# BOOK REVIEWS

*Table of binomial coefficients.* Royal Society Mathematical Tables, Volume 3. Under the Editorship of J. C. P. Miller. Cambridge University Press, 1954. viii + 162 pp. $6.50.

This is the most extensive table of binomial coefficients yet published. The coefficients $\binom{n}{r}$ are given exactly for all integer $r(r \leq n/2)$ and $n$ up to $n = 200$. In addition the table includes larger values of $n$ up to $n = 5000$ but for a limited range of $r$, to $r = 12$ for $200 < n < 500$, to $r = 11$ for $500 < n < 1000$, to $r = 5$ for $1000 < n < 2000$ and to $r = 3$ for $2000 < n < 5000$.

The tables are well arranged especially considering that some coefficients require only one digit whereas some others require almost 60 digits.

The largest previous table of binomial coefficients (by Peters and Stein 1922) was complete only to $n = 60$.

G. NEWELL

*Linear transient analysis.* Volume I. By Ernst Weber. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1954. xiv + 348 pp. $7.50.

At a time when the use and misuse of the Laplace transform is enjoying great popularity, this careful and basic study and critical comparison of a variety of methods for the analysis of transient phenomena is particularly welcome. After a critical examination of the circuit concept as derived from the electromagnetic field relations in the opening chapter, the formulation and classical solution of the integro-differential equations of linear networks are reviewed. One chapter is devoted to Heaviside's operational calculus, with modifications given it by H. Jeffreys which allow a rigorous and systematic treatment. The longest chapter deals with the Laplace transform and its use in electric circuit problems. The expression for the inverse transform is here obtained from Cauchy's integral formula which is developed in an 11-page appendix. The last chapter discusses the frequency spectrum concept, Fourier integrals, and the Fourier transform method. A very good discussion of electrical, mechanical, and thermal analogues is given in one chapter. Six appendices include some of the mathematical background needed in the text as well as a general bibliography. The text is exceptionally well documented by means of footnote references and other reference lists.

This book is intended as a text for seniors or first-year graduate students in electrical engineering. The applications in this volume are essentially limited to two-terminal, lumped-parameter networks; two terminal-pair networks, filters, and transmission lines are to be treated in a second volume. The broad coverage of methods of analysis, the extensive use of examples to introduce or illustrate the methods, the emphasis on physical concepts, and the clear exposition all contribute to make this an excellent textbook.

R. J. SCHWARZ

*Limit distributions for sums of independent random variables.* By B. V. Gnedenko and A. N. Kolmogorov. Translated by K. L. Chung. Addison-Wesley Publishing Co., Inc., Cambridge 42, Massachusetts, 1954. ix + 264 pp. $7.50.

This translation of a recent Russian book will be of most value to advanced students and specialists in the mathematics of probability theory. Much of the book covers material that has previously appeared only in periodicals, many in Russian. Although the first few chapters give a general review of the fundamentals of probability theory (in terms of measure theory), most of the book consists of a rather exhaustive and rigorous study of the restricted field described by the title.

G. NEWELL

*(Continued on p. 340)*

# BOOK REVIEWS

(*Continued from p. 270*)

*Selected papers in statistics and probability.* By Abraham Wald. McGraw-Hill Book Co., New York, Toronto, London, 1955. ix + 702 pp. $8.00.

This book contains reproductions of most papers written by the late Abraham Wald on the subject of statistics and probability except those dealing with topics already included in other books written by Wald. It also contains a short biography, a section describing some of the works of Wald particularly in regard to their connection with the works of other authors, a chronological listing of all his publications and one paper that has not previously been published. The book is a fitting memorial to an exceedingly productive mathematician.

GORDON NEWELL

*A short table for Bessel functions of integer orders and large arguments.* Prepared on behalf of the Mathematical Tables Committee of The Royal Society by L. Fox. University Press, Cambridge, 1954. 28 pp. $1.25.

Tables for computation of the functions $J_n(x)$, $Y_n(x)$, $I_n(x)$, $K_n(x)$ for integral values of $n$ from 0 to 20 and for values of $x$ covering the range $20 \leq x \leq \infty$. The tabulation gives the numerical values of auxiliary functions in terms of the auxiliary argument $1/x$. These can now be used to readily compute the required Bessel functions by means of their asymptotic expansions.

G. W. MORGAN

*Decision processes.* Edited by R. M. Thrall, C. H. Coombs, R. L. Davis. John Wiley & Sons, Inc., New York, Chapman & Hall, Ltd., London, 1954. viii + 332 pp. $5.00.

This collection of papers arises from a seminar, held in the summer of 1952, in which a group of "mathematicians, statisticians, psychologists, economists, and philosophers" studied various aspects of "decision processes." The range of topics treated in this seminar is given by the headings of the four groupings under which the papers are published: Individual and Social Choice, Learning Theory, Theory and Applications of Utility, Experimental Studies. The approaches used include an exposition of some of the elements of real variables, the comparison of stochastic and game theoretic models for individual and group behavior against experimental evidence, the development of stochastic models for learning and the comparison of criteria for making decisions.

Briefly, the purpose of most of the authors seems to be to go beyond the purely elementary descriptive approach, e.g., systems of linear equations, etc., usually found in the social sciences and develop more subtle models for the relationships between the various aspects of observed or potentially observable systems. The problem of bridging the gap between observation and model in the sense of Wald's statistical decision theory is treated only incidentally.

The papers are generally at a high and thoughtful level but the reviewer noticed no new results which appeared to be important from a purely mathematical point of view. Those whose work brings them in contact with theoretically inclined social scientists will, however, find this book useful in giving them a feel for the state of knowledge in learning and organization theory and a starting point for their own research.

ISADORE BLUMEN

*Mathematics in type.* The William Byrd Press, Inc., Richmond, Virginia, 1954. xii + 58 pp. $3.00 (50% discount to staff members of educational institutions).

Most mathematicians and scientists are aware of the appalling cost of mathematical composition, but few have sufficient insight into the mechanics of modern typesetting to realize how much of this

cost is caused by the use of typographically inconvenient notation. The same mathematical idea can, in general, be expressed by any one of a number of systems of notation. All too often an author's choice of one of these is based on considerations involving the ease of writing rather than that of typesetting. Thus, "economic forces are set in motion which tend to limit the ultimate market for books and to reduce the number of pages published in journals". This situation is not likely to improve until authors of mathematical papers and books acquire a rudimentary knowledge of the mechanics of setting mathematical type. The present booklet contains a clear discussion of the difficulties of monotype composition of mathematical formulas; it should be required reading for all authors using more than the most elementary mathematical symbols.

W. PRAGER

*Optics.* By Arnold Sommerfeld. Academic Press, Inc., New York, 1954. xi + 383 pp. $6.80.

This is an English translation by O. Laporte and P. A. Moldauer of Volume IV of Lectures on Theoretical Physics by Sommerfeld. This text on physical optics is divided into six chapters, the first of which is concerned with the reflection and refraction of light and includes the usual treatment of the Fresnel formula, total reflection and metallic reflection. The first chapter also includes an unusually concise treatment of reflection and refraction and interference behavior in thin films and thick plates including the Lummer-Gehrke plate, the Fabry-Perot interferometer. Chapter two is concerned with the optics of moving media and sources. In particular, the Doppler effect, the Michaelson experiment, reflection by a moving mirror, and the effect of the earth's rotation on the propagation of light are some of the topics treated. Chapter three is concerned with the theory of dispersion. This chapter contains the main features of classical theory including discussion of magnetic rotation of polarization plane and the normal Zeeman effect. Phase velocity, signal velocity, group velocity and energy transport are discussed. An outline of the wave-mechanical theory of dispersion is given. Chapter four deals with crystal optics and develops from Maxwell's equations the plane wave behavior in optical crystals. The usual ray surface and wave normal surfaces are found together with their relation to the optic axes. The optical symmetry of crystals is discussed, as well as the rotation of the plane of polarization of a wave propagation. Chapter five and much of chapter six are concerned with the theory of diffraction. In addition to a rather complete discussion of what might be called pre-war diffraction theory, there is in chapter six some discussion of the newer theory of Bethe and Levine and Schwinger. Discussion of a reformulation of Kirchoff's solution of the diffraction problem occupies part of chapter six. There is a very interesting section in Cerenkov radiation where the idea of a Mach cone for a Cerenkov electron is developed. The electromagnetic field is zero outside the Mach cone but exists everywhere in the interior of the cone. When dispersion is taken into account, the difference between the phase and group velocity affects the size of the Mach cone.

Finally, there is a section in chapter six on geometrical optics and the existence of the eikonal. Discussion of curved light rays and rectilinear ray bundles (lens formula) is also included.

It is undoubtedly superfluous to say that this is one of the better texts on physical optics. Looking for mistakes or careless work in something written by Sommerfeld is like looking for a bad meal in France. This book is not a treatise such as Born's or Fosterling's or Volume 20 of the Handbuch der Physik; it is an up-to-date combination of intermediate and advanced textbook on physical optics.

ROHN TRUELL

*Nomographie.* By M. W. Pentkowski. Akademie Verlag, Berlin, 1953. xv + 268 pp. $3.58.

In refreshing contrast to other recent texts in this field, nearly one half of this book is devoted to the theoretical foundations of nomography. The first two chapters treat alignment charts involving three variables. Following Soreau, the concept of the nomographic order is used as a convenient basis for the systematic discussion of these charts. Subsequent chapters deal with network charts, alignment

charts containing networks, composite charts, and special methods of nomographic representation. The second half of the work is devoted to practical methods of constructing various types of charts, special attention being given to the projective transformation of charts. The clear exposition of nomographic theory and practice is supported by numerous skillfully drawn and well-reproduced figures.

W. PRAGER

*Economic activity analysis.* Edited by O. Morgenstern. John Wiley & Sons, New York, 1954. xviii + 554 pp. $6.75.

The thirteen papers of the volume are grouped in three parts labelled Economic Properties of the Input-Output System, Mathematical Properties of Linear Economic Systems, and Meta-Economics. Part I contains two expository papers on models of general economic equilibrium (by J. Balderston) and on the input-output system (by O. Eckstein), which will be particularly welcome to the applied mathematician unfamiliar with economic theory. The remaining papers of Part I are concerned with the role of aggregation in economic models and the treatment of foreign and domestic trade and transportation charges. Part II consists of seven papers of primarily mathematical contents; most of these are concerned with properties of certain types of matrices that occur in linear models of economic systems. For example, the elements of a Minkowski-Leontieff matrix are non-negative, and the sum of the elements in any column cannot exceed unity. Part III contains a translation of slightly revised versions of two papers by K. Menger on the logic of the laws of return. These were published in Vienna in 1936, but have been largely ignored in the English-American literature on economics. A final paper by O. Morgenstern is concerned with experiment and large-scale computation in economics.

W. PRAGER

*Gas dynamics of thin bodies.* By F. I. Frankl and E. A. Karpovich. Interscience Publishers, Inc., New York and London, 1954. viii + 175 pp. $5.75.

This monograph, which is a translation of a Russian text first published in 1948, presents a mathematical treatment of the linearized theory of compressible flow of a perfect, inviscid, and non-heat conducting fluid. Throughout the work only irrotational flows are considered so that the presence of a velocity potential can be assumed. Both subsonic and supersonic steady as well as unsteady flows are discussed.

The book is divided into five chapters, the first of which contains a rather nationalistic historical survey of the subject, along with a formulation of the three-dimensional unsteady problem in terms of the wave equation. The idea of the retarded potential (supersonic source) as the fundamental solution is introduced. Chapter II considers the solution to the steady compressible subsonic and supersonic flow around slender bodies of revolution by the use of the axially distributed source, and in the case of angle of attack, the distributed doublet. The unsymmetrical body which is not too different from a body of revolution, as well as the case of axially accelerated motion, are also briefly treated. The third chapter is concerned with the steady motion of a wing, and in it is to be found a review of the problem of the infinite span wing at subsonic and supersonic speeds, as well as the lifting line theory for finite span wings in subsonic flow. The remainder of the chapter is devoted to finite span wings in supersonic flow. The general drag and lift problem for a wing of zero thickness in supersonic flow is solved by the method of reducing the solution to a sequence of Abel integral equations. After this general treatment, the acceleration potential is applied to the problem of the lifting trapezoidal wing at supersonic speeds. Finally, the concept of the supersonic source is utilized to discuss the flow around symmetric wings with thickness and large sweep. Chapter IV examines the problem of the unsteady motion of airfoils and wings executing small harmonic oscillations. A solution for the oscillating airfoil in subsonic flow is given, and by the introduction of the pulsating supersonic source, the two- and three- dimensional problem at supersonic speeds is also considered. The remainder of this chapter is devoted to an account

of the linearized theory of high speed propellers based on the lifting line theory. In the final chapter the linearized theory of conical flow is presented along with its use in supersonic wing theory through the application of conformal mapping techniques. Some attention is also given to the generalized linear conical flow theories.

This reviewer cannot, for several reasons, agree with the translator's opinion that this ". . . should certainly make a fine text book for a one-year course in linearized supersonic aerodynamics". The primary reason stems from the nature of the approach which is purely mathematical, and which is characterized by an almost complete lack of stated physical assumptions and arguments, a characteristic evident in most Soviet papers on the subject. This trait is exemplified in the discussion of wave drag on p. 29 where a Mach wave (cone) is indicated as being the same as a shock wave. Of course, the second feature that detracts from its use as a text is the nationalistic treatment which results in the exclusion of the names and work of such people as Glauert, Jones, Lighthill, Evvard, Lagerstrom, Stewart, and Ward, to name but a few. As noted by the translator, numerous misprints are prevalent in Soviet texts; however, although an attempt has in this instance been made to correct them, it would appear that a significant number still remain. Finally, because the translation is literal much is left to be desired in the way of readability for those familiar, or those first becoming acquainted with aerodynamic terminology; e.g. the use of "compression layers" for shock waves, "elongated body" for slender body, "wave resistance" for wave drag, etc.

In spite of the preceding criticism, it is this reviewer's opinion that this monograph should prove useful as a general reference work, or as a supplementary text in a course in which the subject matter is covered. It should also prove worthwhile for those workers or students who have some familiarity with the field, and who desire a unified mathematical approach to the topic.

RONALD F. PROBSTEIN

*The dynamics and thermodynamics of compressible fluid flow.* By Asher H. Shapiro. Volume II. The Ronald Press Co., New York, 1954. xi + 535 pp. Volume II: $16.00 (the set $30.00).

This is the second volume written by Professor Shapiro on compressible fluid flow theory. The first volume has been reviewed in a former issue of this *Quarterly*. Thus, the general characteristics of the work have been emphasized previously and the present review can be restricted to the contents of the second volume.

To begin with, the author continues (Part V) the treatment of supersonic flow theory. Three chapters (Axially symmetric supersonic flow, Supersonic flow past wings of finite span, Hypersonic flow) are devoted to this material. Some parts are treated in great detail, others—especially the general method used in linearized theory—are only outlined. The chapter on hypersonic flow will be found useful because the material given there does not appear in the classical books. The next part is devoted to mixed flow, i.e. flow which is partly subsonic and partly supersonic. The first chapter is a very classical—source, vortex, compressible flow with 180° turn are treated by the hodograph method—special attention (perhaps too much) is given to the limiting line. Transonic flow is then considered in the physical plane with special emphasis on similarity laws and applications to the flows through a converging-diverging nozzle and along a wavy wall. A third chapter is concerned with experimental results at transonic speeds and their interpretation; drag, lift, occurrence of shock waves and their interactions with the boundary layer are discussed.

Unsteady motion in one dimension is treated in Part VII. The first chapter deals with linearized theory (acoustical approximation). This provides a good introduction to the more difficult case arising when the disturbances are not small (second chapter); in this case the method of characteristics finds a very elegant application. Special attention is given to the effects of area change, friction, and entropy variations. Specific examples and comparisons of experiments with the theory are presented. Finally, the theory of unsteady one-dimensional shock waves is discussed, in particular the interactions of shocks with each other and with continuous waves is considered. The final part of this work is concerned with flow of real gases, taking account of viscosity and heat conductivity; only the boundary layer effects are considered. In the first chapter the fundamental concepts together with the corresponding differentia.

and integral equations (Momentum and Energy) are introduced; applications are then made to the laminar boundary layer. The next chapter deals with the corresponding problems associated with the theory of the turbulent boundary layer, which is perhaps of greater practical significance but far more difficult to understand. The analysis is based on the so-called "Reynolds stresses". A great deal of significant experimental information is found at the end of this chapter. Finally, a special chapter is devoted to boundary layers in tubes, in particular in the presence of shock waves.

As in the first volume, the material covered in this book is very extensive. Much valuable information in the form of introductions to many theories and their developments, summarization and discussion of various experiments, various tables and charts make this work very useful for the aeronautical engineer who must apply the results of Gas Dynamics to practical problems. It is perhaps not so advisable however to recommend it to a student who has not received a sound basic training or read previously some more elementary book in which the fundamental ideas are introduced. But for the manifold data which can be found in these two books, we must be very grateful to the author.

P. GERMAIN

*A selection of graphs for use in calculations of compressible airflow.* Prepared on behalf of the Aeronautical Research Council by the Compressible Flow Tables Panel (L. Rosenhead, Chairman, W. G. Bickley, C. W. Jones, L. F. Nicholson, H. H. Pearcey, C. K. Thornhill, R. C. Tomlinson). Oxford University Press, 1954. x + 115 pp. $13.45.

This book, which is a companion volume to "Compressible Airflow: Tables", makes available to engineers, physicists, and applied mathematicians a set of graphs which is useful for calculations involving the steady compressible flow of air for Mach numbers not greater than five. There are five sections to the book: (A) isentropic flow, both subsonic and supersonic; (B) normal shocks; (C) oblique shocks, including shock reflection from a plane wall; (D) conical flow, which treats the axisymmetric, supersonic flow past a cone with an attached conical shock at its vertex; (E) the determination of Reynolds number from specified values of Mach number, stagnation pressure, and stagnation temperature, or specified values of Mach number and altitude. The graphs of each section are preceded by a relatively short descriptive passage on the theory involved, in which the plotted equations are clearly indicated. The graphs themselves, which are large (size 8″ x 11″) and easily read, are of two types: single-page graphs, which are suitable for rough calculations; and multiple-page graphs, in which the single-page graphs are plotted to a much larger scale. The table of contents indicates the range of abscissa and ordinate and the values of the parameters used for each graph. Aside from the fact that the section on conical flow is particularly useful, the most important feature of this book is that it presents data graphically. Such a method of presentation makes it possible to consider either variable as being the dependent one and eliminates the necessity of making numerical interpolations.

P. F. MAEDER

*Mathematics and plausible reasoning.* Volume I: Induction and analogy. Volume II: Patterns of plausible inference. By G. Polya. Princeton University Press, 1954. Vol. I: xvi + 280 pp. $5.50. Vol. II: x + 190 pp. $4.50. (The set: $9.00).

It does not seem possible to convey a reasonably complete idea of the contents of these volumes within the space available for this review; the most that the reviewer can hope to accomplish is to induce many readers of this review to discover for themselves the attractions of this charming work. *Plausible reasoning* is a most important tool of the creative mathematician. In publishing his results, however, the mathematician is apt to hide the beautiful grain of plausible reasoning under a glossy lacquer of *demonstrative reasoning*. While this policy may often be dictated by limitations on space, there is no excuse for its adoption in teaching. "If the learning of mathematics reflects to any degree the invention of mathematics, it must have a place for guessing, for plausible inference." "A serious student of mathematics, intending to make it his life's work, must learn demonstrative reasoning; it is his profession and the distinctive mark of his science. Yet for real success he must also learn plausible reasoning; this is the kind of

reasoning on which his creative work will depend." The author warns the reader that "the efficient use of plausible reasoning is a practical skill and is learned, as any other practical skill, by imitation and practice." The first volume contains carefully chosen examples of plausible reasoning drawn from many branches of pure and applied mathematics. In many cases the author must content himself with offering conjectures regarding the steps that led to a discovery; in others he knows what happened either because the examples are drawn from his own work or because the discoverer left sufficient clues for the reconstruction of his chain of thought. Euler's work is particularly noteworthy in this respect because "he takes pains to present the relevant inductive evidence carefully, in detail, in good order," adding "to the discoveries, with which he enriched science, the candid exposition of the ideas that led him to those discoveries" (Condorcet). Chapters on plausible reasoning in different branches of mathematics intersperse chapters on methodology. Each chapter is followed by comments and examples. The latter provide the reader with a welcome opportunity to test his skill; they often form suggestive sequences. In the second, more philosophical, volume, the author undertakes to formulate certain patterns of plausible reasoning and discuss their relation to the Calculus of Probability. The final chapter is devoted to the role of plausible reasoning in invention and instruction. Throughout the work, simplicity seems to have governed the author's choice of examples, but he has by no means overlooked mathematical beauty or historical interest. Altogether, this is a stimulating and challenging work, which, it is hoped, will in time profoundly influence the teaching of mathematics.

W. PRAGER

*Methods of mathematical physics.* Vol. I. By R. Courant and D. Hilbert. Interscience Publishers, New York, 1953. xv + 561 pp. $9.50.

It is indeed a pleasure to have this standard text and reference work available at last in the English language. The translation from the 1931 German original was done by Professor Courant himself, and is entirely clear and readable.

The revisions announced on the title page are not major ones. A short section on the quasi-geometric interpretation of reciprocal quadratic variational problems has been added to the chapter on the calculus of variations, and the discussion of Castigliano's principle in the theory of elasticity has been altered to make use of this. An appendix to the last chapter discusses the transformation of spherical harmonics under rotation of coordinate axes. The references have been brought up to date by the inclusion of a supplementary bibliography at the end of the book.

STEPHEN PRAGER

*Superfluids, macroscopic theory of superfluid helium.* By Fritz London. Vol. II. John Wiley & Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1954. xvi + 217 pp. $8.00.

This second volume on the subject of superfluids represents the last major work of a most distinguished scientist. Despite the designation as Volume II, the book is self-contained and is concerned exclusively with properties of liquid helium. Only occasionally are references made, by way of analogy, to the subject matter of Volume I, superconductivity.

Even though the theory of liquid helium still contains numerous controversies and is still developing at a very rapid rate, most of the basic concepts discussed in this book are expected to remain valid. The book is concerned primarily with the theory although comparisons with experimental data are very plentiful. Despite the fact that the author originated and has been one of the chief promoters of the interpretation of the $\lambda$-point as being due to a Bose-Einstein condensation, the various other conflicting theories are also given careful consideration. The major part of the book deals, however, with the macroscopic rather than the microscopic properties of the superfluid state.

The book certainly gives a very nice survey of literature up to early 1953 with emphasis on physical principles rather than just mathematical expressions and adequately fills a most conspicuous vacancy that has existed in the scientific literature for many years.

G. F. NEWELL

*Heat conduction with engineering, geological, and other applications.* By L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll. Revised edition. The University of Wisconsin, Madison, 1954. xiii + 325 pp. $5.00.

The present book is a revised edition of the well known book on the 'Mathematical Theory of Heat Conduction', first published more than 40 years ago, and later expanded to nearly its present size. The last edition includes two new chapters: Chap. 13 entitled "Theory of earth heat exchangers for the heat pump", and Chap. 14 entitled "Drying. Soil consolidation".

The original aim of the authors was "to develop the subject with special reference to the needs of the student who has neither time nor mathematical preparation to pursue the study at great length . . . [and] . . . to point out the many applications of which the results are susceptible." The authors have succeeded in achieving this aim, particularly in pointing out the numerous problems arising in geology, and in mechanical, metallurgical and civil engineering which require the calculation of heat conduction. The applications discussed in the book range from thawing of frozen soil, cooling of the earth, cooling of lava under water to the removal of shrink fittings, temperature fluctuations in engine cylinders, ground pipe heat sources and others, too numerous to quote here in detail.

No attempt is made to classify the topics into their appropriate fields of application, each topic being discussed to illustrate problems as they arise in their natural sequence.

The treatment is simple and includes remarks on such mathematical topics as Fourier series, a short table of formulae, the use of conjugate functions for the solution of Laplace's equation, short tables of definite and indefinite integrals and several useful functions.

In limiting themselves to elementary mathematical tools, the authors exclude the much more powerful modern methods of handling the heat conduction equation, but they include a useful short discussion of auxiliary methods of treating heat-conduction problems (electrical methods, the use of tables and curves, the Schmidt method, the relaxation method and the step method). In all cases the essentials are explained in a simple manner, and no deeper understanding is sought.

The more specialized reader will find the scope of the work too narrow. On the one hand the applied mathematician will miss the more advanced and more powerful methods of analysis and will probably prefer to turn to Carslaw and Jaeger's classical treatise on the subject ('The conduction of heat in solids', Clarendon Press, Oxford, 1947). The mechanical engineer will acquire an appreciation of the variety of possible applications. He will, however, find that the number of aids in the form of graphs, charts, and tables is inadequate. He will also have to turn to the original references on Schmidt's graphical method or on relaxation methods, before he can make use of them. He will, most probably, prefer to make use of the comprehensive volume by Max Jakob ('Heat transfer', vol. I, John Wiley, New York, 1949).

The book is very well laid out, printed in large clear type, and provided with an adequate Index and with a fairly extensive, but rather selective list of references. In this connection it may be pointed out that Schmidt's second paper on graphical methods (Forschung **13**, 177-185, 1942) seems to have escaped the authors' notice. In describing the work on thermal conductivity the very important contributions due to Keyes (e.g. Trans. ASME **76**, 809, 1954) find no mention, either.

Several small omissions and incongruities might usefully be corrected in the future. The present reviewer prefers the designation "error function" to "probability integral" used by the authors and finds that the Bessel functions (of the first kind) used are not adequately described. He also deplores the use of the symbols fph and cgs to designate indiscriminately individual units in either of the two systems of units.

Within the limitations which the authors have set themselves, they have succeeded in writing a useful first introduction to the subject.

J. KESTIN

*Dynamical theory of crystal lattices.* By Max Born and Kun Huang. Oxford at the Clarendon Press, 1954. xii + 420 pp. $8.00.

This book was written to satisfy a need for a newer, more comprehensive treatise on lattice dynamics and crystal theory. The first three chapters primarily by K. Huang constitute Part I of this book and

they are concerned with atomic forces, lattice vibrations, and elasticity and mechanical stability (of lattices). The last four chapters are due primarily to M. Born and they are entitled Quantum Mechanical Foundation, The Method of Long Waves, The Free Energy, and The Optical Effects.

The text is one on the mechanics of the lattice and crystal dynamics; it is not a text on the general theory of solids (or metals), nor does this book contain any discussion of the diffraction of X-rays, electrons or neutrons by crystal lattices. The interaction of lattice vibrations and scattered particles is also omitted "only with regret". There is no treatment of "defects" of the crystal lattice.

Chapter six is concerned with the thermodynamics of lattices and most parts of it appear to be new. This chapter involves discussion of the elasticity of lattices under finite strains. There is detailed analysis of the elastic, piezoelectric, and pyroelectric properties of dielectrics, including the temperature dependence of these properties.

The first three chapters of this book are considerably more descriptive and physical in nature than the last four chapters which are highly mathematical.

While the material in this book deals mainly with the lattice mechanics of non-conducting materials, the material is obviously fundamental to the whole theory of solids, and it appears to be a very valuable contribution to the more important texts in this field.

ROHN TRUELL

*Formulas for computing capacitance and inductance.* By Chester Snow. National Bureau of Standards Circular 544, issued September 1954. 69 pp. $.40.

This is a collection of explicit formulas for the calculation of (1) the capacitance between conductors having a large variety of geometrical configurations, (2) the inductance (self and mutual) of circuits of various geometries, (3) forces acting between current-carrying coils. Formulas for skin effect and other frequency effects are included. The resulting expressions involve elementary functions, Legendre polynomials, Legendre functions, and elliptic functions.

ROHN TRUELL

*Multipole fields.* By M. E. Rose. John Wiley & Sons, Inc., New York, Chapman & Hall, Ltd., London, 1955. viii + 99 pp. $4.95.

This book is concerned with the angular momentum and transformation properties of multipole electromagnetic fields in physical systems, especially nuclei. Internal conversion, the emission and absorption of electromagnetic radiation, the emission of $\beta$ particles, and the static interactions of nuclear moments with fields caused by ions or other charges, are examples dealt with in this book—examples where the angular momentum properties are important.

The chapter headings in the text are: I. The Classical Field Equations. II. The Multipole Fields. III. Properties of the Multipole Fields. IV. The Retarded Electromagnetic Interaction. V. Internal Conversion. VI. Emission of Gamma Radiation.

ROHN TRUELL

*Quantum mechanics.* By F. Mandl. Academic Press, New York, Butterworth's Scientific Publications, London, 1954. viii + 233 pp. $5.80.

This book arose as the result of a course of lectures on quantum mechanics, primarily for experimentalists, at the Atomic Energy Research Establishment at Harwell in 1952. The first five chapters are used to develop the machinery of quantum mechanics in fairly concise fashion. Chapter six discusses in some details the angular momentum, spin, and symmetry properties of systems of particles. Chapter

eight deals in a very clear way with an introduction to scattering problems. Chapter nine is concerned with an introduction to the ideas of group theory.

One of the attractive features of the book, in addition to its conciseness, is a rather large collection of exercises and problems.

ROHN TRUELL

*Numerische Behandlung von Differentialgleichungen.* By L. Collatz. Second edition. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1955. xv + 526 pp. $8.65.

The first edition, which appeared in 1951, was reviewed in this Quarterly, vol. 12, pp. 92. The present volume exceeds the earlier one by 68 pages. The principal change consists in the addition of an introductory chapter of 45 pages, which is devoted to background material and general principles. References to this chapter frequently enable the author to achieve a more concise presentation in subsequent chapters.

The smaller changes and additions are too numerous to be listed here; they are mostly concerned with extensions of the theory, in particular improvements in the analysis of errors.

W. PRAGER

*Engineering cybernetics.* By H. S. Tsien. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1954. xii + 289 pp. $6.50.

Cybernetics is defined by the author in his Preface as "the science of organization of mechanical and electrical components for stability and purposeful actions." The author distinguishes the science of engineering cybernetics from the practice of servomechanisms engineering, and aims at making "a comprehensive survey of the whole field" of the new science.

A particular sub-field of engineering cybernetics is examined in each one of the 18 chapters, and the best way to show the unusually broad coverage of this work is to list the chapter titles, which follow: 1. Introduction 2. Method of LaPlace Transform 3. Input, Output, and Transfer Function 4. Feedback Servomechanism 5. Noninteracting Controls 6. Alternating-current Servomechanisms and Oscillating Control Servomechanisms 7. Sampling Servomechanisms 8. Linear Systems with Time Lag 9. Linear Systems with Stationary Random Inputs 10. Relay Servomechanisms 11. Nonlinear Systems 12. Linear System with Variable Coefficients 13. Control Design by Perturbation Theory 14. Control Design with Specified Criteria 15. Optimalizing Control 16. Filtering of Noise 17. Ultrastability and Multistability 18. Control of Error.

The treatment of these eighteen subjects is very unequal. The early ones, which may be called 'classical', have been discussed in careful detail in a number of textbooks, and their present cursory treatment is often disappointing. In some of the more advanced chapters, the author frankly states that he is reproducing a treatment already available in book form elsewhere. There is not much one can expect of a one-chapter treatment of non-linear systems! There remains the fact that about one half of the book consists of very valuable reports on advanced methods of design, recently published in aeronautical journals, some due to the author himself. It would be ungrateful to complain of presentational shortcomings; we should rather be thankful for this up-to-date information coming from a competent and very busy expert.

P. LE CORBEILLER

*Studies in mathematics and mechanics.* Presented to Richard von Mises by friends, colleagues, and pupils. Academic Press, Inc., New York, 1954. ix + 353 pp. $9.00.

This dedicatory volume was presented to Richard von Mises on the occasion of his seventieth birthday, April 19, 1953. Professor von Mises survived the event by only three months.

The papers were contributed from among his many students, collaborators, and friends. The breadth of subject matter is itself indicative of the manifold interests of the recipient, one of that handful of scientists who have done so much to give mechanics the shape it has today.

The papers are grouped into five categories: algebra, number theory, and geometry; analysis; theoretical mechanics; applied mechanics; probability and statistics.

The contributing authors are: E. Bompiani, Alfred Brauer, M. Herzberger, Gaston Julia, A. Ostrowski, Hans Rademacher, Franceso Severi, G. Szegö, Olga Taussky, C. Arf, Stefan Bergman and M. M. Schiffer, Garrett Birkhoff, J. B. Diaz and Alexander Weinstein, F. H. van Den Dungen, Zeev Nehari, Mauro Picone, D. C. Spencer, G. Temple, J. M. Burgess, P. R. Garabedian, G. Kuerti, C. C. Lin, Charles Loewner, G. S. S. Ludford, N. Minorsky, G. Pólya, William Prager, J. L. Synge, P. W. Bridgman, Howard W. Emmons, Karl Federhofer, Alfred M. Freudenthal, H. Reissner and E. Reissner, A. Signorini, Arthur H. Copeland, Sr., Maurice Fréchet, François N. Frenkiel and James W. Follin, Jr., Y. Garti et T. Consoli, Hilda Geiringer, Corrado Gini, J. Neyman and E. L. Scott, Nakibe T. Uzgören.

The introduction is by Professor Phillip Frank and includes a list of Professor von Mises published works. It was with delight that the reviewer noted that the list concludes with six publications on the Austrian poet, Rainer Maria Rilke. In this age of specialization they bear silent witness to the stature of this great scholar.

WILLIAM H. PELL

*Automatic feedback control system synthesis.* By John G. Truxal. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1955. xiii + 675 pp. $12.50.

The bulk of the material in this book has been taught in a two-semester graduate servomechanisms course in the Electrical Engineering School at Purdue University. It is not, therefore, a book for the beginner; but it is, by far, the best book I have seen dealing with Automatic Control beyond the elements. Chapters 1 to 6 deal with system synthesis, mostly in the frequency domain; it includes a great deal of material relative to electric network synthesis, without which much of what is said about improving automatic control risks remaining theoretical and inapplicable. The very valuable method of signal flow diagrams, of S. J. Mason, is made the basis of feedback and stability theory. At the same time, the author lays great stress upon the necessity of correlating harmonic steady-state with transients; this requires that the student should have a real understanding of the Laplace transform, instead of the usual scanty knowledge limited to third-order transients. The method of Bubb (based on articles by Schoenberg in QAM, 1946) receives due attention. Contributions by many authors are not simply quoted, but always thoroughly explained and integrated into the general treatment. Chapers 7 to 11 describe more advanced methods of statistical design and analysis of sampled-data and non-linear systems. This very thorough, conscientious and judicious work is confidently recommended.

P. LE CORBEILLER

*The elements of probability theory and some of its applications.* By Harold Cramer. John Wiley & Sons, Inc., New York, and Almquist & Wiksell, Stockholm, 1955. 281 pp. $7.00.

This is a translation and revision of a book originally written in Swedish. It is designed for students at the advanced undergraduate level and should indeed be understandable to readers familiar with elementary calculus and its usual prerequisites. The emphasis is particularly well suited to those who are primarily interested in acquiring enough familiarity with probability and statistics to handle routine practical problems and should be quite valuable as a book for self study by non-mathematicians.

The book is divided into three parts. The first part labeled "Foundations" contains an interesting historical introduction after which the concept of probability is defined operationally in terms of frequency

ratios. A discussion of the elementary calculus of probability is followed by some of the standard combinatorial problems.

The subject of part II is "Random variables and probability distributions" in which only discreet or continuous distributions are considered. The discussion here is already directed heavily toward practical statistics with primary emphasis on the special types of distributions that occur frequently in statistical analysis and such concepts as regression coefficients, correlation coefficients, etc.

The third part is described as "Applications" and deals almost exclusively with problems such as sampling, statistical inference, errors and other such topics dealing with statistical analysis of finite sets of observed values of random variables.

Except possibly for a few sections in which the author attempts with only moderate success to bring some of the more advanced topics down to an elementary level, the difficulty of the book is kept at a fairly uniform and modest level with numerous examples and exercises. The level is perhaps even too elementary for someone with strong mathematical inclinations and is more suited for someone who finds mathematics difficult but necessary. There are numerous suggestions of more advanced topics that the reader may find elsewhere.

G. NEWELL

*Einfuhrung in die Verbandstheorie.* By Hans Hermes. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1955. vi + 164 pp. $3.35.

Prof. Hermes' book makes a very pleasant introduction to some of the most interesting ideas of lattice theory, for those who are curious about this new branch of abstract algebra. It is, perhaps, even more readable than the recent comparable French book "Théorie des treillis", by Mme. Dubreil-Jacotin and MM. Lesieur and Croisot.

After introducing the basic concepts of lattice, modular and distributive lattice, and Boolean algebra, the author explains some important applications of lattice theory to other fields of mathematics, and does this very clearly. Thus, the relations to projective geometry and totally disconnected bicompact topological spaces are treated, together with applications to the structure of abstract groups. "Atomistic" Boolean algebras are treated in some detail, and the relation of lattices to logic is briefly sketched.

Throughout, the emphasis is on a clear presentation of esthetically appealing fundamental ideas; no attempt is made at completeness. The orientation is that of an algebraist and logician; not that of an analyst. Thus lattice-ordered groups, vector lattices, and partly ordered sets are not even mentioned.

Finally, it is not yet clear to what extent the ideas of lattice theory will prove fruitful in "applied mathematics", and the reader who wishes to learn about the subject with such applications in mind must be ready to develop his own techniques.

GARRETT BIRKHOFF

*Mathieusche Funktionen und Sphaeroidfunktionen.* By J. Meixner and F. W. Schaefke. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1954. XII + 414 pp. $7.65.

Both Mathieu functions and spheroidal wave functions arise from a separation of variables of the wave equation in certain systems of curvilinear coordinates. Their importance as a tool in mathematical physics has been recognized long ago, but the development of the theory of these functions has been rather slow. Large portions of the present book by Meixner and Schaefke could not have been written fifteen years ago. It is the first account of a systematic theory of spheroidal functions, and for Mathieu functions the only comparable work is "Theory and application of Mathieu functions" (Oxford, 1947) by N. W. McLachlan.

The book under review consists of four chapters, (approximately equal in length).

In the first chapter, the authors state and prove basic theorems some of which are not directly connected with the theory of special functions. The sections on eigenvalue problems involving two parameters deserve special attention. In the statement and proof of the theorems the terminology and the basic concepts of the theory of Banach spaces are used; a brief explanation of the vocabulary is

given in small print. The results of these sections, including an equiconvergence theorem, are used extensively in the later parts of the book. In another section of Chapter I, series expansions in terms of Hankel functions of order $v + n$ ($v$ fixed, $n = 0, \pm 1, \pm 2, \ldots$) are investigated. A useful lemma is proved stating a sufficient condition under which the asymptotic expansion of the sum of a series can be found by a term-by-term application of the asymptotic expansion for the Hankel functions.

Chapters II and III deal respectively with the theory of Mathieu functions and the theory of spheroidal wave functions. A common feature of both chapters is the underlying point of view; all functions are considered as analytic functions of both the variable and the parameters. This point of view is fully justified by the success achieved by the authors. A remarkable example is furnished by the results about the analytic behavior in the large of the characteristic curves. (In the case of Mathieu functions, these are the curves in the plane of the parameters which separate the regions of stability from those of instability.) Both Chapter II and III contain results of the following types: (1) formulas connecting the solutions defined by initial or boundary conditions for a finite interval with solutions defined by their asympotic behavior; (2) general expansion theorems; (3) addition theorems of great generality which are based on (2) and on the following remarks: The equation $\Delta u + u = 0$ is invariant under notation and translation of the coordinate system, and a solution which vanishes sufficiently strongly at infinity is identically zero.

The derivation of the results sheds a new light even on the well-known addition theorem for Bessel functions. It is not possible to describe in detail the large number of results in these chapters, nor even to point out those which are new or of special importance.

About thirty applications to problems of mathematical physics are briefly but lucidly discussed in Chapter IV. Problems in mechanics, acoustics, electromagnetics and quantum mechanics are treated. They include the diffraction and radiation problems investigated by Bouwkamp and by Stratton and Chu which started the revival of interest in spheroidal wave functions and the rapid development of the theory. Some of the applications are as recent as the problem of the strong-focusing synchrotron by E. D. Courant, M. S. Livingston and H. S. Snyder. Other applications are listed in the introduction.

The book is organized with great care and circumspection; its program and its limitations are clearly defined in the introduction, and the list of references is comprehensive and approaches completeness. The style is clear although of necessity condensed in many places. The reviewer did not feel it necessary to check the formulas for errors or misprints since this has already been done by C. J. Bouwkamp who is not only an authority on the field but also a mathematican with an unsurpassed reputation for reliability.

The work by Meixner and Schaefke can be used both as a textbook (on a reasonably high level) and as a handbook of reference for anyone who is not entirely unaquainted with the subject. It can be expected to be eminently useful and highly stimulating both in applied research and for the pure analyst.

W. MAGNUS