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CONTENTS

into (12) and solving for p, we obtain

$$p = \frac{4\sigma_0 h_0}{a}\left(1 - 2\alpha \log \cos\frac{\theta}{2}\right)\sin\frac{\theta}{2}\cos^3\frac{\theta}{2}. \tag{17}$$

Figure 2 shows the pressure maximum computed from (17) and the corresponding values of θ, h/h_0, and $|\xi| = -\log(h/h_0)$, all versus the strain-hardening parameter α. It is seen that the pressure maximum $p_{\max}$ as well as the corresponding values of θ and $|\xi|$ increase with α, whereas the ratio h/h_0 at the pressure maximum decreases with α. In the considered range of α all these quantities vary with α in a nearly linear manner.

BOOK REVIEWS

Mathematical methods for scientists and engineers. By Lloyd P. Smith. Prentice-Hall, Inc., New York, 1953. x + 453 pp. $10.00.

The material in this book is essentially that taught for a number of years by the author to graduate students in physics and physical chemistry at Cornell University. The unusual feature of this text is the wide range of mathematical methods treated. At the same time it will be found that the treatment of the material is quite adequate for dealing with most physical problems. The discussion is consise and clear.

There is no specific treatment of differential equations except as they arise in the treatment of the other topics.

The reviewer believes that this is one of the most useful books available on intermediate and advanced mathematical methods.

The chapter headings of this text are as follows: Elements of Function Theory; Differential Calculus, Integral Calculus; Space Geometry; Line, Surface, and Multiple Integrals; Theory of Functions of a Complex Variable Residues and Complex Integration; Representation of Functions by Infinite Series of Functions; Applications of Functions of a Complex Variable to Potential and Flow Problems; Algebra of Linear Equations, Transformations and Quadratic Forms; Vector and Tensor Analysis; Orthonormal Function Systems; Orthonormal Functions with a Continuous Spectrum; Integral Equations; Variational Methods; and Elements of Probability Theory.

ROHN TRUELL

Calculus of variations with applications to physics and engineering. By Robert Weinstock. McGraw-Hill Book Company, Inc., New York, 1952. x + 326 pp. $6.50.

According to the preface, this volume presents an introduction to the calculus of variations followed by application of the subject to problems of physics and theoretical engineering.

The first five chapters give the usual elementary treatment of the calculus of variations with no pretense of complete mathematical rigor. Chapters 6-12 present applications to dynamics, elasticity, quantum mechanics, and electrostatics. These applications are, for the greater part, of an elementary nature; modern problems in acoustics, electromagnetic theory, and quantum mechanics are not discussed.

Up to the present time there is no other volume in the English language that offers such a variety of applications of the calculus of variations to problems in physics. The book must therefore be accepted as a worthwhile contribution to the applied mathematician's library.

This reviewer has a few adverse comments to make on the contents of the book; these are not in-

tended to detract from the general acceptability of this volume as a text-book at the first-year graduate level.

(a) It is claimed in the preface that "the reader who has mastered the essence of the material included should have little difficulty in applying the calculus of variations to most of the subjects which have been squeezed out." This is unwarranted optimism on the part of Professor Weinstock. The book does not offer any examples of variational principles for integral equations; there is no mention of Green's functions. How, then, could a reader be expected to understand "with little difficulty" the Levine-Schwinger treatment of scattering problems?

(b) There is no mention of variational techniques for obtaining lower bounds to eigenvalues (work of Weinstein, Aronszajn, etc.). In particular the chapter on quantum mechanics is weak because of this and other omissions.

(c) The work of Pólya and Szegö and their colleagues on "isoperimetric inequalities in mathematical physics" is barely touched on in the last chapter on electrostatics.

(d) Chapter 5 is entitled "Geometrical Optics: Fermat's Principle." This chapter is five pages long and consists exclusively of deriving the two-dimensional principle of Fermat by two different methods. It is unfortunate that the author did not discuss "corner conditions" in this connection. Snell's law could then be taken as an illustration and the chapter expanded to reasonable size. (See e.g. H. Lewy: "Aspects of the Calculus of Variations." University of California Press, Berkeley, California, 1939. This excellent little volume is not listed in Weinstock's bibliography.)

(e) A few serious errors in the text have been pointed out by Synge in his review in "Bulletin of the American Mathematical Society", Vol. 59, No. 4, July, 1953.

Here are two additional non-trivial errors.

1. The second part of theorem (d), p. 157, is incorrect.

2. Introduction, p. 2: "The only values of x at which $y = g(x)$ can possibly achieve a minimum are the roots of $g'(x) = 0$." A minimum can occur at points where $g'(x)$ does not exist, or at an endpoint of an interval. This and similar precautions are neglected throughout the book; for instance, on p. 39, where no condition on the differentiability of $g(x, y)$ is given.

I. STAKGOLD

Numerische Behandlung von Differentialgleichungen. By L. Collatz. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1951. xiii + 458 pp. $11.43.

This book represents a careful and comprehensive study of the numerical methods of solution of ordinary and partial differential equations. It also contains a concise treatment of numerical methods for integral equations. By virtue of the wealth of material covered the book should prove to be a very valuable source of reference to anyone interested in this important subject.

In addition to a description of the various methods the author has included, particularly in the case of ordinary differential equations, analyses of their theoretical foundations. In many instances there is a discussion of the estimation of the error. There are numerous illustrative examples, many of them representing practical physical problems.

Chapter I contains, in addition to introductory remarks, difference formulae and quadrature formulae, a study of initial-value problems for ordinary equations of first and higher order. Chapter II is devoted to boundary-value problems (boundary conditions prescribed at more than one point) for ordinary equations, including eigenvalue problems. In Chapter III we find initial-value problems and combined initial and boundary-value problems for partial differential equations. In chapter IV the author takes up boundary-value problems for partial differential equations. Chapter V is concerned with integral equations and a brief discussion of functional equations.

G. W. MORGAN

Die praktische Behandlung von Integralgleichungen. By H. Bückner. Springer-Verlag, Berlin-Goettingen-Heidelberg, 1952. vi + 127 pp. DM 18.60.

As the author states in the Introduction, the numerical treatment of integral equations is a comparatively new and still developing field of numerical analysis. The present systematic survey is therefore particularly welcome even though it is restricted to equations of the Fredholm type for functions of a single variable. The first chapter recapitulates the principal results of the general theory. Chapter II presents formulas and variational principles that are useful in the calculation of characteristic values. Particular attention is given to methods of bracketing characteristic values. Chapter III is devoted to iterative methods. The use of mixed iteration for the solution of inhomogeneous integral equations is of special interest. Chapter IV is concerned with techniques using approximations to the kernel of the integral equation. Perturbation methods, the methods of Ritz and Galerkin, and methods replacing the integral by a sum are treated as special cases. The brief final chapter deals with special kernels.

W. PRAGER

Mathematical physics. By Donald H. Menzel. Prentice-Hall, Inc., New York, 1953 v + 412 pp. $8.50.

This text contains five chapters entitled: I. Physical Dimensions and Fundamental Units, II. Mechanics and Dynamics, III. Waves and Vibrations, IV. Classical Electromagnetic Theory, V. Relativity. The book is claimed by the author to be designed for use in junior, senior, or graduate courses in mathematical physics; this purpose seems to have been achieved very nicely. The book appears to the reviewer to be a very satisfactory text for the transition from advanced undergraduate physics to first year graduate physics, and it should be welcomed by many students and teachers.

This is not a text on mathematical methods in physics. It is rather a text on theoretical physics with emphasis on the mathematics.

ROHN TRUELL

The principles of the control and stability of aircraft. By W. J. Duncan. Cambridge University Press, 1952. xvi + 384 pp. $8.00.

In view of the importance of the field and the abundance of texts on other aspects of aeronautics, the scarcity of textbooks on aircraft stability and control is surprising. The present volume is therefore a particularly welcome addition to the aeronautical literature. Within the limited space available for this review, the scope of the work is best indicated by the following chapter headings: Introductory survey. Elementary mechanics of flight. The equations of motion of rigid aircraft. Methods for solving the dynamical equations and for investigating stability. Longitudinal-symmetric motion. Lateral-antisymmetric motion. Flap controls in general. The measurement of aerodynamic derivatives. Controls for roll, pitch, and yaw. Static stability and maneuverability. Stalling and the spin (by A. D. Young). The influence of distortion of the structure. The influence of the compressibility of the air. Flaps for landing and take-off (by A. D. Young). Sundry topics.

The exposition is particularly clear and easy to follow.

W. PRAGER

CORRECTION

In the review of Petrovskij, *Vorlesungen ueber die Theorie Integralgleichungen*, this Quarterly 11, 375 (1953) the price of this book was erroneously stated to be $7.80 instead of DM 7.80.

Complex analysis. An introduction to the theory of analytic functions of one complex variable. By Lars V. Ahlfors. McGraw-Hill Book Co., New York, Toronto, London, 1953. xi + 247 pp. $5.00.

This book is a rigorous introductory treatment of the theory of analytic functions of one complex variable. It is very readable and would be an excellent basis on which to teach a course. There are exercises every few pages. Although no physical examples are given, the student who has mastered this book will have no mathematical difficulty applying the knowledge he has gained to any application of complex variable in mathematical physics.

In Chapter 1, complex numbers are introduced, their geometrical representation and linear transformations considered. Chapter 2 is devoted in the main to theorems needed from the theory of a real variable and from the theory of sets. Complex integration is introduced in Chapter 3. Cauchy's integral formula and the calculus of residues follow. Chapter 4 contains convergence and uniform convergence of series, Taylor and Laurent series, infinite products and normal families. The Dirichlet problem is considered in Chapter 5 and multiple-valued functions in Chapter 6.

D. R. BLAND

Proceedings of Symposia in Applied Mathematics—Fluid Dynamics. Volume IV. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1953, for the American Mathematical Society, 80 Waterman Street, Providence, Rhode Island. v + 186 pp. $7.00.

This volume contains thirteen papers and one abstract of a paper which were presented at the Fourth Symposium in Applied Mathematics of the American Mathematical Society held at the University of Maryland on June 22 and 23, 1951. All papers are concerned with fluid dynamics. Topics in turbulence, compressible (including transonic), potential, and viscous flow are treated, and one paper deals with the development of the hydrodynamical equations from the point of view of thermodynamics of irreversible processes.

The papers included are: S. Chandresekhar, "Some aspects of the statistical theory of turbulence". C. C. Lin, "A critical discussion of similarity concepts in isotropic turbulence". A. Busemann, "The nonexistence of transonic potential flow". R. E. Meyer, "On waves of finite amplitude in ducts (abstract)". T. Y. Thomas, "On the problem of separation of supersonic flow from curved profiles". G. F. Carrier and K. T. Yen, "On the construction of high-speed flows". M. H. Martin and W. R. Thickstun, "An example of transonic flow for the Tricomi gas". A. E. Heins, "On gravity waves". S. R. De Groot, "Hydrodynamics and Thermodynamics". J. M. Burgers, "Non-uniform propagation of plane shock waves". T. Theodorsen, "Theory of propellers". G. Birkhoff, D. M. Young and E. H. Zarantonello, "Numerical methods in conformal mapping". J. L. Synge, "Flow of viscous liquid through pipes and channels". A. Weinstein, "The method of singularities in the physical and in the hodograph plane".

GEORGE W. MORGAN

Anfangswertprobleme bei partiellen Differentialgleichungen. By R. Sauer. Springer-Verlag, Berlin-Goettingen-Heidelberg, 1952. xiv + 229 pp. $6.90.

The book treats initial value problems for single partial differential equations and systems of partial differential equations of hyperbolic type. The introductory first chapter discusses the typical boundary value problem for the Laplace equation and the typical initial value problem for the wave equation. The second chapter develops the theory of characteristics for first partial differential equations of the first order. Chapters III and IV which constitute about three quarters of the volume are devoted to the system of quasilinear differential equations of the first order and the equation of the second order, Chapter III treating the case of two and Chapter IV the case of more than two independent variables.

The clear exposition makes the book easy to read. A particularly valuable feature is the constant detailed reference to relevant physical problems (hydrodynamics, acoustics, gas dynamics, plasticity).

W. PRAGER