QUARTERLY

OF

APPLIED MATHEMATICS

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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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stress concentration is localized near the rigid disk. The x- and y- axes are parallel and perpendicular to the grain, respectively.

Shear stresses are probably most important in producing failure in a wooden plate. The shear stress given by (18) evaluated on the boundary of the rigid disk is

$$X_\nu \Big|_{0 \leq \theta \leq \pi/2} = \frac{\alpha \epsilon A_1 \sin \theta \cos \theta}{\alpha^2 \epsilon^2 b^2 \cos^2 \theta + a^2 \sin^2 \theta} \tag{21}$$

$$+ \frac{\beta \epsilon B_1 \sin \theta \cos \theta}{\beta^2 \epsilon^2 b^2 \cos^2 \theta + a^2 \sin^2 \theta}.$$

In Figure 1 the curve for Sitka spruce was plotted utilizing formulas (19), (20) and (21). The isotropic curve was computed for $\sigma = 0.3$.

Throughout the Sitka spruce plate containing a rigid circular disk $|Y_\nu|$ is less than $0.03S$. The maximum value of X_z is $1.23S$ at the point $(a, 0)$.

From these results, it is probable that, for S sufficiently large, failure in the plate will occur approximately along the lines $y = \pm a \sin 77\frac{1}{2}° = \pm 0.976a$ with the crack beginning at the edge of the disk.

BOOK REVIEWS

The mathematical theory of plasticity. By R. Hill. Oxford at the Clarendon Press, 1950. ix + 356 pp. $7.00.

Time, temperature, Bauschinger, hysteresis, and size effects are all explicitly ruled out and the major but not exclusive emphasis is on ideal plasticity. However, a wide range of topics is discussed in this interesting and invaluable treatise. Mathematical proofs, experimental evidence and the practical evaluation of theory are kept in excellent balance from the study of stress-strain relations, variational principles, and the questions of uniqueness, to the solving of practical metal forming problems. A few relatively simple elastic-plastic solutions are included for prismatic bars, and for thick cylindrical tubes and spherical shells. However, most of the chapters deal with plastic-rigid techniques and solutions developed by the author and E. H. Lee. An extensive discussion is given of the slip line fields for plane problems. The necessity for complete solutions is stated strongly and repeated warning is given against the error of thinking in terms of static determinacy only and not considering velocity conditions as well. A number of interesting and truly amazing solutions are given in detail for which the configuration remains geometrically similar as the plastic deformation proceeds. Among the miscellaneous subjects covered are: machining, hardness tests, notched bars, normal and oblique necking, earing and anisotropy. Tensor notation is used throughout without apology.

As is entirely proper for a book on the mathematical theory of plasticity, the physics of metals is covered only by reference to treatises on the subject. A very brief Appendix on suffix notation, the summation convention, and hyperbolic differential equations is considered sufficient for the reader who is assumed to be familiar with the elementary theory of elasticity.

The reviewer regretted to detect an apparent desire on the part of the author to have written a truly definitive text at a stage in the development of plasticity when too many basically new facts are being discovered. As the author himself is in the forefront of many of these developments he could have done a greater service by indicating more clearly the needed fundamentally new directions rather than glossing over our present shortcomings and often embarrassing lack of knowledge. He then might not have allowed himself to get so worried by Southwell and Allen's elastic-plastic solution for a notched bar that

he unjustifiably loses faith in his rigid plastic solutions. Also, there would be no need to keep a distinction between the true yield function and the plastic potential in the derivation of general stress-strain relations, nor to place so much faith in the path independence of the dissipated work.

D. C. DRUCKER

Response of physical systems. By John D. Trimmer. John Wiley & Sons, Inc., New York and Chapman & Hall, Limited, London, 1950. ix + 268 pp. $5.00.

This interesting book was written from material used in a course in "Instrumentation" primarily for physicists and engineers. The book is interesting because it manages to cover in concise form, on a level suitable for seniors, the essentials of system response to sinusoidal and pulse driving. Both mechanical and electrical systems are dealt with and system stability is treated in sufficient detail to give a good idea of the problems. Measuring instruments are discussed as physical systems and measurements as the response of such systems together with measurement errors. The Chapter headings are: A Pattern for Systems, Physical Systems, First Order Systems, Second Order Systems, Sinusoidal Forcing of Linear Systems, Higher Order Systems, Measuring Instruments, Feedback Systems, Parametric Forcing, Distributed Systems, Nonlinear Systems. Appendices: The Laplace Transform in the Study of Linear Systems, Problems and Review Questions.

R. TRUELL

Classical mechanics. By H. C. Corben and Philip Stehle. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London, 1950. xvii + 388 pp. $6.50.

As the authors point out in the Preface "the parts of classical mechanics which are of present-day interest to the physicist are not those which were of paramount interest in the nineteenth century." The authors therefore set out to write a modern book which would reflect this change in emphasis in an old field. In this reviewer's opinion, they have succeeded to a remarkable extent in producing such a text. The space available for this review does not permit a detailed discussion of the contents, and the following list of chapter headings (with some significant section headings added in parentheses) must suffice. Kinematics of particles. The laws of motion. Conservative systems with one degree of freedom. Miscellaneous theorems on systems of particles Lagrange's equations of motion (Charged particle in an electromagnetic field). Applications of Lagrange's equations (Rutherford scattering. The cylindrical magnetron). Linear vector spaces. Small oscillations of conservative systems (Application to molecules. The molecule X_3Y). Rigid bodies. Hamiltonian theory. Contact transformations. Contact transformations which simplify the equations of motion. Poisson brackets (Poisson brackets in quantum mechanics). Infinitesimal contact transformations. Further development of transformation theory. Miscellaneous generalizations and analogies (Analogy with thermodynamics). Introduction to special relativity theory. The motion of particles in high energy accelerators.

W. PRAGER

Methods of mathematical physics. By Harold Jeffreys and Bertha Swirles Jeffreys. University Press, Cambridge, 1950. vii + 708 pp. $15.00.

The first edition of this treatise appeared in 1946 [see this Quarterly 5, 366(1947)]. The fact that a second edition should have become necessary so soon clearly indicates that the work has become a standard reference for mathematical physicists and applied mathematicians. While the material has been

rearranged in some chapters and minor changes have been made throughout the book, the major changes are as follows. A section on block matrices has been added, the chapter on multiple integrals has been thoroughly revised, relaxation methods have been treated in somewhat greater detail, the presentation of the theory of inverse functions has been revised, and the treatment of multipole radiation has been extended. Also, some new problems have been added.

W. Prager

Foundations of aerodynamics. By A. M. Kuethe and J. D. Schetzer. John Wiley & Sons, Inc., New York and Chapman & Hall, Ltd., London, 1950. ix + 374 pp. $5.75.

This is a textbook for a general course in aerodynamics, intended for graduate and senior-undergraduate students whose training has been in engineering, mathematics, or physics, but who need not previously have studied aerodynamics. There are three major parts of the volume, devoted substantially to incompressible-inviscid, compressible-inviscid, and incompressible-viscous flows. It is a major attraction of the textbook, however, that real gases are discussed in the opening chapter. Here, in addition, the plan of the whole book is laid out for the student and the admirable objective of his study is stated: "to provide a background of sound concepts for finding approximate solutions of problems in the flow of a compressible, viscous, inhomogeneous gas."

The chapters based on incompressible-flow theory include, among other matters, classical two-dimensional thin-airfoil theory and Prandtl wing theory. The compressible-fluid chapters include channel flow, shock and expansion waves, Prandtl-Glauert rule, and linearized supersonic airfoil theory. Except for a descriptive section, the last-mentioned is confined to plane flow and the equivalent sweptback infinite wings.

The stress terms in the Navier-Stokes equations are derived in an Appendix, in lucid style. Two chapters concern mostly laminar boundary-layer flow; two more are devoted to turbulence, and another to boundary-layer transition, including descriptions of boundary-layer instabilities of two types. These chapters seem particularly valuable, and are probably unique in the field of fluid-mechanics textbooks.

The closing chapters present material on viscous-compressible flow and some descriptive information on wing properties, such as transonic flow and stalling. The viscous-compressible paragraphs are soundly physical, concerning especially energy balance and aerodynamic heating. There is also some information on interactions between shock wave and boundary layer, and a regrettably short paragraph on "the thickness of a shock wave."

There are problems for the first 13 of the 18 chapters, which seem, to the reviewer, to demand rather little originality from the student. There are gratifying exceptions to this generalization, however.

Throughout, the authors have emphasized the physical picture that their equations describe. Moreover, the reader is frequently reminded of the confirmation, and occasional conflict, of experimental results. Thus, although it will undoubtedly teach most seniors and young graduate students a good deal of applied mathematics, it is really not a book in that field, but one in the field of physics, as it should be.

In view of the purpose and the scope of the volume, it is probably not justifiable to complain about sins of omission; rather, the authors should be commended on their skill in having selected their topics well. It may suffice to remark that no single volume of 374 pages can be adequate for full-dress presentations of theoretical aerodynamics at the graduate level. In its own field—which is one that should be vigorously encouraged—this is certainly the best textbook, and a badly-needed one.

W. R. Sears

Theory of probability. By M. E. Munroe. First Edition. McGraw-Hill Book Company, New York, Toronto, London, 1951. viii + 211 pp. $4.50.

This is an elementary textbook designed to introduce students who have had a first year course in calculus to the theory of probability. The author makes frequent references to Cramér's and Uspensky's

books, and the general purpose of the book seems to be to provide a book more accessible to less advanced students than these books are. As a matter of fact, the present book is more in the spirit of probability theory than is Uspensky's book. Even a version of the strong law of large numbers is discussed. However, strong convergence is defined as convergence in the space of infinite sequences. This is rather sophisticated for students with a year of calculus and an alternate limit definition might have been more intelligible. The book has many illustrative problems and exercises for the student.

J. Wolfowitz

Vector and tensor analysis. By Harry Lass. McGraw-Hill Book Company, Inc., New York, Toronto and London, 1950. xi + 347 pp. $4.50.

This book contains a thorough treatment of vector analysis, with extensive applications. It also contains a short but good introduction to tensors. There is a large number of examples and problems. The style is clear and concise, and there is a sharp segregation between general theory and the many applications. This renders the book suitable for the standard undergraduate course in vector analysis, as well as for more advanced work dealing with an introducton to mathematical physics.

There are nine chapters. Chapters I, II and IV deal with the algebra, differential calculus and integral calculus of vectors. These chapters, comprising about one quarter of the book, contain the theory appearing in the standard undergraduate course in vector analysis. Chapters III, V, VI and VII, which comprise about one half of the book, deal respectively with differential geometry, electricity, mechanics, and hydrodynamics and elasticity. Of course, in a book of this nature the treatments of these topics can be an introduction only, but the treatment is systematic, and particularly in the case of electricity is rather extensive. Chapters VIII and IX, which comprise the remaining one quarter of the book, contain a brief introduction to the theory of tensors, as well as brief applications of tensors to geometry, mechanics and elasticity.

G. E. Hay

Elasticity. Proceedings of the Third Symposium in Applied Mathematics of the American Mathematical Society, Volume III. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1950. v + 233 pp. $6.00.

This volume contains the papers presented at the Third Symposium on Applied Mathematics of the American Mathematical Society, held at the University of Michigan on June 14-16, 1949. Seventeen papers are included, two of these being presented in the form of abstracts.

The subject matter of thirteen of the papers is elasticity, that of the remainder, plasticity. The papers in elasticity include studies in the plane problems of anisotropic materials, finite deflections of axisymmetric shells and asymptotic integration in shell theory, the bending of plates, general theory of finite elastic deformation, the beam of circular cross-section under concentrated loading parallel to its axis, approximate determination of the torsional rigidity of beams, torsion of the axially symmetric shaft, and the boundary layer edge effect in elastic plates. The plasticity contributions are on the stress analysis of indeterminate elastic-plastic structures, the use of characteristics in obtaining graphic solutions of plane problems, and the solution of problems in plane flow with stress discontinuties.

William H. Pell