where Φ is determined in terms of ϕ by

$$\frac{\partial \Phi}{\partial X_i} = \frac{\partial}{\partial X_j}\left\{\frac{\partial^2 \phi}{\partial X_i\, \partial T}\frac{\partial^2 \phi}{\partial X_j\, \partial T}\Big/ \nabla^2\phi\right\}. \tag{12}$$

Expressions (11) and (12) are McVittie's results *but with the dependence of p on ϕ given explicitly*. It is necessary to point out, however, that neither Eq. (10) nor Eq. (12) has a solution unless the right-hand side is an irrotational vector field. Since (12) is equivalent to McVittie's equations, this shows that these too have solutions only for a restricted class of ϕ.

It is now shown how (3), (4), (5), (6), (7) may be reduced to (11) and (12). To eliminate ϕ_1, ϕ_2, ϕ_3 in (5), (6), (7), an expression for the term $\frac{1}{3}\sum_{i=1}^{3}(\nabla^2\phi_i - \partial^2\phi_i/\partial X_i^2)$ in (5) is required in terms of ϕ. However, if the three quantities which are equated in (7) are denoted by E_1, E_2 and E_3, respectively, and $\frac{1}{3}(E_1 + E_2 + E_3)$ is denoted by Ψ, it it seen that (5) is $p = -\partial^2\phi/\partial T^2 + \Psi$; it remains, therefore, to demonstrate that $\Psi = \Phi$. Differentiating E_1 with respect to X_1 we have

$$\frac{\partial \Psi}{\partial X_1} = \frac{\partial E_1}{\partial X_1} = \frac{\partial^3\phi_2}{\partial X_3^2\, \partial X_1} + \frac{\partial^2\phi_3}{\partial X_2^2\, \partial X_1} + \frac{\partial}{\partial X_1}\left\{\left(\frac{\partial^2\phi}{\partial X_1\, \partial T}\right)^2 \Big/ \nabla^2\phi\right\}. \tag{13}$$

Expressions for $\partial^2\phi_2/\partial X_3\partial X_1$ and $\partial^2\phi_3/\partial X_2\partial X_1$ are given in terms of ϕ by (6); hence $\partial\Psi/\partial X_1$ can be given in terms of ϕ alone. We have

$$\begin{aligned}\frac{\partial \Psi}{\partial X_1} &= \frac{\partial}{\partial X_3}\left\{\frac{\partial^2\phi}{\partial X_3\, \partial T}\cdot\frac{\partial^2\phi}{\partial X_1\, \partial T}\Big/ \nabla^2\phi\right\} + \frac{\partial}{\partial X_2}\left\{\frac{\partial^2\phi}{\partial X_2\, \partial T}\cdot\frac{\partial^2\phi}{\partial X_1\, \partial T}\Big/ \nabla^2\phi\right\} \\ &\qquad + \frac{\partial}{\partial X_1}\left\{\left(\frac{\partial^2\phi}{\partial X_1\, \partial T}\right)\Big/ \nabla^2\phi\right\} \\ &= \frac{\partial}{\partial X_j}\left\{\frac{\partial^2\phi}{\partial X_j\, \partial T}\cdot\frac{\partial^2\phi}{\partial X_1\, \partial T}\Big/ \nabla^2\phi\right\}.\end{aligned} \tag{14}$$

$\partial\Psi/\partial X_2$ and $\partial\Psi/\partial X_3$ are obtained similarly and we observe that the expressions agree with (12); hence, Φ and Ψ are indeed the same.

BOOK REVIEWS

An introduction to the theory of differential equations. By Walter Leighton. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1952. viii + 174 pp. \$3.50.

This book constitutes a carefully written exposition of the elements of the theory of ordinary linear differential equations. Accordingly, considerable attention has been given to questions of rigor, and to the understanding of basic concepts as opposed to mere formal facility of operation. To this end the author discusses the nature of solutions of differential equations and states and discusses existence theorems at appropriate points in the text. The proofs of these and other of the more difficult theorems are given, but in appendices for the sake of clarity of exposition.

The reviewer feels, however, that this book will be too difficult for the student who has just emerged from the first course in the calculus as given in most of our colleges. Frequent use is made of results ordinarily not met until a second calculus course is taken, although, where this is done, a statement to this effect is made or a reference given to material in the appendices. In several instances results are borrowed from more advanced differential equation theory, and in two cases reference is made to the theory of differential equations in the complex domain. This is presumably done for the purpose of presenting a less adumbrated picture of the theory than that usually given, but does not seem likely to contribute much to the student's understanding at this stage.

With the exceptions noted below, the topics treated are much the same as those in most elementary texts on differential equations. The manner in which they are treated, however, is more satisfying to the mathematically minded than is usually the case, and the omission of some of the special types of equations and the formalism for treating them makes for a more coherent book. The applications are principally relegated to a chapter on particle mechanics. This would seem to indicate that this text is aimed at the mathematics rather than the engineering student, and although the author does not say so, the general tenor of the book bears this out. The better engineering student will nevertheless find much of profit to him from the point of view adopted in this book.

Topics treated which are considered cursorily or not at all in most first texts in differential equations are: the second order equation with a regular singular point, the method of successive approximations, eigenvalues and eigenfunctions, expansion of functions in series of orthogonal functions, and, in particular, oscillation theorems. The treatment of eigenfunctions, eigenvalues, and the related problem of expansion of functions in series of orthogonal functions is, to the reviewer, too brief. The discussion on oscillation theorems is a distinct novelty in a book on this level.

The main purpose served by the referencing of advanced material and the appendices would seem to be that of stimulating the superior student (for whom the book is most suitable). Would it not then be suitable to mention the Lipschitz condition in connection with the existence theorem in Chapter 1? Similarly, would it not be stimulating to bring in the notion of vectors and matrices in the Chapter on systems of linear differential equations?

This text is one of the publisher's International Series in Pure and Applied Mathematics, and is a worthy addition thereto.

WILLIAM H. PELL

An introduction to relaxation methods. By F. S. Shaw. Dover Publications, Inc., New York, 1953. 396 pp. $5.50.

This book presents an easily read account of how to solve problems by the Relaxation Method. Although many papers and several books have been published during the 20 or so years that have elapsed since R. V. Southwell first exploited the Relaxation Method of solution none have succeeded as well as the present book in presenting to the reader so clear an account of what to do and how to do it.

A brief historical introduction is followed by chapters dealing with successively more difficult problems. A discussion of the solution of Linear Algebraic Equations is followed in succession by a treatment of Linear Ordinary Differential Equations, Laplace's and Poisson's equations for rectangular boundaries and curved boundaries, other more general second order partial differential equations, and fourth order partial differential equations. In Chapter VIII the author presents in detail the method of solving Eigenvalue problems, selecting illustrations from algebraic systems, and ordinary, and partial differential equations. In the last chapter (IX) a brief discussion is presented of problems involving an unknown boundary (a plastic torsion problem given as illustration), the precision of relaxation solutions and how to improve it on a given net, and some dimensional considerations. This last chapter covers too much too rapidly but is probably the best compromise between too long a book and no mention of these topics. It is perhaps permissible to hope that by the time the reader arrives at the last chapter a briefer treatment will serve to extend his knowledge into these additional areas.

The author disavows any attempt to discuss the physical significance (except in the historical introduction) of the equation being solved and of the method of solution itself. As a consequence an occasional discussion is more general and abstract than necessary. Unfortunately several such items appear in the second chapter in the discussion of finite difference approximations. Any reader finding some of the earlier sections difficult can safely skip over them to the chapters discussing specific problems since the latter are reasonably self contained and the problems solved are treated in detail. It is the reviewer's belief that a moderate amount of discussion of physical significance would have increased the readability and understandability of the book.

Anyone wishing to learn the relaxation method in order to solve problems in this way when appropriate would do well to study this book and complete the examples used by the author.

HOWARD W. EMMONS

Foundations of the nonlinear theory of elasticity. By V. V. Novozhilov. Translated from the first (1948) Russian edition by F. Bagemihl, H. Komm and W. Seidel. Graylock Press, Rochester, N. Y. 1953. vi + 233 pp. $4.00.

The author's object is to present an "... exposition of the theory of elasticity without any assumptions restricting the magnitude of elongations, displacements or angles of rotation." The first five chapters are concerned with the development of the kinematics of finite deformations and the derivation of "stress-strain" relations, equations of motion and boundary conditions. The remaining two chapters discuss the problem of elastic stability and the deformation of flexible bodies from the standpoint of the theory developed in the earlier chapters. Much of the content of these two chapters is new to the reviewer and they form significant contributions to the subjects they discuss.

Throughout the book, the formulae and equations are expressed in a notation which would have been more acceptable to Cauchy than it will be to the modern American reader.

Taken as a whole, the book cannot be regarded as a comprehensive account of the present state of development of finite elasticity theory. This cannot be held entirely to the account of the author, since a number of advances have been made after 1948, the year of publication of the original Russian edition. These have been critically discussed by C. Truesdell (J. Rat'l Mechanics and Analysis, 1, 125-300 (1952); 2, 593-616 (1953)).

R. S. Rivlin

Stochastic processes. By J. L. Doob, John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1953. vii + 654 pp. $10.00.

This book contains the first complete and detailed treatment of stochastic processes and is essential for anyone concerned with the mathematical basis of the subject.

The first two chapters deal with the necessary probability background and definition of the various classes of stochastic processes—Gaussian, Markov, stationary, martingales, etc. There follow detailed sections on processes with mutually independent, uncorrelated and orthogonal random variables, discrete and continuous parameter Markov processes, martingales, processes with independent and orthogonal increments and discrete and continuous parameter stationary processes. The final chapter treats the theory of linear least squares prediction.

There is a supplement at the end of the book in which the author outlines those aspects of measure theory most needed for the subject matter of the book. However, a fairly thorough knowledge of measure theory may nevertheless be considered to be an essential prerequisite for the profitable use of this work.

Leonard C. Maximon

Theory of matrices. By Sam Perlis. Addison-Wesley Press, Inc., Cambridge 42, Mass., 1952. xiv + 237 pp. $5.50.

The book gives a concise and clear treatment of the theory of matrices with emphasis on the basic ideas rather than particular applications. It can serve well as a text for a year course, especially if supplemented by applications of the material presented in the book. The approach of the text emphasizes strongly the algebra of matrices, with chapters on vector spaces, equivalence, rank and inverses, congruences and Hermitian congruences. Other topics considered include determinants, polynomials over a field and matrices with polynomial elements, similarity, characteristic roots and vectors, and linear transformations. Exercises are given at the end of each chapter.

The book provides a good background for anyone who is to specialize in either pure mathematics or physics and applied mathematics. Although some prior acquaintance with the basic notions of modern algebra would help the student using this text, the concepts are not presented in a form so abstract that the student without such background will feel lost.

Leonard C. Maximon