QUARTERLY OF APPLIED MATHEMATICS

Vol. XXI JULY, 1963 No. 2

SOLAR HEATING OF A ROTATING SOLID CYLINDER*

BY

W. E. OLMSTEAD AND S. RAYNOR

Northwestern University, Evanston, Illinois

Abstract. This paper is concerned with the quasi-steady temperature distribution of a solid cylinder which rotates about its geometric axis, arbitrarily inclined to the incoming parallel radiation. The differential equation for the temperature with a linearized boundary condition is solved by the method of Green's functions. The numerical results for the case of an aluminum alloy cylinder indicated only small temperature variations for all angular velocities.

Introduction. The current emphasis on satellites and space vehicles has created an interest in the temperature distribution on rotating bodies exposed to solar radiation. The work done thus far has dealt primarily with thin-walled bodies, see references [1], [2], [3]. The next step is the analysis of thick-walled bodies and solid bodies. This paper considers the temperature distribution for a rotating solid cylinder.

As shown in Fig. 1, a solid cylinder of radius b rotates about its geometric axis with angular velocity ω. The geometric axis makes an angle φ with the parallel radiation. The top and bottom of the cylinder are insulated against "end effects." Since the cylinder is located in space, convection effects will be neglected. Radiation from the cylinder surface to space (effective temperature near absolute zero) is assumed to be proportional to the fourth power of the surface temperature.

The temperature distribution which will be determined is that for the quasi-steady state. This state implies that a balance has been attained between the total heat absorbed by the cylinder and the total heat re-radiated into space. Thus each point on the rotating cylinder will have a temperature which varies periodically with time. An alternate viewpoint of this state is that of a time-independent temperature associated with each position fixed with respect to the incoming radiation.

Derivation of the differential equation. In the interior of the cylinder, heat is transmitted only by conduction, hence the periodically varying temperature $\tau(r, \psi, t)$ may be found from the Fourier heat equation

$$\frac{\partial^2 \tau}{\partial r^2} + \frac{1}{r}\frac{\partial \tau}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \tau}{\partial \psi^2} = \frac{1}{\alpha}\frac{\partial \tau}{\partial t}, \tag{1}$$

where r and ψ are the polar coordinates locating a point in a system fixed in the cylinder. The thermal diffusivity is $\alpha = k/(\rho c_p)$, where k is the thermal conductivity, ρ the density and c_p the specific heat.

The local heat transfer per unit area at the surface of the cylinder is given by the

*Received July 18, 1962.

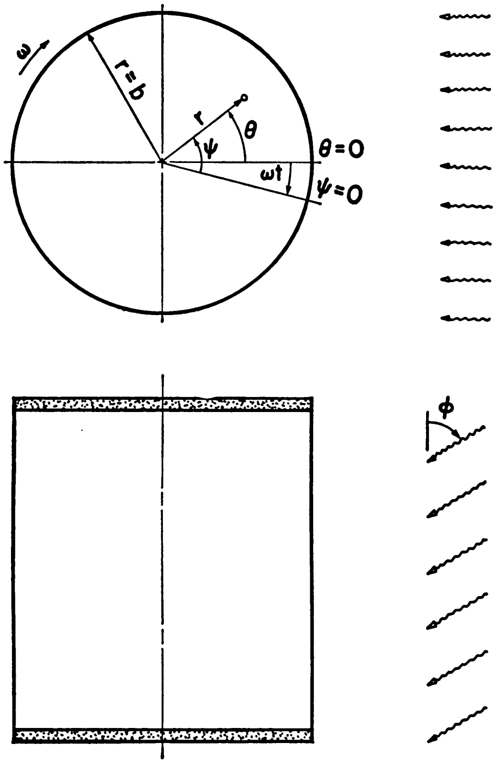

FIG. 1. Coordinate Systems for the Rotating Cylinder.

difference between the local absorbed radiation (normal component) and local re-radiated energy. Thus the boundary condition for equation (1) is

$$k\left(\frac{\partial \tau}{\partial r}\right)_{r=b} = -\sigma e(\tau^4)_{r=b} + aK_s \sin \varphi \cos^+ (\psi - \omega t), \tag{2}$$

where

$$\cos^+ (\psi - \omega t) = \begin{cases} \cos (\psi - \omega t) & \text{for} \quad -\pi/2 \leq \psi - \omega t \leq \pi/2 \\ 0 & \text{for} \quad \pi/2 \leq \psi - \omega t \leq 3\pi/2 \end{cases}$$

and σ is the Stefan-Boltzmann constant, e the average emissivity of the cylinder material, a the average absorptivity of the cylinder material, and K_s the energy per unit area and time received by a plane normal to the incoming radiation.

The above equations can be put into a form which is independent of time. This is done by transforming to the system of coordinate positions fixed with respect to the incoming radiation. This transformation is discussed by Schneider [4] for rectangular coordinates. For polar coordinates, the transformation is

$$\begin{aligned} T(r, \theta) &= \tau(r, \psi, t), & \theta &= \psi - \omega t \\ r &= r, & t' &= t \end{aligned} \tag{3}$$

with the condition that $\partial T/\partial t' = 0$. This transformation yields the following differential equation and boundary condition,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\omega}{\alpha}\frac{\partial T}{\partial \theta} = 0, \tag{4}$$

$$\left(\frac{\partial T}{\partial r}\right)_{r=b} = -\frac{\sigma e}{k}(T^4)_{r=b} + a\frac{K_s}{k}\sin \zeta \cos^+ \theta. \tag{5}$$

In order to linearize the above boundary condition, the following substitution is made

$$T = T_0(\tfrac{3}{4} + T^*). \tag{6}$$

Then assuming small variation of T^* about the reference temperature T_0, it follows that

$$T^4 = T_0^4[1 + (T^* - \tfrac{1}{4})]^4 \approx T_0^4[1 + 4(T^* - \tfrac{1}{4})] = 4T_0^4T^*. \tag{7}$$

Thus the differential equation and linearized boundary condition can be expressed in terms of the nondimensional temperature variation T^*, as

$$\frac{\partial^2 T^*}{\partial r^2} + \frac{1}{r}\frac{\partial T^*}{\partial r} + \frac{1}{r}\frac{\partial^2 T^*}{\partial \theta^2} + \frac{\omega}{\alpha}\frac{\partial T^*}{\partial \theta} = 0, \tag{8}$$

$$\left(\frac{\partial T^*}{\partial r}\right)_{r=b} = \left(\frac{-4\sigma e T_0^3 T^*}{k}\right)_{r=b} + \frac{aK_s}{kT_0}\sin \varphi \cos^+ \theta. \tag{9}$$

Further simplification of these equations is accomplished by defining a nondimensional radius,

$$s = \frac{r}{b}. \tag{10}$$

Also define the following nondimensional groups,

$$\zeta = \omega b^2/\alpha, \tag{11}$$

$$\beta = 4b\sigma e T_0^3/k, \tag{12}$$

$$\gamma = aK_s b \sin \varphi/(kT_0). \tag{13}$$

Now Eqs. (8) and (9) become

$$\frac{\partial^2 T^*}{\partial s^2} + \frac{1}{s}\frac{\partial T^*}{\partial s} + \frac{1}{s^2}\frac{\partial^2 T^*}{\partial \theta^2} + \zeta\frac{\partial T^*}{\partial \theta} = 0 \tag{14}$$

$$\left(\frac{\partial T^*}{\partial s}\right)_{s=1} = -\beta(T^*)_{s=1} + \gamma \cos^+ \theta. \tag{15}$$

Accessory problem. In order to find the solution to Eq. (14) with the boundary condition (15), the accessory problem for the adjoint Green's function, $G(s, \theta; s_0, \theta_0)$ is considered,

$$\frac{\partial^2 G}{\partial s^2} + \frac{1}{s}\frac{\partial G}{\partial s} + \frac{1}{s^2}\frac{\partial^2 G}{\partial \theta^2} - \zeta\frac{\partial G}{\partial \theta} = -\frac{\delta(s, s_0)\delta(\theta, \theta_0)}{s} \tag{16}$$

with the boundary condition

$$\left(\frac{\partial G}{\partial s}\right)_{s=1} = -\beta(G)_{s=1}, \tag{17}$$

where $\delta(s, s_0)$ and $\delta(\theta, \theta_0)$ are Dirac δ-functions.

By multiplying (14) by G and (16) by T^*, then subtracting and integrating over the cross-sectional area of the cylinder using the boundary conditions (15) and (17), it follows that

$$T^*(s_0, \theta_0) = \gamma \int_{-\pi/2}^{3\pi/2} G(1, \theta; s_0, \theta_0) \cos^+ \theta \, d\theta. \tag{18}$$

Hence the solution for $T^*(s_0, \theta_0)$ follows from (18) once the appropriate Green's function has been found.

To solve the accessory problem, assume*

$$G(s, \theta; s_0, \theta_0) = G_0^*(s; s_0) + \sum_{\substack{n \neq 0 \\ n=-\infty}}^{\infty} G_n^*(s; s_0, \theta_0) e^{in\theta}. \tag{19}$$

From (19), the definition of G_n^* becomes

$$G_n^* = \frac{1}{2\pi} \int_0^{2\pi} G(s, \theta; s_0, \theta_0) e^{-ni\theta} \, d\theta \quad \text{for all} \quad n. \tag{20}$$

By multiplying (16) by $e^{-in\theta}/2\pi$ and integrating over θ, a differential equation in G_n^* results:

$$\frac{d^2 G_n^*}{ds^2} + \frac{1}{s} \frac{dG_n^*}{ds} - \frac{n^2}{s^2} G_n^* - in\zeta G_n^* = -\frac{e^{-in\theta_0}}{2\pi} \frac{\delta(s, s_0)}{s} \tag{21}$$

with the boundary condition

$$\left(\frac{dG_n^*}{ds} \right)_{s=1} = -\beta (G_n^*)_{s=1} . \tag{22}$$

For $n \neq 0$, the homogeneous part of Eq. (21) has solutions which are modified Bessel functions with complex arguments. Hence,

$$G_n^* = \left. \begin{cases} A I_n(\lambda_n s) & \text{for} \quad 0 \leq s < s_0 \\ B I_n(\lambda_n s) + C K_n(\lambda_n s) & \text{for} \quad s_0 < s \leq 1 \end{cases} \right\} n \neq 0, \tag{23}$$

where $\lambda_n = i^{1/2} n^{1/2} \zeta^{1/2}$.

For the case of $n = 0$, it follows

$$G_0^* = \begin{cases} A_0 & \text{for} \quad 0 \leq s < s_0 \\ B_0 \log s + C_0 & \text{for} \quad s_0 < s \leq 1. \end{cases} \tag{24}$$

First considering the case for $n \neq 0$, Eq. (23) will satisfy the boundary condition at $s = 1$ when

$$G_n^* = \left. \begin{cases} A I_n(\lambda_n s) & \text{for} \quad 0 \leq s < s_0 \\ B' [M_n I_n(\lambda_n s) - N_n K_n(\lambda_n s)] & \text{for} \quad s_0 < s \leq 1 \end{cases} \right\} n \neq 0, \tag{25}$$

where

$$M_n = \lambda_n K_n'(\lambda_n) + \beta K_n(\lambda_n), \qquad N_n = \lambda_n I_n'(\lambda_n) + \beta I_n(\lambda_n). \tag{26}$$

---

*The term G_n^* $(s; s_0)$ must be treated separately since it will involve functions different from those of $G_n^*(s; s_o, \theta_o)$ where $n \neq 0$.

Then by the requirement that G_n^* be continuous at $s = s_0$, Eq. (25) becomes

$$G_n^* = \begin{cases} A'I_n(\lambda_n s)[M_n I_n(\lambda_n s_0) - N_n K_n(\lambda_n s_0)] & \text{for} \quad 0 \le s < s_0 \\ A'I_n(\lambda_n s_0)[M_n I_n(\lambda_n s) - N_n K_n(\lambda_n s)] & \text{for} \quad s_0 < s \le 1 \end{cases} n \neq 0. \tag{27}$$

The jump in dG_n^*/ds at $s = s_0$ is evaluated from the differential equation (21). Multiplying this equation by s and integrating gives

$$\int_{s_0-}^{s_0+} \frac{d}{ds}\left(s \frac{dG_n^*}{ds}\right) ds - \int_{s_0-}^{s_0+} \left(\frac{n^2}{s} + in\zeta s\right) G_n^* \, ds = -\frac{e^{-in\theta_0}}{2\pi}.$$

Since G_n^* is continuous at $s = s_0$, the second integral above vanishes. Then follows

$$\left(s \frac{dG_n^*}{ds}\right)_{s=s_0+} - \left(s \frac{dG_n^*}{ds}\right)_{s=s_0-} = -\frac{e^{-in\theta_0}}{2\pi}. \tag{28}$$

Substitution of (27) into (28) yields

$$A's_0\lambda_n N_n[I_n(\lambda_n s_0)K_n'(\lambda_n s_0) - I_n'(\lambda_n s_0)K_n(\lambda_n s_0)] = \frac{e^{-in\theta_0}}{2\pi} \quad \text{for} \quad n \neq 0. \tag{29}$$

In Eq. (29), the quantity in the brackets is recognized as the Wronskian of $I_n(\lambda_n s_0)$ and $K_n(\lambda_n s_0)$. Reference [5] gives

$$\text{Wronskian} \quad [I_n(\lambda_n s_0), K_n(\lambda_n s_0)] = -\frac{1}{\lambda_n s_0}. \tag{30}$$

Thus it follows

$$A' = -\frac{e^{-in\theta_0}}{2\pi N_n} \quad \text{for} \quad n \neq 0. \tag{31}$$

Hence,

$$G_n^*(s; s_0, \theta_0) = \begin{cases} -\dfrac{e^{-in\theta_0}}{2\pi N_n} I_n(\lambda_n s)[M_n I_n(\lambda_n s_0) - N_n K_n(\lambda_n s_0)] & \text{for } 0 \le s < s_0 \\ -\dfrac{e^{-in\theta_0}}{2\pi N_n} I_n(\lambda_n s_0)[M_n I_n(\lambda_n s) - N_n K_n(\lambda_n s)] & \text{for } s_0 < s \le 1 \end{cases} \quad n \neq 0. \tag{32}$$

Following a similar procedure as above for the case of $n = 0$ yields

$$G_0^*(s; s_0) = \begin{cases} \dfrac{1}{2\pi\beta}(1 - \beta \log s_0) & \text{for} \quad 0 \le s < s_0, \\ \dfrac{1}{2\pi\beta}(1 - \beta \log s) & \text{for} \quad s_0 < s \le 1. \end{cases} \tag{33}$$

Now (32) and (33) could be substituted directly into (19) to give a general expression for $G(s, \theta; s_0, \theta_0)$. However, for the problem at hand only $G(1, \theta; s_0, \theta_0)$ is needed. Thus it follows from (32) and (33) that

$$G_n^*(1; s_0, \theta_0) = \frac{e^{-in\theta_0}}{2\pi} \frac{I_n(\lambda_n s_0)}{\lambda_n I_n'(\lambda_n) + \beta I_n(\lambda_n)} \quad \text{for} \quad n \neq 0, \tag{34}$$

$$G_0^*(1; s_0) = \frac{1}{2\pi\beta} \qquad \text{for} \quad n = 0. \tag{35}$$

Combining (34) and (35), and substituting into (19) one finds

$$G(1, \theta; s_0, \theta_0) = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{2} \frac{I_n(\lambda_n s_0) e^{in(\theta-\theta_0)}}{\lambda_n I_n'(\lambda_n) + \omega I_n(\lambda_n)}. \tag{36}$$

Ultimately, a real expression for the temperature distribution is desirable, Thus the complex Fourier expansion (36) must be converted into its corresponding trigonometric form:

$$G(1, \theta; s_0, \theta_0) = \frac{a_0(s_0)}{2\pi} + \frac{1}{\pi} \sum_{n=1}^{\infty} a_n(s_0) \cos n(\theta - \theta_0) + b_n(s_0) \sin n(\theta - \theta_0). \tag{37}$$

Since the coefficients of the series (36) for $n < 0$ are complex conjugates of those for $n > 0$, it follows that

$$a_n(s_0) = Re\left[\frac{I_n(\lambda_n s_0)}{\lambda_n I_n'(\lambda_n) + \beta I_n(\lambda_n)}\right], \tag{38}$$

$$b_n(s_0) = -Im\left[\frac{I_n(\lambda_n s_0)}{\lambda_n I_n'(\lambda_n) + \beta I_n(\lambda_n)}\right]. \tag{39}$$

The relationship between the complex function $I_n(i^{1/2}n^{1/2}\zeta^{1/2}s_0)$ and the real functions, $\mathrm{ber}_n\,(n^{1/2}\zeta^{1/2}s_0)$ and $\mathrm{bei}_n\,(n^{1/2}\zeta^{1/2}s_0)$, is

$$I_n(i^{1/2}n^{1/2}\zeta^{1/2}s_0) = i^{-n}[\mathrm{ber}_n\,(n^{1/2}\zeta^{1/2}s_0) + i\ \mathrm{bei}_n\,(n^{1/2}\zeta^{1/2}s_0)] \tag{40}$$

from which follows

$$\lambda_n I_n'(i^{1/2}n^{1/2}\zeta^{1/2}s_0) = i^{-n}n^{1/2}\zeta^{1/2}[\mathrm{b\acute{e}r}_n\,(n^{1/2}\zeta^{1/2}s_0) + i\ \mathrm{b\acute{e}i}_n\,(n^{1/2}\zeta^{1/2}s_0)], \tag{41}$$

where the prime indicates differentiation with respect to the argument. Substitution of these into (38) and (39) yields

$$\begin{aligned} a_n(s_0) = \frac{1}{\Omega_n} \{ &[n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}r}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2})]\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2}s_0) \\ &+ [n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}i}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{bei}_n\,(n^{1/2}\zeta^{1/2})]\ \mathrm{bei}_n(n^{1/2}\zeta^{1/2}s_0)\} \end{aligned} \tag{42}$$

$$\begin{aligned} b_n(s_0) = \frac{1}{\Omega_n} \{ &[n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}i}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{bei}_n\,(n^{1/2}\zeta^{1/2})]\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2}s_0) \\ &- [n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}r}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2})]\ \mathrm{bei}_n\,(n^{1/2}\zeta^{1/2}s_0)\} \end{aligned} \tag{43}$$

where

$$\begin{aligned} \Omega_n = &[n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}r}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2})]^2 \\ &+ [n^{1/2}\zeta^{1/2}\ \mathrm{b\acute{e}i}_n\,(n^{1/2}\zeta^{1/2}) + \beta\ \mathrm{ber}_n\,(n^{1/2}\zeta^{1/2})]^2. \end{aligned}$$

Thus the trigonometric form for $G(1, \theta; r_0, \theta_0)$ is given by (37) with the coefficients $a_n(s_0)$ and $b_n(s_0)$ given by equations (42) and (43) respectively.

Solution for the temperature distribution. The nondimensional temperature variation $T^*(s_0, \theta_0)$ can now be found by substituting (37) into (18):

$$\begin{aligned} T^*(s_0, \theta_0) = \frac{\gamma}{2\pi} a_0(s_0) \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta \\ + \frac{\gamma}{\pi} \sum_{n=1}^{\infty} \left[a_n(s_0) \int_{-\pi/2}^{\pi/2} \cos n(\theta - \theta_0) \cos\theta\, d\theta + b_n(s_0) \int_{-\pi/2}^{\pi/2} \sin n(\theta - \theta_0) \cos\theta\, d\theta \right]. \end{aligned} \tag{44}$$

Upon evaluation of the integrals, Eq. (44) becomes

$$T^*(s_0\,,\,\theta_0) = \gamma\left[\frac{a_0(s_0)}{\pi} + \tfrac{1}{2}(a_1(s_0)\cos\theta_0 - b_1(s_0)\sin\theta_0) + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}(a_{2n}(s_0)\cos 2n\theta_0 - b_{2n}(s_0)\sin 2n\,\theta_0)\right]. \tag{45}$$

Substituting (45) into (6), and noting that $a_0(s_0) = 1/\beta$ one finds the temperature distribution $T(s_0\,,\,\theta_0)$ as

$$T(s_0\,,\,\theta_0) = T_0\left\{\frac{3}{4} + \gamma\left[\frac{1}{\pi\beta} + \tfrac{1}{2}[a_1(s_0)\cos\theta_0 - b_1(s_0)\sin\theta_0] + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}[a_{2n}(s_0)\cos 2n\theta_0 - b_{2n}(s_0)\sin 2n\theta_0]\right]\right\}. \tag{46}$$

The reference temperature T_0 has not yet been specified. The temperature distribution given by (46) is based on a linearization which assumes that variation about T_0 is small. Hence a logical choice for T_0 is the value of temperature at the centerline; also there is the advantage that this value can be found easily from equation (46) as

$$T_0 = T(0,\,\theta_0) = T_0\left\{\frac{3}{4} + \frac{\gamma}{\pi\beta}\right\}. \tag{47}$$

Substituting for β and γ from equations (12) and (13) gives the result*

$$T_0 = \left(\frac{aK_s}{\pi\sigma e}\sin\varphi\right)^{1/4}. \tag{48}$$

With the use of (48), the expressions for β and γ become

$$\beta = \frac{b}{k}\left(\frac{256}{\pi^3}\,\sigma e a^3 K_s^3 \sin^3\varphi\right)^{1/4}, \tag{49}$$

$$\gamma = \frac{\pi}{4}\,\beta. \tag{50}$$

Now the expression for the temperature distribution becomes

$$\frac{T(s_0\,,\,\theta_0)}{T_0} = 1 + \beta\left\{\frac{\pi}{8}[a_1(s_0)\cos\theta_0 - b_1(s_0)\sin\theta_0] + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}[a_{2n}(s_0)\cos 2n\theta_0 - b_{2n}(s_0)\sin 2n\theta_0]\right\}, \tag{51}$$

where the quantities T_0, β, ζ, $a_n(s_0)$, $b_n(s_0)$ are given by (48), (49), (11), (42), and (43), respectively.

Tabulated values of the functions $\mathrm{ber}_n\,x$, $\mathrm{bei}_n\,x$, $\mathrm{b\acute{e}r}_n\,x$, $\mathrm{b\acute{e}i}_n\,x$ for orders other than $n = 0$ are not available at this time. Hence no numerical results were calculated from Eq. (51) directly. However, in the next section, approximate solutions for $\zeta \ll 1$ and

*For the case of incoming parallel radiation normal to the geometric axis (i. e. $\varphi = \pi/2$), equation (48) becomes $T_0 = [aK_s/\pi\sigma e)]^{1/4}$. This is the reference value used by Charnes and Raynor [1] for the cylindrical shell.

$\zeta \gg 1$ and an exact solution for $\zeta = 0$ will be derived. Some numerical results were calculated from these solutions.

Solutions for $\zeta = 0$, $\zeta \ll 1$, $\zeta \gg 1$. For the case of very slow rotation ($\zeta \ll 1$), the modified Bessel functions can be approximated for small argument* by

$$I_n(x) \approx \frac{x^n}{2^n n!}\left[1 + \frac{x^2}{4(n+1)}\right], \tag{52}$$

$$I_n'(x) \approx \frac{x^{n-1}}{2^n n!}\left[n + \frac{(n+2)x^2}{4(n+1)}\right]. \tag{53}$$

Use of these approximations in Eqs. (38) and (39) yields

$$a_n(s_0) = \frac{[16(n+1)^2(n+\beta) + n^2\zeta^2 s_0^2(n+\beta+2)]}{16(n+1)^2(n+\beta)^2 + n^2\zeta^2(n+\beta+2)^2}\, s_0^n\,, \tag{54}$$

$$b_n(s_0) = \frac{4n\zeta(n+1)[(n+\beta+2) - s_0^2(n+\beta)]}{16(n+1)^2(n+\beta)^2 + n^2\zeta^2(n+\beta+2)^2}\, s_0^n\,. \tag{55}$$

Thus the approximate temperature distribution for $\zeta \ll 1$ can be found from (51) by using the coefficients (54) and (55).

With this introduction of $\zeta = 0$ into (54) and (55), the exact result for the temperature distribution in the absence of rotation becomes

$$\frac{T(s_0\,,\,\theta_0)}{T_0} = 1 + \beta\left[\frac{\pi}{8}\frac{s_0}{1+\beta}\cos\theta_0 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}\frac{s_0^{2n}}{(2n+\beta)}\cos 2n\theta_0\right]. \tag{56}$$

For high rotational speed, $\zeta \gg 1$, the modified Bessel functions can be approximated by

$$I_n(x) \approx \frac{e^x}{\sqrt{2\pi x}}, \tag{57}$$

$$I_n'(x) \approx \frac{e^x}{\sqrt{2\pi x}}. \tag{58}$$

Use of these approximations in (38) and (39) for the radial position of $s_0 = 1$ furnishes

$$a_n(1) = \frac{(n\zeta/2)^{1/2} + \beta}{[(n\zeta/2)^{1/2} + \beta]^2 + (n\zeta/2)}, \tag{59}$$

$$b_n(1) = \frac{(n\zeta/2)^{1/2}}{[(n\zeta/2)^{1/2} + \beta]^2 + (n\zeta/2)}. \tag{60}$$

Thus the approximate temperature distribution for $\zeta \gg 1$ can be found from Eq. (51) by using the coefficients (59) and (60).

Numerical example. To compute temperature distributions for the cases of $\zeta = 0$, $\zeta \ll 1$, $\zeta \gg 1$, values for the physical parameters were chosen for an aluminum alloy body analogous to that used by Charnes and Raynor [1]. These values are: radius $b = 1$ ft, thermal conductivity $k = 100$ Btu/ft hr °R, thermal diffusivity $\alpha = 3$ ft^2/hr, solar con-

*The magnitude of the argument varies as $(n\zeta)^{1/2}$. Thus it must be assumed that the series converges rapidly enough so that the significant terms do not violate the condition $(n\zeta)^{1/2} \ll 1$. In approximating the modified Bessel functions for small argument the first and second terms of the expansion were included, since using only the first term leads to the result for $\zeta = 0$.

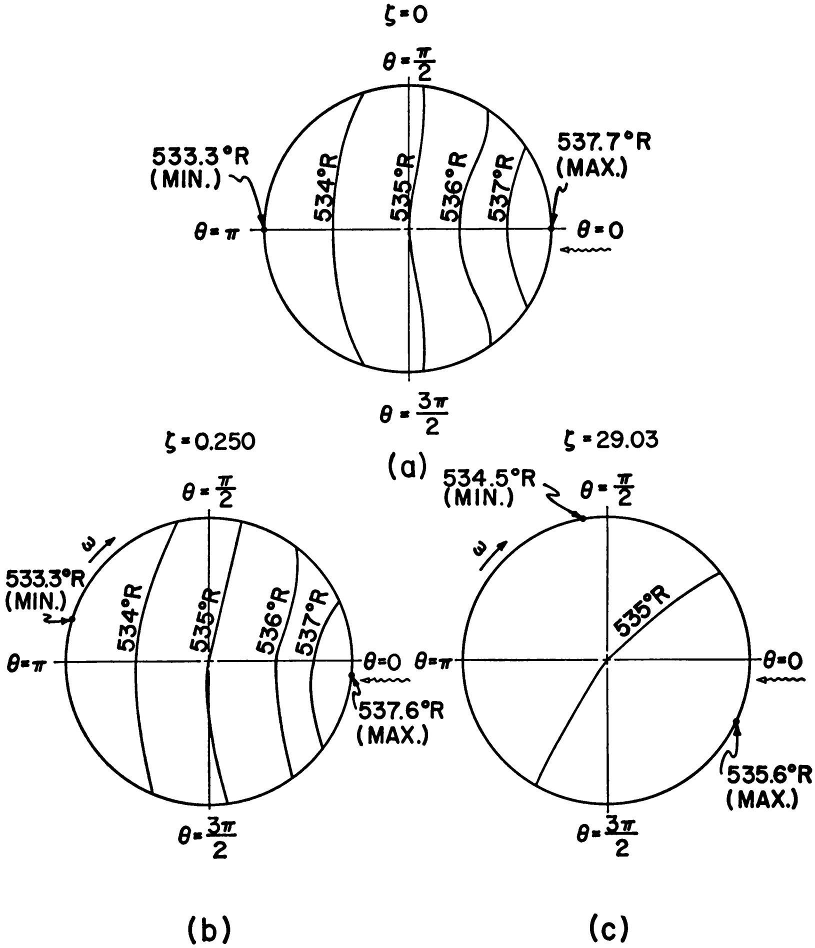

FIG. 2. Temperature Distributions for (a) No Rotation, (b) Slow Rotation, (c) Rapid Rotation.

stant $K_s = 442$ Btu/ft^2 hr, radiation constant $\sigma = 0.1717 \times 10^{-8}$ Btu/ft^2 hr °R^4, absorptivity $a = 1$, emissivity $e = 1$, angle of inclination $\varphi = \pi/2$. From this choice of values follows

$$T_0 = 535.03°\text{R}, \qquad \beta = 0.0105, \qquad \zeta = \omega/3,$$

where ω is given in radian per hour.

In the absence of rotation ($\zeta = 0$) the temperature distribution was computed from Eq. (56). The results, in the form of isotherm lines, are shown in Figure 2a.

For very slow rotation, $\omega = 0.750$ radians/hr or $\zeta = 0.250$, the temperature distribution was computed using the approximate coefficients (54) and (55). These results are shown in Figure 2b. It should be noted that even for this slow rotation there is a noticeable shift in the positions of maximum and minimum temperature.

For a high rotational speed,* $\omega = 87.10$ radians/hr or $\zeta = 29.03$, the asymptotic forms yield the results shown in Figure 2c.

*The case of $\zeta = 29.03$ corresponds to $V_0 = 10$ in the results of Charnes and Raynor [1].

Comparison of these numerical results with those of Charnes and Raynor [1] shows that the temperature distribution of a solid cylinder is considerably more uniform (maximum temperature difference of 4°F) than that of an analogous cylindrical shell (maximum temperature difference of 280°F).

Bibliography

1. A. Charnes and S. Raynor, *Solar heating of a rotating cylindrical space vehicle,* ARS Journal **30** (1960) 479
2. W. E. Olmstead and S. Raynor, *Solar heating of a rotating spherical space vehicle,* Internat. J. Heat Mass Transfer, to be published.
3. L. D. Nichols, *Surface-temperature distribution on thin walled bodies subjected to solar radiation in interplanetary space,* NASA Tech. Note D—584, (1961)
4. P. J. Schneider, *Conduction heat transfer,* Addison-Wesley, Reading, Mass., 1957, p. 284
5. G. N. Watson, *A treatise on the theory of Bessel functions,* 2nd ed., Cambridge, 1958, p. 80