{1/2}$ with a number of special cases of this last type. Irrational integrands which can be transformed into rational integrands, Elliptic integrals in Legendre canonical form and in Weierstrass canonical form. Rational integral functions of x and $y = (a_0x^3 + 3a_1x^2 + 3a_2x + a_3)^{1/2}$, Reduction to the Legendre canonical form. Rational integral functions of x and $y = (a_0x^4 + 4a_1x^3 + 6a_2x^2 + 4a_3x + a_4)^{1/2}$, Reduction to the Legendre canonical form. Rational integral functions of x and $y = (a_0x^3 + 3a_1x^2 + 3a_2x + a_3)^{1/3} = [a_0(x - \alpha_1)(x - \alpha_2)(x - \alpha_3)]^{1/3}$, Reduction to the Weierstrass and Legendre canonical form. Rational integral functions of x and $y = (x^2 \pm 1)^{1/3}$, Reduction to the Legendre canonical form. Hyperelliptic integrals.

The section of the book on transcendental integrands contains integrals having integrands of the form $R(\exp \lambda x)$, $f(x) \exp \lambda x$, $f(x) \exp (ax^2 + 2bx + c)$, $f(\log x)$, $R(x) \log^m x$, $f(x) \log^m x g(x)$, $R(\sin x, \cos x)$, $R(\sin(ax + b), \cos(cx + d), \dots)$, $x^p \sin^m x \cos^n x$, $\exp ax \sin^m bx \cos^m cx$, $R(x, \exp ax, \sin bx, \cos cx)$, $R(\frac{\sin}{\cos}(ax^2 + 2bx + c), x)$, $R(x, \arcsin x)$, $R(x, \arctan x)$, $R(\sinh x, \cosh x)$, $R(\sinh(ax + b), \cosh(cx + d), \dots)$, $x^p \sinh^m x \cosh^n x$, $R(\sinh(ax + b), \sin(cx + d), \dots)$, $R(x, \arcsinh x)$ and $R(x, \operatorname{arctanh} x)$. Finally there are integrals of Weierstrass elliptic functions and Jacobi elliptic functions.

This table of integrals is labeled Part I of two parts. Part two on definite integrals is said to be in preparation.

ROHN TRUELL

Fluid dynamics. By Victor L. Streeter. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1948. xi + 263 pp. \$5.00.

According to the preface "this book is planned to introduce the reader to the general theory of fluid flow" and is expected to be useful as a text for a second course for seniors or beginning graduate students. The mathematical background assumed extends through elementary calculus, additional mathematics being developed as needed. The author states he has made every effort "to clarify the concepts and to include those exasperating steps in deviations which are usually omitted."

Except for a part on viscous flow, the book is restricted to the study of an "ideal" fluid, by which the author means incompressible as well as nonviscous. Although some of the equations in the text hold for a compressible fluid, this subject is essentially not touched. No wave motion is discussed and turbulence receives but a brief mention. No mention is made of the Froude number or of the effects of surface tension. Thin airfoil theory and lifting line theory are not presented. Hence the title and preface are completely misleading as to the real scope of the book. Within the field of incompressible nonviscous flow the treatment is quite complete, particularly in two-dimensional flow where complex variable theory is used extensively. Flows described by inverse transformations and jet and wake flows are well presented.

The conceptual presentation is basically limited by the scope of the book. Since no thermodynamics is used the concept of the energy equation as distinguished from Bernoulli's equation is omitted. Similarly the basic concepts of barotropy and piezotropy and their relation to circulation are not included. The fundamental fluid dynamic concepts required in compressible flow, dynamic meteorology, chemical or heat power engineering, and open channel flow are all absent.

Within the scope of the book the concepts are well treated as a rule. Weaknesses occur, however, generally as omissions. Uniqueness theorems are given for cases where the velocity potential is single-valued with no mention of the important cases where the potential is multi-valued. Euler's momentum

integral is omitted. In the theory of vortex motion the law for the motion of an individual vortex is not given, and hence there is no discussion of the effect of solid boundaries. The author discusses qualitatively the motion of one or two circular vortices apparently oblivious of the fact that the self-induced velocity of such a vortex is infinite. The logic used in the discussion of percolation is faulty.

All in all, this book is a good text for the potential flow of an incompressible fluid with an adequate first treatment of viscous effects. The mathematics is restricted in level to the complex variables developed in the text. The short cut notation of vector analysis is not used and the "exasperating steps" are included. As such it should serve as a senior text. The plan of the book to introduce the general theory of fluid flow is not fulfilled, however, and the title should have been worded differently.

WALLACE D. HAYES

Tables of inverse hyperbolic functions. By the Staff of the Computation Laboratory. Harvard University Press, Cambridge (Mass.), 1949. xx + 290 pp. \$10.00.

The volume contains the following four tables: I) $\tanh^{-1} x$ for $x = [0(.001) 0.5(.0005) 0.75(.0002) 0.90(.0001) 0.95(.00005) 0.975(.00002) 0.99(.00001) 1]$, II) $\sinh^{-1} x$ for $x = [0(.002) 3(.005) 3.5]$, III) $\cosh^{-1} x$ for $x = [1(.00001) 1.004(.00002) 1.01(.00005) 1.025(.0001) 1.05(.0002) 1.15(.0005) 1.4(.001) 1.8(.002) 3.5]$, and IV) $\sinh^{-1} x$ and $\cosh^{-1} x$ for $x = [3.5(.005) 6.5(.01) 15(.02) 35(.05) 65(.1) 150(.2) 350(.5) 650(1) 1,500(2) 3,500(5) 6,500(10) 15,000(20) 23,000]$. The first and second differences are given in all these tables; the functions are tabulated to 9 decimal places.

W. PRAGER

Tables of Bessel functions of fractional order. Prepared by the Computation Laboratory of the National Applied Mathematics Laboratories, National Bureau of Standards. Volume II. Columbia University Press, New York, 1949. xviii + 365 pp. \$10.00.

The first part (pp. 1-275) contains tables for $I_\nu(x)$ for $\nu = \pm 1/4, \pm 1/3, \pm 2/3, \pm 3/4$ and x ranging from 0 to 13 for the negative values of ν and from 0 to 25 for the positive values of ν . For small values of x (up to 1.00 for $\nu = -3/4, -2/3$, to 0.80 for $\nu = -1/3, -1/4$, to 0.60 for $\nu = 1/4, 1/3$ and to 0.50 for $\nu = 2/3, 3/4$) the interval is 0.001; for larger values of x the interval is 0.01. The second part (pp. 277-332) gives the values of $e^{-x}I_\nu(x)$ for the same values of ν and for $x = [25(.1) 50(1) 500(10) 5,000(100) 10,000(200) 30,000]$. All functions are tabulated to ten decimal places or ten significant figures.

A number of auxiliary tables facilitate interpolation in the x and ν directions.

W. PRAGER

Electromechanical transducers and wave filters. By Warren P. Mason. Second Edition. D. Van Nostrand Company, Inc., Toronto, New York, London. xii + 419 pp. \$6.00.

The second edition of this book differs from the first mainly in the addition of new topics in which interest has grown as a result of many developments during the war. Those who have read the first edition will recall that the subject of linear vibratory systems was developed from the point of view of electrical network theory. After a review of electrical network theory the governing equations for lumped mechanical, acoustic, elastic, piezoelectric, and miscellaneous electromechanical systems are discussed and the electrical network equivalences are developed.

In the present edition the new material is added in the form of a series of appendices, with the paragraphs numbered in such a way as to follow in a natural way some section of the first edition. The new material covers some one hundred pages and treats in a more general manner than did the first

edition, the motion of a vibrating bar, wave propagation in a viscous medium, and piezoelectric phenomena. In addition, there is discussion of the electromechanical properties of systems incorporating gyroscopic elements, a discussion of the behavior of electrical wave guides and acoustic delay lines as both lumped circuit elements and radiation devices.

The text is extremely clear and is quite readable for engineers, mathematicians and physicists. It is not only useful as a discussion of this subject, but also equally valuable as a concise reference on electromechanical systems and electrical-mechanical analogies.

J. A. KRUMHANSL

Advanced dynamics. By S. Timoshenko and D. H. Young. Volume I. McGraw-Hill Book Company, New York, Toronto, London, 1948. xii + 400 pp. \$5.50.

This book deals with methods of solution of dynamical problems in engineering. Chapter I gives a detailed presentation (106 pp.) of various methods of graphical and numerical solution of certain non-linear problems, which can be treated in terms of a single particle. Chapter II (84 pp.) deals with certain problems of systems of particles, and treats particularly the theory of balancing of reciprocating engines. In both of these chapters only elementary notions of single particle dynamics, or of energy and momentum relations of a system of particles are employed. In Chapter III (54 pp.) generalized coordinates are introduced, and Lagrange's equations are derived. A number of problems of systems involving constraints are treated, and an approximate solution of a problem of forced vibration of a system with a non-linear spring characteristic is based on Hamilton's principle. The fourth chapter (82 pp.) introduces the general theory of small vibrations of a system of particles, with detailed discussion of vibrations of systems with two degrees of freedom. There is a short discussion of stability of vibrations about a state of steady motion, and an introduction to the theory of variable-speed vibration dampers. Finally, in the fifth chapter (55 pp.) Euler's equations are developed and a treatment of gyroscopes is based on them. Although the content of this book is not the "advanced dynamics" that a mathematician or physicist might expect from the title, the book should prove most valuable as a basis for courses in mechanical vibrations and dynamics applied to engineering problems on the senior or early graduate level in engineering schools.

P. S. SYMONDS

Numerical methods of analysis in engineering (successive corrections). A publication resulting from a Symposium held at Illinois Institute Of Technology, Chicago, Illinois. Arranged and edited by L. E. Grinter. The MacMillan Company, New York, 1949. xvi + 207 pp. \$5.80.

Most of the chapters of this book originated as papers presented at a Symposium on Numerical Methods held at Illinois Institute of Technology in 1948. Since Hardy Cross's now classical paper on "Analysis of Continuous Frames by Distributing Fixed-End Moments" provided an important impetus to the application by engineers of numerical methods, this paper is reproduced as Chapter 1, and the book is dedicated to Professor Cross. The viewpoint throughout most of the book is practical rather than mathematical; the authors are concerned with the quickest methods of obtaining "engineering solutions", and the emphasis is on methods which yield successive corrections.

Chapter 2 by L. E. Grinter presents a method of solution of plane problems of elasticity by replacement of the continuum by a grid of columns and beams which may be analyzed for example by distribution methods. Chapters 3 and 4 by F. S. Shaw and R. V. Southwell, respectively, are concerned primarily with relaxation methods applied to boundary value problems; the former reviews techniques, and the latter discusses the problem of improving accuracy, contrasting the "mathematician's approach" by means of more elaborate finite difference formulas with the "engineer's approach" in which the main

concern is to reduce the intervals of the net. In Chapter 5 M. M. Frocht reviews the "linear rosette" method for obtaining good initial values in numerical solutions of Laplace's equation. Chapters 6 and 7 are concerned with solutions of typical problems of heat conduction by numerical methods; these are by L. M. K. Boelter and Myron Tribus, and by G. H. Dusenberre, respectively.

The concluding group of three papers are concerned with general comparisons between numerical methods. Chapter 8 by F. Baron discusses the relations between methods which have been applied to problems in diverse fields. In the following chapter N. M. Newmark classifies and compares methods which have been applied particularly to problems of elasticity. The final chapter 10, by T. J. Higgins, consists essentially of an annotated bibliography of papers in which numerical methods are applied to a solution of Saint Venant's torsion problem; the viewpoint here is somewhat more theoretical, and such methods as those of Ritz, Trefftz and Galerkin are briefly summarized as well as the finite difference methods with which the preceding chapters were mostly concerned.

The book provides a valuable cross-section of current applications of certain types of numerical methods to engineering problems. Many gaps, of course, exist. The beginner should perhaps be warned that numerous misprints are present.

P. S. SYMONDS

Table for use in the addition of complex numbers. By Jørgen Rybner and K. Steenberg Sørensen. Jul. Gjellerups Forlag, København, 1948. xiv + 95 pp. \$5.50

The object of the table is to facilitate the calculations of addition or subtraction with complex numbers by tabulating the complex relation

$$Re^{i\alpha} = 1 + re^{i\phi}$$

$$\text{or } R\angle\alpha = 1 + r\angle\phi$$

The table lists the numerical value of R and the phase angle α of the sum as functions of r and ϕ for $0 \leq r \leq 1$ at intervals of 0.01 and for $0 \leq \phi \leq 180^\circ$ at intervals of 1° . α is expressed in degrees and decimal fractions thereof. Interpolation differences are given. The authors have prepared this numerical table because in a previously published work entitled *Nomograms of Complex Hyperbolic Functions* by Jørgen Rybner it was not possible to construct nomograms with an accuracy sufficient for practical calculations. (The tables may be purchased from Scandinavian Book Service, P. O. Box 99, Audubon Sta., New York 32, N. Y.)

ROHN TRUELL

A concise history of mathematics. By Dirk J. Struik. Dover Publications, Inc., New York, 1948. Vol. I, xviii + 123 pp. Vol. II, 175 pp. \$1.50 per Vol. \$3.00 per Set.

This new history of mathematics is a volume in the Dover Series in Mathematics and Physics and which has thus far largely included translations into English of volumes in the well-known collection known as *Sammlung Götschen*. The latter contained a history of mathematics of about the same length as Struik's written by the great historian of mathematics, Heinrich Wieleitner, and published in 1922. It was a happy thought to have Struik write a wholly new work rather than simply translate Wieleitner's, because the latter's *Geschichte der Mathematik* is "out of date" and, furthermore, is not that author's best work (his *Mathematische Quellenbücher*, for example, are much better). Some of the particular merits of Struik's presentation can best be seen by comparing it to Wieleitner's.

One of the first features that strikes the reader of the work under review is the considerable amount of bibliographical material, at once leading the student to further reading and showing him that the study of the history of mathematics is an active field in which new discoveries are being made, just as they are being made in the field of mathematics itself. Struik's interesting and extended discussion of the mathematics of primitive peoples is an important novelty. The section on the ancient Orient, especially

rich on the mathematics of the Babylonians, is another important feature. The stress placed by Struik on Babylonian mathematics and primitive mathematics (the first treated only summarily by Wieleitner and the second ignored by him altogether) is an indication of features of the history of mathematics which were not present when Wieleitner wrote his history. The new discoveries in the field of Babylonian astronomy and mathematics, due chiefly to O. Nöugebauer, have been made largely in the last 25 years, and the general interest in anthropology as related to the history of science is also of recent date. Whereas Wieleitner divided the modern period by topic, so that one has, for the 18th and the first half of the 19th centuries, snippets on each possible mathematical subject, Struik has wisely couched his exposition of 19th-century mathematics in terms of mathematicians and schools. The result is that Struik's account takes on humane perspectives and is eminently readable, whereas Wieleitner's history tends to be a catalogue. Finally, Struik has included a considerable amount of social history, both ordinary social and economic history, as well as an interpretation of the social history of mathematics. This general approach places the mathematical history in a context of the greatest interest for the reader, even though at times it sets forth a point of view or interpretation with which all readers will not agree.

Following the opening account of the formation of mathematical concepts among non-mathematical people, is a splendid discussion of pre-Greek mathematics, setting the stage for the great accomplishments of the Greeks. (Here one must give Struik special credit for his attempt to distinguish between established fact, hypothesis, and tradition.) This chapter is followed by a short discussion of the Islamic period which leads him naturally into the revival of mathematics in western Europe, and completing the first volume. The second volume, containing three major sections—the 17th century, the 18th century, and the 19th century—will probably prove of greater interest to active mathematicians. Nevertheless, in contrast to the first volume, in which the major ideas are carefully explained, in this one Struik has allowed the mathematics to speak for itself. Only a fully rounded mathematician who knows the major fields of mathematics will be able to fully appreciate much of this volume.

Struik's book thus clearly fills a long-felt need for a short readable history of the main ideas and major individuals in the history of mathematics. It is a tribute to his mastery of the craft of exposition that he has been able to include so much related material on astronomy and mathematical physics and so much up-to-date information concerning the history of mathematics. Struik has not only read the original writings of the great mathematicians whose work he recounts, but he has also mastered the vast secondary literature of biography, interpretation, and commentary.

In a work at once as concise and as comprehensive as this one is, certain omissions are bound to occur to each reader. Nevertheless, I feel sure that no one will be able to find a major mathematician omitted. Nor in general can anyone but admire Struik's selection of topics. Anyone who reads this concise history of mathematics will, I am sure, join with me in expressing the wish that the work under review be considered only an earnest of a major work to come, in which Struik will have the opportunity to explore further the social history of mathematics.

I. BERNARD COHEN

Numerical calculus. (Approximations, interpolation, finite differences, numerical integration, and curve fitting.) By William Edmund Milne. Princeton University Press, Princeton, N. J., 1949. x + 393 pp. \$3.75.

Numerical analysis has been sadly neglected in our traditional undergraduate and graduate curricula in mathematics because it was not considered worthy of the attention of serious mathematicians. Now that the availability of electromechanical and electronic calculators has revived interest in this field, the scarcity of adequate textbooks, a consequence of previous neglect, confronts mathematics departments which are ready to increase the course offerings in this field. The appearance of the present text is therefore particularly timely.

The scope of the book is best indicated by the following list of chapter headings: simultaneous linear equations, solution of equations by successive approximations, interpolation, numerical differentiation and integration, numerical solution of differential equations, finite differences, divided differences, reciprocal differences, polynomial approximation by least squares, other approximations by least squares, simple difference equations. These eleven chapters fill 348 pages; the remaining pages contain a table

comparing the various notations used in the calculus of finite differences, a classified guide to the formulas and methods presented in the book, a bibliography, and several brief numerical tables.

The presentation has been kept elementary; the usual three semester sequence of elementary calculus and ordinary differential equations should provide sufficient background for the users of this book. The remarkably low price of the cloth-bound volume has been made possible by photo-offset reproduction from a carefully typed manuscript.

W. PRAGER

Quantum theory of fields. By Gregor Wentzel. Interscience Publishers, Inc., New York and London, 1949. ix + 224 pp. \$6.00.

The advanced student of modern physics must certainly welcome this English edition of the most complete and probably the best introduction to the quantum theory of fields. The translation from *Einführung in die Quanten theorie der Wellenfelder* (Wien 1943) was made by C. Houtermans and J. M. Jauch; it includes a preface to the English edition by G. Wentzel. While the most recent theoretical developments in quantum electrodynamics have not been incorporated in the references, the basic information is provided to enable the reader to follow the current literature. Some changes have been made in the section on the applications of field theory to problems of nuclear physics, and in various sections of the book references to recent publications have been added. An Appendix has been added dealing with the definition of a symmetrical energy-momentum tensor for any kind of a field with an invariant Lagrange function.

The contents of the book fall under the chapter headings: General Principles, Scalar Fields, The Vector Meson Field, Quantum Electrodynamics, The Quantization of the Electron Wave Field According to the Exclusion Principle, Supplementary Remarks, and the Appendix mentioned.

ROHN TRUETT

Non-linear problems in mechanics of continua. Proceedings of Symposia in Applied Mathematics, Volume I. American Mathematical Society, New York, 1949. vii + 219 pp. \$5.25.

This volume contains the papers presented at the First Symposium on Applied Mathematics of the American Mathematical Society, held at Brown University in the summer of 1947. The subject of the Symposium was the same as the title of this book.

The contents are divided into two groups, one under the subtitle of Hydrodynamics and the other under that of Elasticity and Plasticity. The majority of the contributions in the first group are in the field of compressible flow, with studies of two-dimensional flows, shock wave problems, and the general three-dimensional case. Included also are contributions on incompressible flows with free boundaries, turbulence, and boundary layers. In the second group investigations are reported in general elasticity, large deflection plate theory, and plasticity.

W. HAYES

Tables of the Bessel functions of the first kind of orders fifty-two through sixty-three. By The Staff of the Computation Laboratory. Harvard University Press, Cambridge, Massachusetts, 1949. 544 pp. \$8.00.

The Bessel functions $J_{52}(x)$, $J_{53}(x)$, \dots , $J_{63}(x)$ are tabulated to ten decimal places. The argument varies in steps of .01 for $0 \leq x < 100$.

G. F. CARRIER

Tables of supersonic flow around cones of large yaw. By The Staff of the Computing Section Center of Analysis. Under the direction of Zdenek Kopal. Massachusetts Institute of Technology, Cambridge, Massachusetts, 1949. xviii + 125 pp. \$1.50.

Some results concerning the flow of a non-viscous gas past a cone with yaw are tabulated in this volume. The theory is as yet unpublished, but the compilation of formulas given in the introductory pages indicates that this theory utilizes the first two terms in a perturbation expansion in powers of the yaw angle. Cones of semi-apex angle 5° , 7.5° , 10° , 12.5° , 15° , 20° , 25° , are considered. The pertinent quantities (which define the pressure, velocity, . . .) are plotted versus the polar angle. The increments in this independent variable are .1 near the shock and near the body and are larger in the intermediary region. No comments concerning applicability and accuracy can be made until the details of the theory are available.

G. F. CARRIER

The Aeronautical Quarterly. Published by the Royal Aeronautical Society, 4 Hamilton Place, London, W.1. Volume I: Part I (May 1949), 122 pp., 10s. 0d., Part II (August 1949), 72 pp., 10s. 0d.

As is stated in the foreword to the first issue of this new Quarterly, aeronautical science has grown more rapidly in recent years than the means of disseminating the knowledge acquired during this growth. The new Quarterly was founded as a means of remedying this situation; its Editorial Board encourages research workers in aeronautics to submit papers describing new and original work, or papers reviewing the progress in some specialized field.

The first two parts of Volume I contain a diversified group of papers treating, among many others, such topics as downwash behind a supersonic wing, flutter of systems with many freedoms, displacements of elastic systems under transient loads, and laminar layers in compressible flow.

The addition of this new Quarterly to the family of journals devoted to problems of engineering science is welcomed.

G. F. CARRIER

Mathematical foundations of statistical mechanics. By A. I. Kinchin. Dover Publications, Inc., New York. viii + 179 pp. \$2.95.

This book forms an excellent introduction to the difficult and important discipline of Statistical Mechanics. It is clear, concise and rigorous. There is a very good chapter on the ergodic theorem (with a complete proof!) and a traditional but highly lucid chapter on statistical foundations of thermodynamics. The treatment is somewhat reminiscent of that of P. Hertz (misspelled Kertz in the book) and is based on the notion of the microcanonical ensemble. The scope is extremely limited since the author deals only with systems composed of many independent parts (ideal polyatomic gas). It is because of this limitation that a variant of the central limit theorem can be successfully applied. One gathers, unfortunately, the impression that the author tries to replace once and for all the 'abstruse' method of steepest descent by the use of the central limit theorem. The reviewer fails to see how the central limit theorem can be helpful in cases when non-negligible interactions between the components are present. It is a pity that in a book entitled 'Foundations' no mention is made of the Boltzmann-Zermelo polemic concerning the meanings of the H -theorem (in fact, there is no mention of the H -theorem) and there is no discussion of non-equilibrium problems. The translation is very good and only in few places did the reviewer find his knowledge of Russian helpful.

The book should prove useful to teachers of Statistical Mechanics and to mathematicians who would like to get acquainted with the problems and methods of this fascinating branch of physics.

M. KAC