BOOK REVIEWS

Rocket exploration of the upper atmosphere. Edited by R. L. F. Boyd and M. J. Seaton, in consultation with H. S. W. Massey, F.R.S. Interscience Publishers, Inc., New York, and Pergamon Press Ltd., London, 1954. viii + 376 pp. $11.00.

This volume contains the forty-four papers presented at a conference (August 1953) arranged by the upper atmosphere rocket research panel of the U. S. and by the Gassiot Committee of the Royal Society of London. The contributions are segregated into the categories: Rocket techniques; Pressures, temperatures, and winds; Composition of the high atmosphere; The ionosphere, solar radiation, and geomagnetic variations; Rocket cosmic ray measurements; Laboratory studies; Theoretical considerations and suggested experiments. These papers provide a wealth of information concerning the experimental techniques and the information derived thereby. The book should be of interest to a broad group of scientists and engineers.

G. F. Carrier

Theory of ordinary differential equations. By Earl A. Coddington and Norman Levinson. McGraw-Hill Book Company, Inc., New York, Toronto, London, 1955. xii + 429 pp. $8.50.

This book is a welcome addition to the literature on the subject. Its general aim is to provide a theoretical foundation for the applications of the theory of ordinary differential equations. Being developed from courses given by the authors, the selection of material naturally reflects their interests. There has been no attempt to give the historical origin of the theory, but this is partly compensated by a chapter list of references placed at the end of the book, which serves as a guide to the original sources. One finds much new material and novel and original treatments of classical theory. The sets of problems (at the end of each chapter save the last) have been devised and formulated with great care, and their presence enhances greatly the value of the book as a class room text on the subject. Main results are stated clearly and precisely in the form of theorems, and the accompanying proofs are easy to follow. The basic prerequisite for the book is an acquaintance with matrices and the fundamentals of functions of a complex variable, although the notion of the Lebesgue interval is needed in chapters 2, 7, 9 and 10. A brief survey of the contents of the book follows: Chapters 1 and 2 are concerned with existence and uniqueness of solutions of the initial value problem for the first order equation: $dx/dt = f(t, x)$, $x(\tau) = \xi$, where t is a real (or complex) variable, and x, f, and ξ are n-rowed column vectors. Chapter 3 treats the linear "system" $dx/dt = A(t)x + b(t)$, where A is the n by n "matrix of coefficients". Chapters 4 and 5 analyze the linear system $dw/dz = A(z)w$, where z is complex, and A is an n by n complex-valued matrix with at most an isolated singularity (usually a pole) at some complex number z_0; while chapter 6 considers systems $dx/dt = \rho^r A(t, \rho)x$, containing a large parameter, which are of interest in boundary layer theory. Self-adjoint eigenvalue and boundary value problems on a finite interval occupy chapter 7, while the corresponding singular problems (i.e. when either the interval is infinite or the coefficients of the differential operator have a sufficiently singular behavior at an end point) for the second and the nth order case are treated, respectively, in chapters 9 and 10, and non-self adjoint boundary value problems on a finite interval in chapter 12. Chapter 8 contains oscillation and comparison theorems for second order linear equations and chapter 11 discusses algebraic properties of linear boundary value problems on a finite interval. Chapter 13 deals with the asymptotic behavior of nonlinear systems (stability), and is restricted to the behavior of solutions starting near a known solution of a system. Chapters 14 and 15 are devoted to perturbation theory (i.e., the discussion of the behavior of a system $dx/dt = g(t, x) + \mu h(t, x, \mu)$, for small $|\mu|$, based on the behavior of the corresponding system for $\mu = 0$) of systems having a periodic solution and of two dimensional real autonomous systems, respectively. The Poincaré-Bendixson theory of two dimensional autonomous systems is the subject of chapter 16. The final chapter is dedicated to differential equations on a torus, i.e. to the study of solutions, in the large, of the single differential equation $dx/dt = f(t, x)$, where f is a real, continuous function defined for all real (t, x), having period 1 in each variable separately, and such that through each point of the (t, x) plane passes a unique solution of the differential equation.

J. B. Diaz

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Vorlesungen Über Orthogonalreihen. By Francesco G. Tricomi. Springer-Verlag, Berlin, Gottingen, Heidelberg, 1955. viii + 264 pp. $8.95.

This is another outstanding text by the author, whose expository skill is well known to mathematicians. The present monograph, as several of Tricomi's monographs, grew out of his lectures on the subject at the University of Turin. The main purpose of the present work is to furnish a rapid introduction to the theory of Fourier series and orthogonal polynomials, and in this the author has succeeded admirably. The arguments of the proofs are clearly presented, and one finds at every turn illuminating comments relating seemingly unrelated (at first glance) portions of the subject matter. Chapter I is devoted to the general theory of orthogonal sets of functions, and covers such topics as approximation in mean square, Fischer-Riesz theorem, and completeness and closure of a set of functions. Of special interest are the criteria of Lauricella, Vitali and Dalzell for the completeness of a set of functions and the proof of the completeness of the set of the trigonometric functions, which is based on Dalzell's criterion. Chapters II and III are concerned with Fourier series, the general theory and the convergence properties, respectively. In chapter II occur Riemann's theorem for the local behavior of a Fourier series and the convergence criteria of Dirichlet, Dini and Lipschitz. In chapter III one finds the Lusin-Denjoy theorem on absolute convergence of a Fourier series, together with a consideration of Cesàro, Abel and Riemann summability, plus a brief discussion of the Fourier integral theorem. The remaining three chapters are devoted to orthogonal polynomials proper. Chapter IV presents the general properties of orthogonal sets of polynomials. There is a unified treatment of the so-called "classical" orthogonal polynomials, leading to a generalized "Rodriguez formula". Chapter V is dedicated to orthogonal polynomials on a finite interval: Jacobi, Gegenbauer, Tschebyscheff, Legendre, while Chapter VI deals with orthogonal polynomials on an infinite interval: Laguerre, Hermite, Jacobi.

J. B. Diaz

Aeroelasticity. By Raymond L. Bisplinghoff, Holt Ashley, and Robert L. Halfman. Addison-Wesley Publishing Co., Cambridge, 1955. ix + 860 pp. $14.50.

This is an important and valuable book, treating a subject characterized by a paucity of adequate instructional and reference texts. With truly ambitious objectives and considerable courage, the authors have assembled a near compendium of current unclassified knowledge in the art and science of aeroelasticity.

When considering an outline for a book on aeroelastic design principles, one is at once confronted by the difficulty of treating a subject which embraces at least three of the classical disciplines of engineering science, as well as one in which the practical art must not be divorced from rather advanced theoretical concepts. Aeroelasticity, in its broadest sense, deals with those problems in aeronautical engineering which arise by virtue of the inevitable flexibility of real structures. One branch of the subject deals with the changes in aircraft structural and flight performance due to airframe deformations caused by essentially *static* aerodynamic loads. Unfortunately for the designer, these loads are themselves affected by the airframe deflections. A second branch treats the aircraft behavior when airframe components are excited into vibrational motions; in such cases, the inertial effects cannot be ignored.

Thus the aeroelastic carousel forms—structures, aerodynamics and dynamics—each following on the other's heels in a tight circle. In some cases, aeroelastic effects cause only a modification of the aircraft performance as compared with the hypothetical rigid airframe case; in others, auto-excited phenomena appear, so that stability considerations become paramount. In all cases, the designer is faced with the realization that theory and calculation fall far short of coping with the complexity of today's problems, so that final recourse must be made to model tests, full-scale tests, and art.

With this background in mind, a background born of personal and intimate experience in the field, the authors approached their task. To satisfy the need for knowledge in three separate disciplines, they have simply set for themselves the task of writing three treatises within the covers of one book—one on

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advanced aircraft structures, a second on the elements of dynamics, and the third on advanced steady and unsteady linear aerodynamic theory. With these tools in hand, the study of aeroelasticity commences—starting with static aeroelastic phenomena, continuing on to flutter analyses and dynamic load studies, and concluding with a thorough account of model and full-scale testing theory and practice. The aircraft configurations discussed range from conventional, high aspect-ratio types to a variety of modern, low aspect-ratio designs; the speed ranges dealt with are subsonic and supersonic flight.

Within the 860 pages of this book, the reader is introduced to almost every theoretical and practical development of modern aeroelasticity. The presentation achieves an admirable level of lucidity, and the authors have generally shown good judgment in their choice of subject balance.

Perhaps the most outstanding feature of the book is its comprehensive treatment of unsteady aerodynamic theory. In no other source has the reviewer encountered so complete and integrated an account of current knowledge in this area. The discussion of model design, construction and testing techniques is likewise a unique and valuable one. In dealing with static and dynamic aeroelastic phenomena, the authors perform a valuable service by providing the reader with a truly practical, as well as theoretical perspective. Adequate illustrative examples point out the important elements of aeroelastic considerations from the viewpoint of both design analysis and synthesis.

Because of the ambitious scope of the book, any recounting of the details of its contents is quite out of the question in this review. It is perhaps also unfair, for the first edition, to quibble over the merits of individual topical coverages. The university student, unless well trained along advanced lines, may feel that the tempo of the book is too rapid. The applied mechanics specialist, seeking an insight into aeroelastic practice, might prefer that the authors had placed greater reliance on other well-known reference texts in the classical fields; by greater selectivity in choice of material, a more concise yet equally effective presentation would perhaps have resulted.

These minor points, however, are in no sense important to the moment. The authors deserve our thanks for this splendid text, which will benefit the educator, research scientist and aeronautical engineer for many years to come. The aeronautical engineering library, whether personal or institutional, can scarcely afford not to benefit from this worthy contribution to the technical literature.

M. Goland

Operations research for management. Edited by Joseph F. McCloskey and Florence N. Trefethen. Introduction by Ellis A. Johnson. The Johns Hopkins Press, Baltimore, 1954. xxiv + 407 pp. $7.50.

Based on lectures presented at an informal seminar on operations research at The Johns Hopkins University, the volume consists of an introduction ("The Executive, The Organization, and Operations Research") and three parts ("General", "Methodology", and "Case Histories"). From the mathematical point of view, the second part is most interesting. It contains the following articles, which provide a useful introductory survey of the mathematical tools of operations research: "Progress in Operations Research" by P. M. Morse, "Statistics in Operations Research and Operations Research in Statistics" by R. L. Ackoff, "Queueing Theory" by B. O. Marshall, Jr., "Information Theory" by D. Slepian, "Suboptimazation in Operations Problems" by C. Hitch and R. McKean, "Symbolic Logic in Operations Research" by W. E. Cushen, "The Use of Computing Machines in Operations Research" and "Linear Programming and Operations Research" by J. O. Harrison, Jr., and "Game Theory" by D. H. Blackwell.

W. Prager

Probleme der Plastizitätstheorie. By William Prager. Verlag Birkhäuser, Basel and Stuttgart, 1955. 100 pp. $3.00.

In four chapters totaling a hundred pages, the author discusses developments in the field of perfectly plastic solids since the publication of plasticity texts by Hill ("The mathematical theory of plasticity," Oxford University Press, 1950) and Prager and Hodge ("Theory of perfectly plastic solids," John Wiley, 1951). The first chapter is entitled "Mechanical behavior of plastic materials" and introduces an ingenious kinematic model of a perfectly plastic material. Generalized stress and strain coordinates are introduced, and the theory of the plastic potential is discussed. Chapter 2 is entitled "Mechanical behavior of structures" and is primarily concerned with the analysis of a truss with a single degree of indeterminacy. The yield condition for an elastic-perfectly plastic material is represented by a yield polygon in stress space, and this concept is used to discuss the load carrying capacity and shakedown of the truss. Chapter 3 is entitled "Limit analysis". The basic theorems are derived in terms of generalized coordinates with an arbitrary yield condition. Applications are then given to portal frames, arches, circular plates, and circular cylindrical shells. The effect of progressive deformations on the load carrying capacity of circular plates is also discussed. The final chapter is entitled "Finite plastic deformation" and is primarily concerned with plane strain. Emphasis is placed upon numerical-graphical methods of solution of the equations.

The material covered is limited in scope, in that recent researches into the dynamic response of plastic materials and into the behavior of strain hardening materials are not even mentioned. Since the author played (and is playing) an important role in both of these new directions, such omission must have been intentional. In the reviewer's opinion it is fully justified, since current research is so active that a book might well be out of date before it is printed. The theory of perfectly plastic solids, on the other hand, seems to have reached a comparatively stable plateau and hence to be well suited to text book writing. The present work is a welcome addition to the field.

The book is based upon a series of lectures given at the Eidgenoessische Technische Hochschule in Zurich during the fall of 1954. Since the lectures were necessarily given in German, the book is written in that language. In view of the fact that the vast majority of recent work in the field has been done in the English language (for example, of 70 references in the book to work published since 1945, 59 are to English language publications), it is to be hoped that the author or some other qualified person will soon make this valuable work available in English.

P. G. Hodge, Jr.

Vorlesungen über Approximationstheorie. By N. I. Achieser. Akademie-Verlag, Berlin, 1953. ix + 309 pp. $6.91.

Several important Russian texts are now being published in Germany, and this book is a very welcome addition to this collection. The original appeared in 1947. Professor Achieser is a leading Russian mathematician and his book is a masterful exposition of an important branch of analysis to which he himself has contributed.

Of the six chapters three contain background material and form attractive introductions to important chapters in analysis. The first chapter deals with normed linear spaces and states the approximation problem in its abstract setting. This chapter is also a good introduction to the theory of function spaces. Chapter III contains a beautiful introduction to classical harmonic analysis and Chapter IV gives some information on entire functions of exponential type. Approximation theory proper is dealt with in the remaining three chapters. Chapter II treats approximation in the sense of Tschebyscheff, that is in the maximum norm and contains results of Tschebyscheff, de la Vallée-Poussin, Haar, and Markoff. Chapter V deals with the best approximation and contains classical and recent results associated with the names of Bernstein, Steckloff, Sz. Nagy, Privaloff, Krein, the author and others. The last chapter centers around the Tauberian theorem of Wiener. A very valuable appendix contains many examples and problems. There is a good index and an extensive bibliography.

To prevent misunderstanding it should be pointed out that questions relating to practical numerical approximations are not treated. But a numerical analyst who would want to understand the theoretical bases of approximation practices will read this book with profit. The presentation is essentially self-contained, though the theory of Lebesgue integration is, of course, presupposed.

Lipman Bers

Harmonic analysis and the theory of probability. By Solomon Bochner. University of California Press, Berkeley and Los Angeles, 1955. viii + 176 pp. $4.50.

This is a monograph dealing with the interrelation between the two subjects of the title and the developments in these fields during the last twenty-five years that have resulted from the stimulation each has given the other. The book begins at an advanced level assuming the reader is familiar with most of the basic mathematical concepts of such subjects as measure theory, probability, harmonic analysis, topological and Banach spaces, to mention but a few. Furthermore, the subject matter of the book is in a highly concentrated form which requires very careful reading.

The main feature of the book is a discussion of characteristic functionals and stochastic processes in the last two chapters. Also interesting is the use of complex valued measures throughout. This is not a book for beginners.

G. F. Newell

Mécanique Vibratoire. By Robert Mazet. Librairie Polytechnique Ch. Béranger, Paris et Liége, 1955. xix + 280 pp. $14.35.

The author is a Professor in the Science Faculty at Poitiers and Scientific Director in the Office National d'Études et de Recherches Aéronautiques and it is his declared intention to discuss the dynamics of so-called "linear" mechanical systems, introducing for this purpose analogies from different branches of physics involving linear differential equations with constant coefficients: and to take as many examples as possible from aeronautical problems. After a brief review of fundamental notions with particular reference to electrical analogies, he deals in turn with undamped and damped systems with one degree of freedom; the idea of coupling and two or more degrees of freedom, open loop systems, filters, damped systems with several degrees of freedom, and systems associated with an independent source of energy. His final chapter deals briefly with non-linear factors.

It will be realised that there is little or nothing in this which is novel. In spite of the suggestion that aeronautical topics are to be given special attention, no such bias is evident until the last two chapters are reached; and the treatment there given to such topics as flutter is quite elementary. The chief claim to novelty is the application to mechanical problems of methods such as operational calculus more frequently used by electrical engineers, and it seems doubtful if these methods are particularly advantageous at this level of treatment.

The almost complete lack of literature references is astonishing; and it is noticeable that such references as are given are almost entirely to French papers. One very minor matter which, however unjustifiably, annoyed the reviewer was the curious convention adopted to indicate a mechanical spring.

John L. M. Morrison

Arithmetic operations in digital computers. By R. K. Richards. D. Van Nostrand Co., Inc., Toronto, New York, London, 1955. iv + 397 pp. $7.50.

This excellent book explains in a fundamental and elementary manner the basic working principles of digital computers. By this must be understood not the analysis of actual components and circuits used in computers, nor the discussion of existing or projected machines, but a review of the fundamental concepts underlying all computer design. A large amount of space is devoted to methods of performing arithmetic operations and of representing quantities symbolically. Applications of Boolean algebra to computer components, the performance of operations in binary and in decimal codes and methods of binary-decimal conversion are covered in detail. A chapter is devoted to switching networks, which are any devices from which output signals may be obtained as some prescribed function of input signals.

A chapter on computer organization and control briefly reviews the history of digital computers, both externally and internally programmed. The problems of programming stored-program machines are discussed with reference to program loops, sub-routines, interpretive programs and other important topics. A useful bibliography is appended.

The author, an I.B.M. development engineer, is to be congratulated on producing a most readable account of a field of ever-increasing importance which could so far only be followed in specialised periodicals. It is to be hoped the principles of programming will be further explored in forthcoming publications.

Walter Freiberger

The geometry of geodesics. By Herbert Busemann. Academic Press Inc., New York, x + 422 pp. $9.00.

The principal aim of this text is to provide an axiomatic treatment of geodesics and a class of general spaces in which these geodesics lie (G-spaces). The basic idea is to obtain the properties of geodesics by topological methods without using differentiability hypotheses.

The geodesics are defined so that they possess the basic property of prolongation; the G-spaces are axiomatically defined so that they possess the following properties: are metric, are finitely compact, are convex, and are such that prolongation is locally possible and is unique. In the first part of the text, the metric, convexity, etc. of the space are prescribed and the properties of the resulting G-space are analyzed. For instance, it is shown that if prolongation is possible in the large (straight G-spaces), the two-dimensional space is homeomorphic to the plane. In the second portion of the text, the universal covering space of a G-space as well as spaces of non-positive curvature (defined by a triangle inequality) are considered. To a geometer, this is the most interesting section of the text. The last section of the text deals with homogeneous spaces (spaces which admit a transitive group of motions).

As a classical differential geometer, the reviewer was amazed at the number of interesting geometric results obtained without any assumptions as to differentiability. However, this text is not easy reading. Real understanding of the material calls for a fair background in differential geometry and topology as well as an acquaintance with the reasoning methods of modern mathematics. For the reader with adequate preparation, this text contains a wealth of information and ideas.

N. COBURN

Experimental design—theory and application. By Walter T. Federer. The MacMillan Co., New York, 1955. xix + 544 pp. $11.00.

The purpose of this book is to acquaint experimenters and practicing statisticians with the available techniques of design and analysis of experiments. The author therefore gives a comprehensive discussion and description of all designs of major importance and their variants. The purpose of the text and the background of the readers to which it is addressed does not permit a precise mathematical formulation of the underlying assumptions, much less a mathematical presentation of the justification of the methods recommended. The author does however give sufficient references for readers interested in the mathematical foundations.

The first chapter presents a discussion of various statistical and non-statistical aspects of the problems arising in the planning of experiments. Chapters II and III describe various methods of design and analysis, many of them for the first time incorporated in a book. Chapters IV to XIII and Chapter XV are devoted to the description of the standard designs as Randomized block designs, Latin squares, Incomplete block designs, etc. The discussion of these designs is very thorough and various aspects and many variants of these designs are considered. Chapter XIV describes designs in which different treatments are applied at different times to the same experimental unit. Included in this chapter are designs which allow measurements of residual effects. The last Chapter discusses methods dependent on the measurement of covariates.

The methods described are illustrated by an adequate number of relatively simple examples.

H. B. MANN

Methods in numerical analysis. By Kaj L. Nielsen. The MacMillan Co., New York, 1956. xiii + 382 pp. $6.90.

The current surge of interest in numerical methods has encouraged publication of numerous books with varying choice of emphasis and varying degrees of sophistication. Dr. Nielsen has written a text book for the practical man, presenting methods rather than proofs, assuming no previous knowledge of numerical analysis, and demanding a minimum of mathematical background. "The author is a firm believer in systematic procedures, and many illustrative examples, calculating forms, and schematics are discussed". The treatment is simple, direct, and economical in words.

The selection of material is confined to the basic topics of numerical analysis, as will be seen from the following list of chapters; 1 Fundamentals, 2 Finite differences, 3 Interpolation, 4 Differentation and integration, 5 Lagrangian formulas, 6 Solution of equations and systems of equations both linear and non-linear, 7 Differential equations and difference equations, both ordinary and partial, 8 Least squares, the Nielsen-Goldstein method, and orthognal polynomials, 9 Harmonic analysis, exponential type curve fitting, differential corrections, data fitting and the autocorrelation function.

The practical computer will be pleased to find a total of 19 useful tables covering 48 pages providing values for coefficients needed in interpolation, differentiation, integration, and least squares.

In the reviewer's opinion the author might have pointed out more emphatically the essential *unity* of polynomial interpolation so that the unwary reader would not be confused by the bewildering variety of *methods* of interpolation. But in the main the author has succeeded well in his goal of providing a useful and practical text.

W. E. Milne

Carl Friedrich Gauss: *Titan of Science*. By G. Waldo Dunnington. Exposition Press, New York, 1955. xi + 479 pp. $6.00.

This painstaking compilation of minute personal and professional detail culminates a lifelong enthusiasm of the author. The scientific reader may be surprised by the quotations from Gauss's letters, expressed in an extravagant romantic emotionalism reminiscent of Goethe, Wordsworth, and Berlioz. While there are extensive and sincere attempts to discuss scientific work, a mathematician not already familiar with the material is likely to find them mystifying. The author presents many details concerning minor scientists whom Gauss knew, but the book is written in the idolizing superlatives often found in biographies of literary figures, and the reader is little likely to form a just idea of the enormous debt Gauss owed to his great predecessors. While not filling the need for a biography of Gauss, this work will retain value as a handy archive.

C. Truesdell

Tables of Weber parabolic cylinder functions. Computed by Scientific Computing Service Ltd. Mathematical Introduction by J. C. P. Miller. Her Majesty's Stationery Office, London, 1955. 233 pp. $11.35.

The functions tabulated in this book arise in connection with problems involving the solution of the two-dimensional wave equation in parabolic coordinates, such as wave propagation in bays with parabolic shores or around parabolic capes and wave transmission in certain stratified media.

The book contains a 90-page mathematical introduction and the tables. The former discusses approximate solutions to differential equations in general, integral forms of solution, asymptotic developments, and in detail the solutions of the equations $d^2y/dx^2 + (\pm x^2/4 - a)y = 0$ as well as their relations to Bessel, Confluent Hypergeometric and other functions. The tabular material is restricted to solutions of one form of the equation only, viz. that with the sign of $\frac{1}{4}x^2$ positive.

The tables have been prepared by the Scientific Computing Service Ltd. in conjunction with the National Physical Laboratory, London, and reflect in excellence of layout and typography the late Dr. L. J. Comrie's interest in such matters. Dr. J. C. P. Miller's introduction will be helpful to anyone interested in these functions, whether or not likely to use the tables.

The book is available from the British Information Service, 30 Rockefeller Place, New York 20, New York.

Walter Freiberger

Studies in the economics of transportation. By M. Beckmann, C. B. McGuire, and C. B. Winsten. With an introduction by T. C. Koopmans. Yale University Press, New Haven, 1956. xix + 232 pp. $4.00.

According to the introduction, the studies presented in this volume are of exploratory character and aim at developing concepts, methods, and models, that may serve as useful points of departure for the assessment of the capacity and efficiency of a transportation system. The first part (pp. 3-110) is devoted to highway transportation and contains chapters on Road and Intersection Capacity, Demand, Equilibrium, Efficiency, and Unsolved Problems. The second part (pp. 113-218) is concerned with railroad transportation and contains the following chapters: The Time Element in Railroad Transportation, Freight Operations and the Classification Policy, A Single Classification Yard, Assignment of Classification Work in a System of Yards, Division of Sorting Work Between Yards, Scheduling of Trains to Minimize Accumulation Delay, and Short-Haul Routing of Empty Boxcars. Since the traffic on a network of roads is the outcome of the individual decisions of a large number of drivers, whereas the traffic on a railroad network is centrally directed, entirely different mathematical problems arise in the two kinds of transportation system. As is natural in a pioneering work, there is a good deal of variation in the completeness with which the different topics are treated: in some cases an adequate mathematical model is constructed and concrete results are obtained; in other cases the model may assist in the formulation of useful concepts but is unlikely to yield valid numerical results. This reviewer found the book extremely stimulating; his only complaint is that the analysis is almost exclusively concerned with static situations.

W. PRAGER

Principes de mécanique analytique. By André Mercier. Gauthier-Villars, Paris, France, 1955. xi + 131 pp.

This is a detailed review of analytical dynamics using the methods of Lagrange and Hamilton· Transformation theory (canonical transformations) and the connection with integral invariants are discussed in Chapter III. Chapter IV is concerned with the Hamilton-Jacobi partial differential equation and multiply periodic systems. Chapter V discusses the connection between dynamics optics including matter waves. Chapter VII is concerned with analytical mechanics and statistical mechanics involving mainly a discussion of phase space, Liouville's theorem and the ergodic problem. Chapter VII considers the behavior of a system under the influence of external parameters which change very slowly (quasi static invariants). The book is written in an "easy to read" manner and should be particularly useful to students.

ROHN TRUELL

Numerical analysis. By Zdenek Kopal. John Wiley & Sons, Inc., New York, 1955. xiv + 556 pp. $12.00.

The emphasis of this treatise is on applications of numerical techniques to problems of infinitesimal calculus in a single variable. The principal topics are: polynomial interpolation; numerical differentiation; ordinary differential equations; boundary-value problems: algebraic, variational, iterative and other methods; mechanical quadratures; integral and integro-differential equations.

Within these self-imposed limitations of subject matter the book is excellent. It is written with a rare didactic skill, stimulating the reader's interest in and enhancing his enjoyment of the subject by numerous relevant asides and a wealth of cleverly chosen examples. The style of presentation is leisurely and conciseness has been sacrificed to readability and breadth of treatment: laudable in a subject of such wide significance.

WALTER FREIBERGER

Singular integral equations (Boundary problems of function theory and their application to mathematical physics). By N. I. Muskhelishvili. P. Noordhoff, N. V., Gronnigen, Holland, 1953. 447 pp. $7.50.

An integral equation is called singular if the unknown function appears in an integral which does not converge absolutely but exists as a Cauchy principal value. In the most important examples of singular integral equations the singular integrals are actually of the Cauchy type:

$$\int \frac{\mu(t)\, dt}{t - \tau}$$

taken over a curve or system of curves in the complex plane. Such equations arise very naturally in many problems in pure and applied mathematics. The foundations of the theory were laid by Poincaré and Hilbert, and some of the fundamental papers were written by Carleman and Noether in the early twenties. But up to now no systematic account of this beautiful and important theory was available in a western language. The publication of an English translation of Muskhelishvili's monograph is therefore especially welcome.

The author has devoted many years to the exploration of singular integral equations and to their applications to various problems in mathematical physics, in particular to problems in the theory of elasticity. He has also created a whole school of mathematicians working with these methods, among whom I. N. Vekua is particularly prominent. The present volume gives an exhaustive and eminently readable presentation of the theory and of its most important applications. It can be read by anybody familiar with the elements of complex function theory and with the theory of Fredholm integral equations. The presentation is pleasantly detailed and leisurely; many examples are worked out in detail, many applications are considered, and an excellent bibliography is appended. The translation and the physical appearance of the book are excellent.

Professor Muskhelishvili presents the subject from the point of view of a classical analyst. The connection with Hilbert space theory, for instance, is not even mentioned. Readers interested in the more abstract approach to singular integral equations may consult the monograph by Mikhlin (English translation, American Mathematical Society Translations, No. 24, 1950) and the paper by Halilov in Math. Sbornok, Vol. 25, 1949.

L. Bers

Proceedings of the Second International Congress on Rheology. Edited by V. G. W. Harrison. Academic Press Inc., New York, and Butterworths Scientific Publications, London, 1954. ix + 451 pp. $10.00.

The Second International Congress of Rheology was held at Oxford from 26-31 July, 1953, under the presidency of Sir Geoffrey Taylor. The range of topics covered in the proceedings is wide and many of them will be of little interest to the applied mathematician at the present time. On a long term basis, there are, of course, very few topics in rheology which could not pose significant and interesting problems for the applied mathematician. Sir Geoffrey Taylor, in his presidential address, discussed broadly the type of contribution that the mathematician can make to rheology, with conclusions which are, I feel, unduly pessimistic. Indeed the lack of adequate well-reasoned mathematical theories is evident in many of the papers contributed.

In addition to the presidential address, there are six general lectures, although in some cases their generality is limited to title only, twenty-two papers on high polymers, nineteen on viscosity and plasticity, three on biology and four on oils and greases.

As might be expected, the level and character of the papers varies considerably. Some of the papers show a lack of clarity in the formulation of the relevant scientific problem and a lack of depth both in the choice of problem and in the manner in which it is discussed. This is no doubt largely due to the circumstance that the work described in them is carried out with an industrial, rather than a scientific, objective as the principal motivation. In certain cases, they are nevertheless stimulating and may well form the basis for further researches in a more rigorous manner.

R. S. Rivlin

Applications of spinor invariants in atomic physics. By H. C. Brinkman. North-Holland Publishing Co., Amsterdam, and Interscience Publishers, Inc., New York, 1956. 72 pp. $3.25.

This interesting little book gives a survey of the theory of spinors as developed by H. A. Kramers and his students. No explicit use is made of the theory of group representations; the methods are, in fact, quite similar to those of the classical theory of invariants. Throughout, the spinors and their transformations correspond to the rotation group of Euclidean 3-space E_3, and the applications of the theory are to atomic physics and to the Pauli electron. No mention is made of spinors which correspond to the Lorentz group of Minkowski space, of the Dirac electron or of other elementary particles.

Chapter I deals mainly with the basic mathematical theory. Spinors with components u, v are introduced as a (double-valued) parametric representation of complex vectors of zero length. The real and imaginary parts of such a null vector form an ordered pair of equal orthogonal vectors, which can be extended to an orthogonal triad; this simple geometrical picture supplies the relation between spin transformations and rotations of E_3. The basic invariant $u_1v_2 - u_2v_1$ of two spinors is introduced; it is the scalar product of van der Waerden's spinor calculus. Two fundamental theorems are established: Any invariant polynomial of spinor components is a polynomial in the basic invariants; the homogeneous linear transformations of the monomials $M_j^m = u^{j+m}v^{j-m}$, $m = -j, -j + 1, \ldots, +j$ [there is an error of sign in Eq. (45)], provide an irreducible representation of the rotation group of E_3. There is no proof of the fact that there are no other finite-dimensional irreducible representations.

Chapter II outlines the range of applications of the theory of spinor invariants. It deals with accidental degeneracy and term splitting, angular momentum and the classification of wave functions, the calculation of matrix elements, electron spin, many electron systems. Chapter III applies the results obtained to some detailed calculations for the following special problems: two atomic electrons in electrostatic interaction (e.g., the Helium atom), the spin-orbit interaction, the intensities of atomic spectral lines.

Within its self-imposed limitations, the book gives a clear and straightforward account of the subject matter. The reviewer regrets one omission, the mention and demonstration of the simple fact that the rotation group of 3-space is doubly connected with an irreducible cycle of period 2. It is this property of the rotation group (also valid in higher dimensions) which motivates the consideration of double valued spin representations, and which shows that there exist no 3- or higher-valued continuous representations.

A. SCHILD

Mathematical theory of elasticity. By I. S. Sokolnikoff. Second Edition. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1956. xi + 476 pp. $9.50.

The second edition differs considerably from the first and is now eminently suitable as the basis for a first year applied mathematics course in elasticity. Although almost all of the first edition has been taken over intact, the flavor of the book has been altered appreciably by the inclusion of an extensive new chapter of 80 pages on the Muskhelishvili approach to two-dimensional elastostatic problems, a 50 page chapter on three-dimensional static and wave problems, and an expanded and greatly improved chapter on variational and absolute minimum principles. A brief historical sketch has been added as have many up-to-date references to Russian and other work. Linear elasticity alone is treated and attention is devoted almost exclusively to small displacement theory expressed in cartesian tensor or xyz notation. Engineers will be pleased by the inclusion of Mohr's circle and several other new pictorial representations. They will be made even more unhappy than before by the badly distorted stress-strain curves drawn for illustration and by the continued use of the word beam for a bar under tension or twist. These are minor criticisms as the book is on the *mathematical* theory of elasticity. It presents a very valuable unified analytical approach to many branches of well-established theory and to several topics in the forefront of research.

D. C. DRUCKER

Tables of the function arc sin z. By the Staff of the Computing Laboratory, Harvard University Press, Cambridge, Mass., 1956. xxxviii + 586 pp. $12.50.

This tabulation of the inverse sine for complex argument was undertaken at the request of the Aeronautical Research Laboratory of the United States Air Force. The introduction (pp. xi to xxxviii) treats the properties of the function arc sin z, the methods used for computing the tables, and means of interpolating within them. Each of the eleven tables covers a region consisting of one, two, or three rectangles; each of these regions contains the branch point $z = 1$, and the mesh is coarser for the tables covering the larger regions. The following table indicates the rectangles covered by the various tables and the corresponding mesh lengths.

TABLE	x_0	x_1	y_0	y_1	δ
I	0.916	1.090	0.000	0.140	0.002
II	0.810	0.915	0.000	0.350	0.005
	0.920	1.085	0.125	0.475	
	1.090	1.195	0.000	0.350	
III	0.54	0.81	0.00	0.70	0.01
	0.82	1.19	0.35	1.05	
	1.20	1.47	0.00	0.70	
IV	0.00	0.54	0.00	1.40	0.02
	0.56	1.46	0.70	2.10	
	1.48	2.38	0.00	1.40	
V	0.00	2.35	1.25	4.75	0.05
	2.40	4.15	0.00	3.50	
VI	0.0	4.1	3.5	10.5	0.1
	4.2	13.5	0.0	7.0	
VII	0.0	13.4	7.0	21.0	0.2
	13.6	21.0	0.0	14.0	
VIII	0.1	20.5	12.5	47.5	0.5
	21.0	39.5	0.0	35.0	
IX	0	39	35	105	1
	40	139	0	70	
X	0	138	70	210	2
	140	198	0	140	
XI	0	195	125	475	5
	200	475	0	350	

W. Prager