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Contents

BOOK REVIEWS

The theory of storage. By P. A. P. Moran. Methuen Co., London, and John Wiley & Sons, Inc., New York, 1959. 111 pp. $2.50.

This little book, the first in a new series of "Methuen Monographs on Applied Probability and Statistics" presents on 105 pages a remarkably compact discussion of what is usually regarded as a rather rambling field: the mathematical theory of inventory control and water storage. In this country most attention has been focused on the first variant of this problem, the management of stocks of repair parts or other items with probabilistic demand so as to maximize some economic criterion of efficiency. This book offers a useful complement in that the dam interpretation of the problem, and those aspects of inventory control that bear most directly on water storage—on which the author is a leading authority—receive the most detailed treatment.

An introductory chapter presents some probability concepts; but the reader should have some knowledge of the probability calculus beforehand. The chapter on the inventory problem emphasizes the calculation of probabilities of states, such as shortages, but pays little regard to the discovery and calculation of optimal policies. A useful and interesting feature is the systematic discussion of various inventory problems that do not involve stochastic processes but can be disposed of as one-period problems. Segerdahl's theory of insurance risk is also presented in outline.

Throughout, the discussion is in terms of specific problems, and solutions are obtained for particular distributions, typically those of the Gamma family. Continuous time problems are analyzed by going to the limit with discrete approximations. The Monte Carlo or simulation method is used to deal with the complicated problems that arise in the study of sequences of dams.

In spite of its great variety of topics the book cannot achieve an adequate survey in its limited space. However, it is an excellent introduction on an elementary level to a field of operations research where much work is going on at the present.

Martin Beckmann

Handbook of supersonic aerodynamics—mechanics of rarefied gases. By Samuel A. Schaaf and Lawrence Talbot. Johns Hopkins University Applied Physics Laboratory, Silver Spring, Maryland, 1959. ii + 85 pp. $1.25.

The present section of the Handbook of Supersonic Aerodynamics is part of a series, the first one of which was issued in 1950. This section deals, as the title indicates, with rarefied gas flows. It opens with a general survey of rarefied flow regimes. Unfortunately, the original description of rarefied flow regimes as put forward by Tsien is still propagated. Following this is a useful chapter on free molecule flow calculations, in which the aerodynamic and heat transfer coefficients for some bodies of simple geometry are presented. The following chapter presents a discussion of slip flow which this reviewer considers to be somewhat dated in the light of more recent advances. This little compendium does have a considerable amount of information, much of it in graphical form, that may be useful to those interested in a cursory view of the field.

Ronald F. Probstein

German-English mathematics dictionary. Compiled and edited by Charles Hyman. Interlanguage Dictionaries Publishing Co., New York, 1960. 131 pp. $8.00.

This useful dictionary contains more than 8500 entries from applied as well as pure mathematics. Extensive sampling did not reveal any incorrect entries. On the other hand, the temptation to increase the number of entries in a trivial manner has not always been resisted. For example, all of the entries "Axiom, Axiomatik, axiomatisch, axiomatisieren, Axiomatisierung, Axiomensystem" could well have been omitted, and the space gained could have been used to list less trivial correspondences, for instance "Drall—moment of momentum" or "Wirbelmoment—vortex strength."

W. Prager

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The numerical treatment of differential equations. By L. Collatz. Third Edition. Springer-Verlag, Berlin, Gottingen, Heidelberg, 1960. xv + 568 pp. $23.56.

Though labelled "third edition", this is essentially an English translation of the second German edition (see this QUARTERLY, vol. **13**, p. 348). The references have been brought up to date, many improvements have been made, and the number of examples has been increased.

WILLIAM PRAGER

Operations research and systems engineering. Edited by C. D. Flagle, W. H. Huggins, and R. H. Ray. The Johns Hopkins Press, Baltimore, 1960. x + 889 pp. $14.50.

Part I (Perspectives) is concerned with the purpose and the historical development of operations research and systems engineering. Part II (Methodologies) is primarily devoted to the mathematical techniques. It contains chapters on simplified models (E. Naddor), basic statistics (A. J. Duncan), statistical quality control (A. J. Duncan), digital computers (W. C. Gore), inventory systems (E. Naddor), linear programming (V. V. McRae), queuing theory (C. D. Flagle), simulation techniques (C. D. Flagle), theory of games (E. Naddor), symbolic logic (W. E. Cushen), design of experiments (W. G. Cochran), human engineering (A. Chapanis), information theory (W. C. Gore), flow-graph representation of systems (W. H. Huggins), system dynamics (W. H. Huggins), and feedback and stability (N. H. Chosky). In Part III, the use of these principles and methods in operations research and systems engineering is illustrated by case studies.

W. PRAGER

Progress in solid mechanics. Volume I. Edited by I. N. Sneddon and R. Hill. North-Holland Publishing Co., Amsterdam, 1960. xii + 448 pp. $15.50.

This is the first volume of a new series, which is to be primarily devoted to the "basic principles and mathematical techniques of continuum mechanics, in all its aspects, together with experimental work of a fundamental kind". As the space available for this review is totally inadequate for a critical discussion of the eight articles, the following highly condensed table of contents will have to indicate the character of the volume.

Viscoelastic waves, by S. C. Hunter (Linear viscoelastic solid—Propagation of uniaxial stress pulses—Experimental investigations on pulse propagation—General equations of an isotropic viscoelastic solid).

Matrices of transmission in beam problems, by K. Marguerre (Vibrating beam—Matrices for points of discontinuity—Rigid supports,—matrix—"Stiff" supports—Application of matrix method to more complex problems).

Dynamic expansion of spherical cavities in metals, by H. G. Hopkins (Methods of approach and objectives—Hopkinson's size-scaling law—Strong discontinuities—Elastic deformations—Small elastic-plastic deformations—Large elastic-plastic deformations).

General theorems for elastic-plastic solids, by W. T. Koiter (Basic assumptions and stress-strain relations—Uniqueness theorems—Minimum principles—Plastic collapse theorems and limit analysis—Shakedown theorems—Existence of solutions).

Dispersion relations for elastic waves in bars, by W. A. Green (Exact solutions—Elementary approximate theory—Approximate methods—High frequency solutions for a circular cylinder).

Thermoelasticity, The dynamic theory, by P. Chadwick (Thermoelastic equations—Plane harmonic thermoelastic waves—Thermoelastic boundary value problems).

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Continuous distributions of dislocations, by B. A. Bilby (Burgers vector and torsion tensor—Shape, lattice and dislocation deformations—Zero lattice pure strain—Dislocation density and stress—Generalized spaces).

Asymmetric problems of the theory of elasticity for a semi-infinite solid and a thick plate, by R. Muki (Solution of the equations of equilibrium by Hankel transforms—Solution of the thermo-elastic equations by Hankel transforms—Stresses in a semi-infinite elastic solid under the compressive action of a rigid body—Stresses in a semi-infinite elastic solid with a tangential load on its surface—Thermal stresses in a semi-infinite elastic solid and a thick plate under steady distribution of temperature).

W. Prager

Dynamic programming and Markov processes. By Ronald A. Howard. Technology Press of M. I. T., and John Wiley & Sons, New York and London, 1960. viii + 136 pp. $5.75.

Consider a physical system S represented at any time t by a state vector $x(t)$. The classical description of the unfolding of the system overtime uses an equation of the form $x(t) = F(x(s), s \leq t)$, where F is a prescribed operation upon the function $x(s)$ for $s \leq t$. In certain simple cases, this reduces to the usual vector differential equation $dx/dt = g(x)$, $x(0) = c$.

For a variety of reasons, it is sometimes preferable to renounce a deterministic description and to introduce stochastic variables. If we take $x(t)$ to be a vector whose i-th component is now the probability that the system is in state i at time t, and allow only discrete values of time, we can in many cases describe the behavior of the system over time quite simply by means of the equation $x(t + 1) = Ax(t)$. Here $A = (a_{ij})$, $i, j = 1, 2, \cdots, N$, is a transition matrix whose element a_{ij} is the probability that a system in state j at time t will be found in state i at time $t + 1$. Processes of this type are called Markov processes and are fundamental in modern mathematical physics.

So far we have assumed that the observer plays no role in the process. Let us now assume that in some fashion or other the observer has the power to choose the transition matrix A at each stage of the process. We call a process of this type a *Markovian decision process.* It is a special, and quite important, type of dynamic programming processes; cf. Chapter XI of *Dynamic Programming,* Princeton University Press, 1957.

Let us suppose that at any stage of the process, we have a choice of one of a set of matrices, $A(q) = (a_{ij}(q))$. Associated with each choice of q and initial state i is an expected single-stage return $b_i(q)$. We wish to determine a sequence of choices which will maximize the expected return from n stages of the process. Denoting the maximum expected return from an n-stage process by $f_i(n)$, the principle of optimality yields the functional equation

$$f_i(n) = \max_q \left[b_i(q) + \sum_{j=1}^{N} a_{ij}(q) f_j(n - 1) \right].$$

In this form, the determination of optimal policies and the maximum returns is easily accomplished by means of digital computers; cf. S. Dreyfus, *J. Oper. Soc. of Great Britian,* 1958. Problems leading to similar equations, resolved in similar fashion, arise in the study of equipment replacement and in continuous form in the "optimal inventory" problem; see Chapter Five of the book mentioned above and K. D. Arrow, S. Karlin, and H. Scarf, *Studies in the Mathematical Theory of Inventory and Production,* Stanford University Press, 1959.

As in the case of the ordinary Markov process, a question of great significance is that of determining the asymptotic behavior as $n \to \infty$. It is reasonable to suspect, from the nature of the underlying decision process, that a certain steady-state behavior exists as $n \to \infty$. This can be established in a number of cases.

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The author does not discuss these matters at all. This is unfortunate, since there is little value to steady-state analysis unless one shows that the dynamic process asymptotically approaches the steady-state process as the length of the processes increases. Furthermore, it is essential to indicate the rate of approach.

The author sets himself the task of determining steady-state policies under the assumption of their existence. Granted the existence of a "steady-state," the function $f_i(n)$ has the asymptotic form $nc + b_i + o(1)$ as $n \to \infty$ where c is independent of i. The recurrence relations then yield a system of equations for c and the b_i.

This system can be studied by means of linear programming as a number of authors have realized; cf. A. S. Manne, "Linear programming and sequential decisions," *Management Science*, vol. 6, 1960, pp. 259–268.

Howard uses a different technique based upon the method of successive approximations, in this case an approximation in policy space. It is a very effective technique as the author shows by means of a number of interesting examples drawn from questions of the routing of taxicabs, the auto replacement problem and the managing of a baseball team.

The book is well written and attractively printed. It is heartily recommended for anyone interested in the fields of operations research, mathematical economics, or in the mathematical theory of Markov processes.

RICHARD BELLMAN

Handbook of supersonic aerodynamics—shock tubes. By I. I. Glass and J. Gordon Hall. Section 18. Supt. of Documents, Washington 25, D. C. xxxviii + 604 pp. $3.75.

An extensive handbook of shock tube technique. The first part, on the theory and performance of simple shock tubes, is by I. I. Glass. The second part, on the production of strong shocks and the application, design and instrumentation of shock tubes, is by J. G. Hall.

R. E. MEYER

The theory of thin elastic shells. Edited by W. Koiter. Proceedings of I. U. T. A. M. Symposium, Delft, 1959. North-Holland Publishing Company, Amsterdam, 1960. ix + 496 pp. $9.00.

The volume contains the papers presented at the Symposium on the Theory of Thin Elastic Shells sponsored by the International Union of Theoretical and Applied Mechanics (I.U.T.A.M.) and held at the Technological University of Delft in August 1959. All papers at the Symposium were delivered by invitation only and attention was confined to two particular aspects of shell theory, viz. nonlinear theory and problems lacking axial symmetry. An Introduction to the Symposium by Prof. C. B. Biezeno is followed by the contributions of the following authors: W. T. Koiter, W. Zerna, D. G. Ashwell, B. Budiansky, H. Ebner, E. I. Grigolyuk, J. H. Haywood, G. Czerwenka, W. Schnell, M. Kuranishi and J. Niisawa, A. L. Bouna, Y. Tsuboi and K. Akino, A. van der Neut, I. N. Vekua, M. W. Johnson, P. M. Naghdi, W. A. Nash and J. R. Modeer, B. R. Seth, P. Seide, N. J. Hoff and J. Singer, J. W. Choen, E. Reissner, H. Neuber.

R. T. SHIELD