Theorem II has an important consequence. Form the characteristic equation for (7):

$$\sum_{k=0}^{n} \alpha_{2k} p^{2k} = 0. \tag{11}$$

Because this is an even function, if p_1 is a root, $-p_1$ is also a root. Hence, either (1) all the roots are pure imaginaries, or (2) there is at least one root with positive real part. If we interpret the system described by the Euler equation as a linear "filter" or decision rule, the behavior produced by application of the rule will be dynamically unstable.

The two theorems we have established may be interpreted as follows:

1. From Theorem I it appears that linear filters will have desirable qualities when the norm or "error" we wish to minimize is quadratic. If the norm is decidedly non-quadratic this suggests that we can improve filter performance by introducing appropriate non-linearities.

2. From Theorem II and the consequence derived from it, it appears that straightforward application of the calculus of variations to the filter design problem leads to the prescription of an unstable filter, and hence is not practicable.

It can easily be shown that point (2) is related to the fact that the Euler equations give only a necessary and not a sufficient condition for a minimum. Hence it does not follow that a path, $y(t)$, that satisfies (3) will thereby minimize (1). The specification of appropriate initial and terminal conditions to guarantee a *bona fide* minimum requires information about the future of the signal, and therefore, in the face of an incompletely predictable signal the method breaks down.

The difficulty cannot be avoided by minimizing in the domain of the Laplace transform of ϕ instead of in the time domain. For, in general, by Parseval's theorem:[2]

$$\int_{-\infty}^{\infty} [\phi(t)]^2 \, dt = \int_{-\infty}^{\infty} [\phi^*(p)]^2 \, dp, \tag{12}$$

where $\phi^*(p)$ is the transform of $\phi(t)$.

Hence, any function that is a stationary value for the right-hand side of (12) is simply the transform of the function that is a stationary value for the left-hand side.

It is beyond the scope of the present note to discuss newer methods that avoid this difficulty without reverting to cut-and-try procedures or the use of arbitrary figures of merit.

[2]Professor A. Charnes pointed out to me the relevance of Parseval's theorem for this problem. Minimization in the domain of the transform is essentially the method of Ritz for the "direct" solution of variational problems (Courant-Hilbert, I: p. 150).

BOOK REVIEWS

History of strength of materials with a brief account of the history of theory of elasticity and theory of structures. By S. P. Timoshenko. McGraw-Hill Book Co., Inc., New York, Toronto, London. 1953. x + 452 pp. $10.00.

Professor Timoshenko's original contributions to the subject of Strength of Materials are outstanding in contemporary literature; the same is true of his various books on Theoretical Mechanics. It is therefore with considerable interest that a new book by him is read by persons sharing his interests. This is perhaps even more true of the present book. The author has delayed writing it until he had the opportunity to devote an uninterrupted period of time in uncovering all the necessary historical material.

The book is founded upon lectures on the History of Strength of Materials which the author has given over the last twenty-five years to engineering students with some knowledge of the strength of materials and theory of structures. In writing this book the author has had in mind not only such students but also other people whose immediate interests are not specifically in this field. The histories of the three subjects, strength of materials, theory of elasticity, and theory of structures, are so closely interwoven as to be almost inseparable from each other. Much has been written in this historical field from various points of view. The author has followed the example of Saint-Venant in his "Historique Abrégé . . ." rather than that of Todhunter and Pearson in their "A History of the Elasticity and Strength of Materials". The result is an informal account of the history of strength of materials. Included are closely-related parts of the histories of the theory of elasticity and the theory of structures, purely mathematical developments being omitted from the former and purely technical developments from the latter. The presentation, with few deviations, is chronological. Seven chapters are devoted to the strength of materials, two to the theory of structures, and four to the theory of elasticity. These last chapters omit the mention of some well-known names, such as Volterra, and should not be read separately.

A brief introduction describes the empirical approach to the strength of materials current from Ancient Times right down to the Renaissance, and includes comments on the failure of early Renaissance engineers to appreciate da Vinci's work. The story proper begins with Galileo and unfolds through many equally famous names right down to the present time. The wealth of material may be judged from these statistics. About 675 names are mentioned in the text, and biographical material is given for nearly 80 of these. There are 245 figures, and about 40 of these are reproductions of likenesses of famous men. The reader can hardly fail to be stimulated by the biographical accounts often enlivened by anecdotes. The presentation inevitably presents a more difficult task as knowledge accumulates through the efforts of an ever-increasing body of research workers. This is especially true of the period 1900-50 dealt with in the final three chapters. This period is of course the one with which the author has been intimately concerned, and the developments are ably described. Here the coverage is extremely wide; thus fatigue of metals, approximate methods of solving elasticity problems, and the theory of ship structures are some examples of topics discussed. Many original references are cited throughout the book, and discussion is often given of the influence of great teachers. Comments are also made on the circumstances that either stimulated or retarded progress in the strength of materials in different countries over similar periods of time.

A stranger to the subject would not suspect, although outside the covers of this book it is well-recognized, that the genesis of the modern development of the strength of materials in the United States is largely attributable to Professor Timoshenko himself. In the text the author simply refers briefly to himself as a pupil of Prandtl; his name does not occur in the index, and is relegated exclusively to footnote references to original papers.

The reviewer agrees with the author that there is a place for formal lectures on such historical developments in any well-balanced engineering curriculum. For pedagogic reasons it is often not possible to follow the chronological development of a subject in a course of lectures to students. Engineering students must therefore often fail to appreciate the painstaking way in which our present knowledge of the strength of materials has been gradually built-up since Galileo's time. The current plea is a reflection of the need for the increased teaching of the history and philosophy of the natural sciences in undergraduate curricula. The reviewer regrets the author's strict preference for the presentation of the research worker's results in a manner directly of use to the engineer.

The reviewer has detected but few misprints and errors in statements, although some must be almost inevitable in a work of this kind. Kelvin's college is stated to be St. Peter's; the present name is Peterhouse.

H. G. Hopkins

Anniversary volume on applied mechanics dedicated to C. B. Biezeno by some of his friends and former students on the occasion of his sixty-fifth birthday, March 2, 1953. N. V. De Technische Uitgeverij H. Stam, Haarlem, 1953. 328 pp. and 2 fold-in plates. $5.60.

The beautifully made-up volume contains a brief biography and a list of publications of C. B. Biezeno and the following articles: W. Boomstra, Triangles équilatères inscrits dans une conique donnée; H. Bremekamp, Sur la théorie de Sturm-Liouville; Th. v. Kármán and G. Millan, The thermal theory of constant pressure deflagration; J. M. Burgers, Some remarks on detonation and deflagration problems in gases; C. Koning, Some interference problems; R. V. Southwell and Gillian Vaisey, A problem suggested by Saint-Venant's Mémoire sur la torsion des prismes; R. Grammel, Nichtlineare Schwingungen mit unendlich vielen Freiheitsgraden; R. J. Legger, The d'Alembert principle; A. D. de Pater, La stabilité d'un dicône se déplacant sur une voie en alignement droit; A. van der Neut, The local instability of compression members built up from flat plates; J. A. Haringx, Stresses in corrugated diaphragms; A. van Wijngaarden, Ut tensio sic vis; D. Dresden, Shrink-fit used to transmit a torque; W. T. Koiter, On partially plastic thick-walled tubes; G. G. J. Vreedenburgh and O. Stokman, Some new elements in the calculation of flat slab floors; J. P. Mazure, Statical problems in the code of practice for steel windows; J. J. Koch, The Laboratory for Applied Mechanics at the Technological University of Delft; R. G. Boiten, The design of diaphragms for pressure measuring devices based on the use of wire-electrical straingauges.

W. Prager

The dynamics and thermodynamics of compressible fluid flow. Volume I. By Ascher H. Shapiro. The Ronald Press Co., New York, 1953. xiii + 647 pp. $16.00.

Professor Ascher H. Shapiro of the Massachusetts Institute of Technology has undertaken a very important and useful task in writing a treatise on compressible fluids. As the title suggests, it is impossible in such a study to separate the dynamical from the thermodynamical point of view and this accounts for both the difficulty and the interest of this branch of mechanics of continua. The subject dealt with in this book is of great practical importance. At the present time, aeronautical engineers, mechanical engineers, chemical engineers, applied physicists and applied mathematicians frequently need to investigate and to apply the theory of compressible fluids. They will find in this book material of much interest. The author, a professor of mechanical engineering, has given a number of practical applications to various branches of engineering science in each chapter. He has planned this treatise in two volumes, the whole divided into 8 parts and 28 chapters. So far only the first volume containing the first four parts and part of the fifth has been published. The size of the volume (647 pp.) gives some indication of the amount of information contained in it.

Part I sets forth the basic concepts and principles from which the remainder of the book proceeds. The first chapter—foundations of fluid dynamics—contains definitions of a fluid and of the continuum properties (density, pressure, viscosity) and discussions of the conservation of mass and momentum theorems applied to a control volume. In the second chapter—foundations of thermodynamics—the author reviews the most important definitions and concepts of thermodynamics and states the two laws. The second law is introduced by means of the definition of the thermal efficiency of heat engines and is related to the impossibility of obtaining positive work from a system passing through a complete cycle while exchanging heat with only a single source. The thermodynamic properties of the continuum are then defined (internal energy, enthalpy, etc.) and the theorem of conservation of energy is obtained by application of the first law to a control volume. The chapter ends with the basic formulae for a perfect gas. In the third chapter which concludes Part I are introduced the principal ideas concerning compressible fluids. The speed of sound is defined by its physical significance (velocity of propagation of a plane pressure pulse) and the physical differences between subsonic and supersonic flows are then emphasized by statement of the three Kármán rules for supersonic flow and are discussed with the help of various schlieren and interferometer photographs. This chapter also includes a study of the similarity parameters and a survey of the optical methods which were used in the experiments (interferometer, shadowgraph and schlieren methods).

Part II is devoted to "one-dimensional flow"; although it is restricted to the steady case (the unsteady case will be treated in Volume II) it is almost 200 pages long. In the first chapter of this part the fluid is assumed to be continuous and isentropic, and a complete discussion for converging or converging-diverging nozzles and supersonic diffusers is given, including a special study of the choking effect. In the next chapter, the theory of normal shock waves and the basic formulae relating to this theory are found. Special attention is given to the physical explanation of the formation of shock waves, and some discussion is given of the thickness of these shocks. The ideas developed in this chapter are applied to nozzles, supersonic diffusers and the supersonic Pitot tube. Then a chapter is devoted to the detailed study of flow in constant-area ducts with friction. It is shown how friction can produce choking even though the duct area remains constant. Information on isothermal flow in long ducts is also given. The following chapter is devoted to flow in ducts with heating or cooling, i.e., to the case in which, though the area is kept constant and the friction is neglected, the stagnation enthalpy may be changed for various reasons. There exist many problems in engineering in which such a situation arises. Finally, in the last chapter of Part II, more general problems are discussed in which two or more of the previously mentioned phenomena may appear.

Part III contains an introduction to flow in two and three dimensions. It gives the basic definitions and theorems: circulation, rotation and their physical significance; Euler's equation; Kelvin's and Bernoulli's theorems; continuity equation; equation of velocity potential; etc.

Part IV deals with subsonic flow. The author begins with the so-called linearized theory in which small perturbations are superposed on a uniform flow. After the classical example of flow past a wave-shaped wall, the fundamental similarity rules are given. These allow us to relate the subsonic compressible flow past a certain profile to the incompressible flow past a second profile derived from the first by means of an affine transformation. The famous Prandtl-Glauert rule is discussed and many applications are made, in particular to wind tunnel corrections. The next chapter is devoted to the hodograph method for two-dimensional subsonic flow, which in fact allows us to linearize the problem without introducing new simplifications. The tangent-gas approximation of Chaplygin is discussed and applications lead to the Kármán pressure correction formula. In the following chapter miscellaneous methods and results are given—method of expansion in series in terms of the Mach number (Rayleigh-Janzen), method of expansion in series in terms of a shape parameter, relaxation method, etc. Comparisons of theoretical results with experiments are made. The last chapter of Part IV is devoted to three-dimensional subsonic flow—flow past spheres, ellipsoids, bodies of revolution, wings of finite span and sweptback wings. The extension of the similarity rules is made with special care.

Part V deals with two-dimensional supersonic flow (only the first three chapters of this part are included in Volume I). The linearized theory is treated in great detail in order to facilitate its application to practical problems and to provide a good introduction to the following chapter which is devoted to the method of characteristics. This last method is presented as a natural generalization of the basic results of linearized theory. Application of the method is made to the jet problem and to the design of a supersonic wind tunnel; the case in which rotation is present is also considered. In the last chapter, the theory of oblique shock waves is developed (shock equations, reflection and interaction of shocks). Many examples of flow containing shocks are found, together with experimental results. The book ends with an appendix devoted to a mathematical formulation of the theory of characteristics and with some useful tables (standard atmosphere, isentropic flow, normal shock, one-dimensional flow with friction or with change in stagnation enthalpy, hodograph characteristic functions for supersonic flow).

Each chapter in the book begins with a clear introduction advising the reader of the aim of the study; it is followed by a nomenclature list explaining the significance of the symbols used in the chapter. After each article, "working formulae and charts" are given which can be consulted when a specific application is planned. At the end of each chapter there are several well selected problems most of which are drawn from practical situations arising in engineering science. A selected bibliography is appended to each chapter.

On the whole, the material contained in this book is well selected and clearly presented. The second part which is devoted to one-dimensional flow is particularly well developed. Because the book is written primarily for engineers, the viewpoint of physics and engineering predominates, emphasis being placed on results rather than on proofs. For the most part, the author attempts to make each chapter a self-contained unit with the result that some formulae or theorems are proved more than once. The order is not always the most logical. For example, the equation of continuity for two- and three-dimensional flows is given on p. 283—after Euler's equation, Bernoulli's theorem, Kelvin's theorem and Crocco's theorem;

the reviewer suspects that this equation may have been used implicitly before its formal presentation. Moreover, in discussing the linearized theory, the linearization of the boundary conditions does not appear to be carried out systematically. Also, it is not immediately clear why the author claims that the first of the three similarity laws for two-dimensional subsonic flow given on p. 323 is the most exact. However, only a few such criticisms can be made.

The general level of the book is perhaps too high for it to be of value as a classical textbook but those who teach the subject of compressible fluid flow will undoubtedly find it immensely useful. Some years ago, Courant and Friedrich's classical book provided a mathematical approach to the theory of compressible fluids; this new book provides a more physical and technical approach to the same subject. Together, these two extensive treatises constitute a firm basis for the study of this subject.

P. GERMAIN

Methods of theoretical physics. By Philip M. Morse and Herman Feshbach. Volumes I and II. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1953. xxii + 997 pp. (Vol. I), xviii + 980 pp. (Vol. II). $30.00 set.

In more than 1900 pages and two volumes the authors have packed a formidable amount of material on the subject of mathematical methods in physics. This treatise is largely concerned with fields and boundary value problems of all types. The discussion of techniques in connection with field problems naturally leads to the newer aspects of perturbation theory and variational methods; the reader's attention is called particularly to Chapters 9 and 11 in this connection (see topics listed below). The section on Green's functions is really excellent as are many other sections of this work. Anyone with strong interest in the use of advanced mathematical methods will find this work extremely valuable. One of its advantages is that it is rather complete and self-contained.

The advantages mentioned become disadvantages when one considers using this treatise as a text. On the other hand the work was the outgrowth of a course given for quite a few years by the authors at M. I. T.

The chapter headings are listed below together with section headings within each chapter and the space devoted to each chapter. Chapter 1 (117 pages) Types of Fields: Scalar Fields, Vector Fields, Curvilinear Coordinates, The Differential Operator Del, Vector and Tensor Formalism, Dyadics and Other Vector Operators, The Lorentz Transformation, Four-Vectors, Spinors. Chapter 2 (155 pages) Equations Governing Fields: The Flexible String, Waves in an Elastic Medium, Motion of Fluids, Diffusion and Other Percolative Fluid Motion, The Electromagnetic Field, Quantum Mechanics. Chapter 3 (73 pages) Fields and the Variational Principles: The Variational Integral and Euler Equations, Hamilton's Principle and Classical Dynamics, Scalar Fields, Vector Fields. Chapter 4 (133 pages) Functions of a Complex Variable: Complex Numbers and Variables, Analytic Functions, Derivatives of Analytic Functions, Multivalued Functions, Calculus of Residues, Asymptotic Series: Method of Steepest Descent, Conformal Mapping, Fourier Integrals. Chapter 5 (164 pages) Ordinary Differential Equations: Separable Coordinates, General Properties and Series Solutions, Integral Representations, Table of Separable Coordinates in Three Dimensions, Second-Order Differential Equations and Their Solutions. Chapter 6 (115 pages) Boundary Conditions and Eigenfunctions: Types of Equations and of Boundary Conditions, Difference Equations and Boundary Conditions, Eigenfunctions and Their Use, Table of Useful Eigenfunctions and Their Properties, Eigenfunctions by the Factorization Method. Chapter 7 (100 pages) Green's Functions: Source Points and Boundary Points, Green's Functions for Steady Waves, Green's Function for the Scalar Wave Equation, Green's Function for Diffusion, Green's Function in Abstract Vector Form, Table of Green's Functions. Chapter 8 (100 pages) Integral Equations: Integral Equations of Physics, General Properties of Integral Equations, Solution of Fredholm Equations of the First Kind, Solution of Integral Equations of the Second Kind, Fourier Transforms and Integral Equations, Tables of Integral Equations and Their Solutions.

VOLUME II. Chapter 9 (172 pages) Approximate Methods: Perturbation Methods, Boundary Perturbations, Perturbation Methods for Scattering and Diffraction, Variational Methods, Tabulation of Approximate Methods. Chapter 10 (158 pages) Solutions of Laplace's and Poisson's Equations: Solutions in Two Dimensions, Complex Variables and the Two-Dimensional Laplace Equation, Solutions for Three Dimensions, Trigonometric and Hyperbolic Functions, Bessel Functions, Legendre Functions.

Chapter 11 (252 pages) The Wave Equation: Wave Motion on One Space Dimension, Waves in Two Dimensions, Waves in Three Space Dimensions, Integral and Variational Techniques, Cylindrical Bessel Functions, Weber Functions, Mathieu Functions, Spherical Bessel Functions, Spherical Bessel Functions, Spheroidal Functions, Short Table of Laplace Transforms. Chapter 12 (173 pages) Diffusion, Wave Mechanics: Solutions of the Diffusion Equation, Distribution Functions for Diffusion Problems Solutions of Schroedinger's Equation, Jacobi Polynomials, Semi-cylindrical Functions. Chapter 13 (143 pages) Vector Fields: Vector Boundary Conditions, Eigenfunctions and Green's Functions, Static and Steady-state Solutions, Vector Wave Solutions, Table of Spherical Vector Harmonics.

With each chapter a set of problems is offered.

The seventeen tables at the end of Volume II have the following headings: Trigonometric and Hyperbolic Functions; Hyperbolic Tangent of Complex Quantity; Inverse Hyperbolic Tangent of Complex Quantity; Logarithmic and Inverse Hyperbolic Functions; Spherical Harmonic Functions; Legendre Functions for Large Arguments; Legendre Functions for Imaginary Arguments; Legendre Functions of Half-integral Degree; Bessel Functions for Cylindrical Coordinates; Hyperbolic Bessel Functions; Bessel Functions for Spherical Coordinates; Legendre Functions for Spherical Coordinates; Amplitudes and Phases for Cylindrical Bessel Functions; Amplitudes and Phases for Spherical Bessel Functions; Periodic Mathieu Functions; Normalizing Constants for Periodic Mathieu Functions and Limiting Values of Radial Mathieu Functions.

Rohn Truell

The aim and structure of physical theory. By Pierre Duhem. Translated from the French by Philip P. Wiener. Princeton University Press, Princeton, N. J., 1954. xxii + 344 pp. $6.00.

This book is a translation of the original work of the author entitled *La Théorie Physique: Son Object, Sa Structure* (Marcel Rivière et Cie, Paris, 1914) and is the first of Duhem's writings on the philosophy of science to appear in the English language. Actually the French version from which the translation has been made is the second edition of a still earlier work, published in 1906.

Pierre Duhem (1861-1916) was a well known French thermodynamical physicist of the late nineteenth century and made notable contributions to the application of thermodynamics to physical chemistry in developing the ideas of Gibbs, which indeed he was among the first to popularize in France. At the same time he took a profound interest in the history and philosophy of physical science. Though somewhat overshadowed in this field by his more brilliant and versatile contemporary, Henri Poincaré, he nevertheless established a well justified reputation as an incisive critic of physical methodology along with men like Mach, Helmholtz, Clifford and Pearson.

The methodology of physics has been the subject of serious investigation during the past half century and to a certain extent this inevitably serves to date the treatise of Duhem. Nevertheless, it appears he still has something helpful to say to his successors and his essentially pragmatic, positivistic viewpoint of the deductive character of physical theory will prove very stimulating to the present day reader.

The author's treatment is divided into the two main sections reflected in the title. In the first he discusses the relation between physical theory and metaphysics, physics and the "natural" classification of experience, the history of the development of theories, and the relation between abstract theories and mechanical models. The second part is devoted to an analysis of the structure of a physical theory from the standpoints of the notions of quantity and quality, the use of mathematics, the relation of theory to experiment, and the choice of hypotheses. There are two appendices, the first of which on "The Physics of a Believer", though not at all a detailed discussion of the relation between physical science and religion presents some interesting side lights on this perennial question.

The clarity and vividness of Duhem's style have been very ably caught and preserved by the translator, and the thoughtful reader will never mistake the author's meaning. Though in general rather judicious in his estimates, he was sufficiently strong-minded to develop a few obsessions which give a personal flavor to his work. For example, Duhem firmly believed that English physicists approach the building of physical theories in quite different fashion from their Continental colleagues. His arguments are persuasive but will scarcely stand up in the light of more recent history. Moreover his fervent optimism that physical theory progresses by steady evolution to an ideal end, the "natural classification" of experience, apparently failed to foresee the unparalleled creation of new experience in modern physical laboratories which is giving the present day theorist nightmares. Of course we can all agree with Duhem that physical theories will continue to be constructed and that some of them will be successful, but it seems undesirable to hamper ourselves by any preconceived metaphysical notion of an "ideal end".

This book can be recommended in unqualified terms to all who are interested in the logical structure of physical science.

R. B. Lindsay

Applied elasticity. By Chi-Teh Wang. McGraw-Hill Book Co., New York, Toronto, London, 1953. ix + 357 pp. $8.00.

This elasticity text is clearly intended for the use of the advanced engineering student, as is stated in the preface. The classical theory of elasticity is well presented in a manner that presupposes no extensive knowledge of advanced calculus. Where necessary, an attempt is made to introduce whatever higher mathematics is needed. For the most part, this attempt is successful so far as both rigor and technique of solution is concerned, keeping in mind that the main purpose is the latter. However, the introduction to the theory of complex variables is, I believe, much too hurried and sketchy; and at some points certainly does not possess the rigor the author hopes is not sacrificed by concentration on methods of solution.

Chapters I and II develop the concepts of stress and strain, respectively, in straightforward fashion. Chapter III is concerned with stress-strain relations and the concept of strain energy. Chapter IV sets up the plane-stress and plain-strain problem in terms of Airy's stress function and solves problems by choosing solutions to the biharmonic equation. Rotating disks and thermal stresses are also discussed. Chapter V treats the torsion problem in terms of the warping function. The stress function is introduced to make use of the membrane analogy in the solution of the torsion problem of thin open sections and thin tubes.

Since one of the laudable purposes of the book is to provide techniques of solution where exact analytic solutions are intractable, much space is devoted to numerical methods. Chapter VI discusses finite-difference approximations and relaxation methods, with application to the torsion problem as an example. In Chapter VII, energy principles and variational methods are discussed analytically. The results of Chapters VI and VII are used to obtain numerical solutions of buckling problems in Chapter X, the analytic formulation and solution of which are discussed in Chapter IX.

Chapter VIII uses complex variables to obtain solutions to torsion problems and plane problems.

Chapters XI and XII discuss bending and buckling of thin plates, and the theory of thin shells, respectively. The latter is preceded by a comparatively extensive discussion of the differential geometery of a surface.

The book is clearly written, easy to read, and on the whole does, I believe, fulfill the author's purpose "... to provide the student with the necessary fundamental knowledge of the theory ... to acquaint him with the most useful analytical and numerical methods (of solution)."

Harry J. Weiss

Stability theory of differential equations. By Richard Bellman. McGraw-Hill Book Co., New York, Toronto, London, 1953. xiii + 166 pp. $5.50.

This attractive little book gives an original and useful survey of various aspects of behavior, as $t \to \infty$, of solutions of systems of ordinary differential equations. After an introductory chapter introducing the convenient vector-matrix-norm notation for treating such systems, the author defines the sense in which he uses the word stability, as follows:

"*Definition.* The solutions of

$$\frac{dy}{dt} = A(t)y \tag{7}$$

are stable with respect to a property P and perturbations $B(t)$ of type T if the solution of

$$\frac{dz}{dt} = (A(t) + B(t))z \tag{8}$$

also possess property P. If this is not true, the solutions of (7) are said to be unstable with respect to property P under perturbations of type T."

This definition can be applied to many problems, concerning non-linear (when rephrased) as well as linear differential equations. In many cases (e.g., if $A(t)$ is constant), the pattern followed by the author consists in showing that the qualitative asymptotic behavior of known special solutions is unaffected by "small" perturbations of the coefficients—Liapounoff's famous stability theorem is a special case. In other cases, striking counterexamples to plausible guesses are given. In still others, ingenious isolated results are "rescued from oblivion" by displaying them in easily accessible form (e.g., the Fowler-Emden equation, to which Chap. VII is devoted).

The reader who is looking for results of the type just described should first consult Bellman's book. If he does not find them there, he will probably locate them by consulting the well-organized bibliography.

On the other hand, there are a number of important topics, whose omission is not suggested by the somewhat misleading title of this book. For example, the basic Routh-Hurwitz stability criteria are nowhere mentioned, being replaced throughout by the phrase "Let all solutions of $dy/dt = Ay$ tend to zero as $t \to \infty$." Again, no discussion is given of topological arguments or of the stability of limit cycles in the Rayleigh-van der Pol equation. In treating the simple second-order equation $(ku')' + ru = 0$, deeper methods (saddle-point method, asymptotic expansions in λ, etc.) are not explained.

Perhaps because of his exclusive concern with a single problem and with formal methods, the author achieves an admirable clarity and uniformity of style. The result is a very useful survey of the qualitative asymptotic theory of ordinary differential equations.

GARRETT BIRKHOFF

Handbook of elliptic integrals for engineers and physicists. By P. F. Byrd and M. T. Friedman. Springer Verlag, Berlin-Gottingen-Heidelberg, 1954. xii + 355 pp. $8.58 (paperbound), $9.44 (clothbound).

This useful handbook contains over 3000 integrals and formulas and 28 pages of numerical tables designed to aid engineers and physicists in the evaluation of elliptic integrals that occur in practical problems. The explanatory material is written on an elementary level and does not require previous acquaintance with elliptic integrals or functions. The notations of Legendre and Jacobi are used in preference to that of Weierstrass. The typographical presentation, of particular importance in a work of this character, is excellent. A considerable saving of space has been achieved by the use of 9-point type in numerators and denominators of fractions. The resulting compactness of formulas greatly facilitaties the use of the handbook.

W. PRAGER

Proceedings of the Eastern Joint Computer Conference. The Institute of Radio Engineers, Inc., New York, 1954. 125 pp. $3.00.

This report contains the Papers and Discussions presented at the Joint I.R.E.-A.I.E.E.-A.C.M. Computer Conference held in Washington, D. C., in December 1953. The Conference was the third of its kind and the theme chosen for it was, "Information Processing Systems—Reliability and Requirements." The papers can be classified in three groups: (1) Statements of data processing and calculation requirements. Descriptions of requirements for the Life Insurance Business, Numerical Weather Prediction, Air Traffic Control and Linear Real-Time Systems are given. (2) Papers describing the operational and performance characteristics of existing machines. Operating experiences with the OARAC, the Los Alamos 701, a large REAC installation, the UNIVAC, the SEAC and the ORDVAC are described. Two papers deal with performance tests, the "National Bureau of Standards Performance tests" and the "Acceptance Tests for the Raythorn Hurricane Computer." Two further papers can be placed in this group. They are descriptions of the MIT Magnetic-Core Memory and the Electrostatic Memory of ILLIAC. (3) The third group of papers is descriptive of work carried out in connection with the reliability of components; Magnetic Tape, Electrolytic Capacitors, Resistors, Electron Tubes, and other electronic units are discussed.

Two very interesting papers, one by H. H. Goldstine and the other by J. W. Mauchly, do not conveniently fall into one of the above catagories. Goldstine gives a brief resumé of von Neumann's work on logical design and Mauchly discusses the advantages of built-in checking.

The report as a whole is well printed and readable.

J. Foulkes

Linear operators spectral theory and some other applications. By Richard G. Cooke. MacMillan and Co., Ltd., London, 1953. xii + 454 pp. $10.00.

The topics presented in this book range over the spectral theory of operators in Hilbert space, an introduction to quantum mechanics, a valuable discussion of Banach algebras and certain ideas on sequence spaces.

The Hilbert space theory is introduced by a discussion of $\mathfrak{l}_2$ and of abstract Hilbert space. The usual elementary operator theory is covered and the spectral theory due to von Neumann using the Cayley transform. The Stone theory of spectra and resolvent, a development of the Hallinger theory due to Lengyel, the Cooper theory based on integrating the differential equation for the Stone group, and the Riesz-Lorch proof are given.

The presentation tends to be non-geometrical, with a preference for analytical and matrix methods. This permits a simple and direct connection with quantum mechanics. The harmonic oscillator is treated both in the Heisenberg and Schrödinger formulation and perturbation theory is given also in matrix terminology.

The analytic and non-geometric viewpoint is further developed in Chapter 6, which is entitled "Projective convergence and limit in matrix spaces and rings." The basis of this chapter is certain definitions of limits for sequences, which are given in another book of the author, i.e., R. G. Cooke, "Infinite matrices and sequence spaces," MacMillan and Co., (1950), and not repeated in the present work. A number of topologies for sequence spaces are introduced and these in turn lead to topologies for the matrices associated with linear transformations between sequence spaces. The work of H. S. Allen is compared with that of Köthe and Toeplitz and Banach's results, specialized to sequence spaces. Greater generality is claimed for Allen's notions, but they apply, of course, only to the sequential situation. However, the discussion of Chapter 6 seems to the reviewer to be of a far more specialized character than that of the other chapters and of less general interest.

Chapter 7 is on Banach algebras and is quite fascinating. The initial sections contain Zron's Lemma, the Hahn Banach Theorem, a discussion of weak convergence of linear functionals and an algebraic discussion of maximal ideals. The objective is the development of the Gelfand theory in the case of commutative Banach algebras. This theory is applied to yield certain results of Wiener's on the convergence of the Fourier series and the Fourier integral for a reciprocal of a function. It is also used as a tool in the investigations of topological spaces by means of the set of continuous functions defined on them. This chapter also contains a discussion of Radon measure and a proof of Wiener's theorem on the density of the translations of a function in L.

It should be clear from the above that this represents on the whole an interesting and useful collection of topics in the mathematical theory of operators. Most graduate students should be able to read the book without difficulty. A useful set of references for further reading is also given.

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