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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime (′), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

BOOK REVIEWS

A collection of mathematical problems. By S. M. Ulam. Interscience Publishers, Inc., New York and London, 1960. xiii + 150 pp. $5.00.

The question as to which is of greater value to the development of mathematics, a good problem or a good solution, is very much like the chicken-and-egg paradox. The best of problems generate the best of solutions, and conversely. At the present time, dozens of universities of ever-increasing mathematical level are turning out hundreds of talented and highly trained practioners capable of working on and resolving clearly formulated mathematical problems. No comparable effort is being devoted to developing new areas for mathematical research, nor to new applications of known techniques in other parts of science.

Consequently, it is extremely refreshing and significant to see a book devoted purely to problems for which no solutions exist as yet. Even more important is the discussion of problem areas which still await precise formulation. The author frankly admits that the selection is subjective—as any such set must be. His stature as a mathematician guarantees its intrinsic worth.

To those interested in analysis and mathematical physics, which includes most of the readers of this journal, the book reads best from back to front. Probably the most fascinating chapter is the last, "Computing Machines as a Heuristic Aid." The description of this chapter as one devoted to the use of computers as experimental devices for mathematicians does not do justice to the many intriguing variations on this theme which Ulam presents. This chapter, at very least, should be required reading for any acolyte taking the vows of abstraction and obscurantism of the Bourbaki.

The seventh chapter, "Physical Systems," contains much of interest to those concerned with making mathematical models of physical processes, with particular reference to the mechanics of the continua and branching processes.

The sixth chapter, "Some Questions of Analysis," focuses on the basic problem of the stability of a mathematical model in a far deeper fashion than is customary. Although Hyers and Ulam have contributed some important results in this field, most of the basic questions are unanswered, and many have not even been asked.

The first five chapters are entitled in order, "Set Theory," "Algebraic Problems," "Metric Spaces," "Topological Spaces," and "Topological Groups." They contain a large number of explicit problems centering about point set topology, set theory and abstract spaces. With due respect for their historical importance, one has the feeling that there is little mathematical vitality left in these areas. Compared to the intensity that pervades the last three chapters, these chapters are rather the remembrance of things past.

Regardless of which parts of this book are particularly rewarding, everyone concerned with the present and future development of mathematics will find sources of inspiration in this scintillating volume.

RICHARD BELLMAN

Advances in computers. Edited by F. L. Alt. Associate Editors: A. D. Booth and R. E. Meagher. Vol. 1, Academic Press, New York, London, 1960. ix + 316 pp. $10.00.

According to the preface, this new series is to bring together articles about diverse aspects of the design of electronic computing systems and their components, about methods of machine programming and numerical analysis, and about computer application to scientific and engineering problems, to business management and data processing, to statistics, control of operations and "pseudological" problems, thereby making computer mathematicians, computer engineers, and computer users aware of each other's problems. The present volume contains the following articles: General-purpose programming for business applications, by C. C. Gotlieb—Numerical weather prediction, by N. A. Phillips—The present status of automatic translation of languages, by Yeho-shua Bar-Hillel—Programming computers to play games, by Arthur L. Samuel—Machine recognition of spoken words, by R. Fatehchand—Binary arithmetic, by G. W. Reitwiesner.

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BOOK REVIEWS

(Continued from p. 284)

Proceedings of the 1959 *Computer Applications Symposium.* Armour Research Foundation. ix + 155 pp. $3.00.

The volume contains 14 papers presented at a symposium held in October 1959, as well as brief accounts of two panel discussions. Leaving aside a number of papers on business applications, which are presumably of lesser interest to the readers of this QUARTERLY, we mention the following papers: Some aspects of computer technology in the U.S.S.R. (S. N. Alexander); Linear programming on the Bendix G-15 Computer (J. R. Wall); The international algebraic language and the future of Programming (C. Katz); Training for engineering and scientific applications via compilers, interpreters, and assemblers (W. F. Atchison); Scientific design procedures utilizing a small computer (T. I. Harris); Fortran experience and remote operation by non-computer specialists (F. Engel, Jr.).

Self-organizing systems. Edited by M. C. Yovits and S. Cameron. Pergamon Press, New York, 1960. ix + 322 pp. $8.50.

These are the Proceedings of an interdisciplinary conference held in May 1959 under the joint sponsorship of the Office of Naval Research (Informations Systems Branch) and the Armour Research Foundation. Aside from the Welcome Address (H. A. Leedy) and the Opening Address (F. J. Weyl) and an after-dinner speech (A. M. Uttley), the volume contains 13 papers arranged in the following groups: *Perception of the environment* (B. G. Farley, H. von Foerster, W. K. Estes, F. Rosenblatt)—*Effects of Environment Feedback* (R. Auerbach, S. Goldman, G. H. Bishop)—*Learning in finite automata* (A. Newell, J. C. Shaw, and H. A. Simon, P. M. Milner, D. T. Campbell)—*Structure of self-organizing systems* (G. Pask, W. S. McCulloch, A. W. Burks). There are also short accounts of two panel discussions.

An introduction to linear programming and the theory of games. By S. Vajda. Methuen & Co. Ltd., London, and John Wiley and Sons, Inc., New York, 1960, 76 pp. $2.25.

This brief introductory text was not written for mathematicians. It is directed rather towards students of the business and physical sciences who would like to take a cursory look into the subjects of linear programming and game theory. Mathematicians would be better advised to read Dr. Vajda's previous volume, *The Theory of Games and Linear Programming* (Methuen & Company and John Wiley and Sons, 1956).

The exposition in the present book is almost exclusively in terms of the most simple numerical examples and provides, therefore, only a superficial view of the subject matter.

Part I contains a brief discussion of linear programming. A wide variety of subjects are covered including the transportation problem, the general linear programming problem and its geometrical representation and the notion of duality. However, the nature of the descriptions is such that the reader must not expect to solve any linear program after having read this text alone.

Part II on game theory is even more brief. The graphical solution of zero-sum two-person games is discussed, and their reduction to linear programs is exhibited. Non-zero-sum games, many-person games and infinite games are mentioned but only in passing.

The duality theorem of linear programming and the main theorem of game theory are proved in the appendices.

Despite the fact that the book is not primarily intended for them, mathematicians might profit from reading certain portions of the text. In particular, the discussion of "shadow prices" and their interpretation on pages 43–44 will be of value to many mathematicians unversed in economics. The exposition in Appendix I of Dantzig and Orden's proof of the duality theorem is also recommended.

Unfortunately, the proof of the duality theorem contains numerous typographical errors. The

(Continued on p. 350)

BOOK REVIEWS

(Continued from p. 314)

most important of these are: (1) in the tableau on page 46 the first entry of the third column should read "z_{10}"; (2) the last equation of page 46 should be

$$\sum_{j=1}^{n} a_{ij}y_{n+j} - y_i = b_i \; ; \qquad i = 1, \cdots, \cdot$$

(3) the last term in the second from the last equation of page 48 should read

$$\sum_{i=1}^{m-s} b_i \sum_{j=1}^{m} d_{ij}c_j$$

The reader is also cautioned against missing subscripts and missing or erroneous indices on summation signs both in Appendix I and in a few isolated places in the main text.

W. S. DORN

Applied Boolean algebra—an elementary introduction. By Franz E. Hohn. The Macmillan Co., New York, 1960. xx + 139 pp. $2.50.

This is an elementary book about Boolean algebra and its applications to switching theory. It is intended for students of either mathematics or engineering and assumes very little background in either field. The book is too short to be the main textbook of a course. However, it might be used at the start of an undergraduate course in switching circuits or as a sidelight to a course in logic.

The introduction develops Boolean algebra formally from a large set of axioms. There follow chapters on relay circuits, propositional logic, the algebra of sets, minimization (Karnaugh-Veitch charts), and appendices on binary notation and semiconductor logic elements. Only combinational circuits are considered. Sequential circuits should have been mentioned briefly at least. This book can give the impression that switching circuits do only such simple things as may be expressed in terms of Boolean functions.

The author has a clear expository style and uses many illustrative examples. There are 70 exercises.

E. N. GILBERT

Advances in applied mechanics. Volume VI. Edited by H. L. Dryden, Th. von Kármán, and G. Kuerti. Academic Press, New York and London, 1960. x + 294 pp. $9.00.

The volumes of this series generally collect articles which neither fit into the standard periodicals nor, somehow, can be published, as one would wish them to be, as individual books or pamphlets. In the volume here under review, all articles deal with topics in fluid mechanics, but since that field is large, and the articles are very specialized, this does not add much to their common denominator.

It is accordingly difficult to find authoritative reviewers, and the present one admits all the more freely his incompetence to review one of the major articles, by Ludwig and Heil, on Boundary Layers with Dissociation. Most of this article concerns the Kinetic Theory of a pure diatomic gas capable of dissociation and ionisation, of which an extensive treatment is given. This is followed by a derivation and discussion of the Boundary Layer Equations with Dissociation not differing greatly from those found in other recent books.

The other major article, by Oswatitsch, is a book on the aerodynamics of thin wings and slender bodies at all but the lowest flight speeds, from the point of view of the similitude concepts. It suffers from compression into less than 120 pages. The full story of similitude is shown to be anything but uncomplicated, and the vast array of facts assembled by Oswatitsch makes the account highly valuable to the engineer concerned with this class of aerodynamic problems.

Stewartson's article on Unsteady Laminar Boundary Layers is addressed to the applied mathematician working in, or about to enter, the field. The main problems discussed are the compressible fluid motion caused by the impulsive motion of a solid surface so as to remain coincident with itself

in a geometrical sense, the effect of fluctuations in the boundary conditions on steady boundary layers, and boundary layer formation in shock tubes. These problems, notable for both their significance and difficulty, have been attacked by a number of experts with a partial success which has led to learned controversy. Stewartson, himself one of the foremost contributors to knowledge in the field, now offers a concise essay on the known results and their relation to each other and to the questions which one would like to see answered. The article may not end all controversy, but it can be overlooked by workers in this field only at their own peril.

The article on Shock Propagation along Ducts by Chester reproduces much of Whitham's stimulating paper (J. Fluid Mech. 4, 1958, 337) and extends his conjecture to shocks travelling into a gas in steady flow, rather than only a gas at rest, but adds little to the elucidation of the puzzles pointed out by Whitham and Chisnell. Chester also offers a clear exposition of the asymptotic states resulting after a shock has passed from one long, straight duct to another through a contradiction or expansion, making available in the literature the results of Laporte.

The volume closes with a brief and very pretty essay by Wille on what we know, and what we should like to know, about Karman Vortex Streets.

R. E. MEYER

Elements of the theory of Markov processes and their applications. By A. T. Bharucha-Reid. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1960. xi + 468 pp. $11.50.

The purpose of this book is "first, to present a nonmeasure-theoretic introduction to Markov processes, and second, to give a formal treatment of mathematical models based on this theory which have been employed in various fields". The prerequisites are an understanding of elementary probability and statistics plus about graduate-level maturity in mathematics generally. The first three chapters describe separately the theory for Markov processes which are 1) discrete space and time, 2) discrete in space and continuous in time, and 3) continuous in space and time. The remaining six chapters give selected applications to biology, physics, astronomy, chemistry, and queues. The emphasis is almost everywhere on topics of current interest and the bibliography is very extensive, particularly for recent works.

Although there is a great need for a book with the features described above, this work does not fill the need as well as one might hope mostly because it shows every symptom of having been written in a hurry. Errors of all sorts occur with excessive frequency, not only the expected misprints but incorrect or ambiguous statements of theorems and errors due to sudden changes in notation which can easily go unnoticed. Despite the attempt to cover a wide selection of topics, there is a tendency to treat some problems in considerable detail and only briefly mention others. There is particular emphasis on the branching processes with their applications to population and epidemic studies, the theory of cascades and the theory of queues. Since much of the material is taken from original papers and has not yet appeared in other books, this book will certainly serve a useful purpose as an introduction or survey of the literature on selected topics.

G. F. NEWELL

Lineare Planungsrechnung (linear programming). By Martin J. Beckmann. Fachverlag für Wirtschaftstheorie und Ökonometrie, Ludwigshafen Am Rhein, 1959. X + 118 pp. $3.32.

Although the literature in English on linear programming is certainly plentiful, we are envious nevertheless that Beckmann chose to write this short volume in German. He has achieved an admirable balance between verbal and mathematical presentation in an essentially expository monograph.

Beckmann assumes his readers are basically interested in economic theory, and consequently he draws many of his examples from classical economics literature; in so doing, he concomitantly provides the non-specialist with a helpful introduction to this branch of mathematical economics.

The principal theme of the book might be stated as "the linear programming model: applications and implications." In pursuing this, Beckmann concentrates on a variety of examples, theoretical and numerical, each about two or three pages in length. As might be anticipated, references to the dual

theorem and its derivatives are frequent. In contrast, the only computational technique mentioned in any detail is the simplex method, and this appears ten pages from the end of the book. Given the decision to shy away from the specifics of computation, Beckmann carefully designs the few pages on the simplex method so as to make the method apparent; by keeping solely to the mathematical principles involved, he overcomes here, as elsewhere in the book, the common confusion emanating from an explanation based on an involved and detailed numerical example.

The material presentation is not meant for mathematically unsophisticated readers, but the level of difficulty is certainly not beyond, say, those graduate students in economics, engineering, and mathematics who would feel a need to learn about linear programming. Beckmann's book represents a valuable addition to the literature of mathematical economics and would also be welcomed in English translation.

HARVEY M. WAGNER

Einführung in die Differentialgeometrie. By W. Blaschke and H. Reichardt. Second edition. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1960. vii + 173 pp. $5.77.

The principal changes made in this second edition are as follows: a section on the differential geometry of subspaces of the n-dimensional Euclidean space written by H. Reinhardt has been added, and relevant literature published since the appearance of the first edition has been taken into account.

Stresses in shells. By W. Flügge. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1960. xi + 499 pp. $14.14.

For a quarter of a century, the author's "Statik und Dynamik der Schalen" (Berlin, 1934) has been a standard reference in shell theory. Though having slightly more than twice the number of pages, the present work follows the same general plan as the earlier volume, the major changes being as follows: the chapters on the membrane theory of translation surfaces and affine shells have been replaced by a chapter on the membrane theory of arbitrary shells; the chapter on folded structures has been replaced by sections added to the chapters on the membrane theory and the bending theory of cylindrical shells; the chapter on vibrations of shells of revolution has been omitted; and a brief Appendix on forces and deformations in circular rings has been added.

The introductory chapter on the general properties of stress systems in shells is substantially unchanged, except that the section on stressing reinforced concrete shells has been omitted and a section on Mohr's circle has been added. When the rearrangements mentioned above are taken into account, it is found that all other chapters have nearly doubled in size. Most of this increase can be traced to the considerably greater variety of shell geometries and types of loading that are discussed, a feature which greatly enhances the value of the book to design engineers.

W. PRAGER

Mathematical methods in the social sciences. (Proceedings of the First Stanford Symposium, 1959.) Edited by K. J. Arrow, S. Karlin, and P. Suppes. Stanford University Press, 1960. viii + 365 pp. $8.50.

These Proceedings of a symposium held at Stanford University in June 1959 contain 23 papers organized in three sections as follows.

Economics: Price-quantity adjustments in multiple markets with rising demands, by K. J. Arrow—Topological methods in cardinal utility, by G. Debreu—Optimality and informational efficiency in resource allocation processes, by L. Hurwicz—Matrices with dominant diagonals and economic theory, by L. McKenzie—A reconsideration of the Walras-Cassel-Leontief model of general equilibrium, by Michio Morishima—Efficient paths of capital accumulation in terms of calculus of variations, by P. A. Samuelson—Investment and technical progress, by R. M. Solow—The operational implications of imperfect models, by H. Theil and T. Kloek—Preference and rational choice in the theory of consumption, by Hirofumi Uzawa.

Management Science: General convex objective forms, by G. B. Dantzig—A stationary inventory model with Markovian demand, by S. Karlin and A. J. Fabens—Geometric ergodicity and the theory of queues, by D. G. Kendall—The optimality of (S, s) policies in the dynamic inventory problem.

Psychology: Some stochastic process models for intelligence test models, by T. W. Anderson—A theory of stimulus discrimination learning, by R. C. Atkinson—Some two-person interactions, by C. J. Burke—Some properties of Luce's beta model for learning, by R. R. Bush—A random walk model for choice behavior, by W. K. Estes—Some comments on stochastic models and psychological theories, by E. Galanter and G. A. Miller—Response latencies and probabilities, by R. D. Luce—Binary choice constraints and random utility indicators, by J. Marshak—Measures of worth in item analysis and test design, by H. Solomon—Stimulus-sampling theory for a continuum of responses, by P. Suppes.

Combinatorial analysis. Edited by R. Bellman and M. Hall, Jr. Proceedings of Symposia in Applied Mathematics, Vol. 10. American Mathematical Society, Providence, R. I., 1960. vi + 311 pp. $7.70.

These Proceedings of a symposium held at Columbia University in April 1959 contain the following contributions: Current studies on conbinatorial designs, by M. Hall, Jr.—Quadratic extensions of cyclic planes, by R. H. Bruck—On homomorphisms of projective planes, by D. R. Hughes—Finite division algebras and finite planes, by A. A. Albert—The size of the 10×10 orthogonal Latin square problem, by L. J. Paige and C. B. Tompkins—Some combinatorial problems on partially ordered sets, by R. P. Dilworth—An ennumerative technique for a class of combinatorial problems, by R. J. Walker—The cyclotomic numbers of order ten, by A. L. Whiteman—Some recent applications of the theory of linear inequalities to extremal combinatorial analysis, by A. J. Hoffman—A combinatorial equivalence of matrices, by A. W. Tucker—Linear inequalities and the Pauli principle, by H. W. Kuhn—Compound and induced matrices in combinatorial analysis, by H. J. Ryser—Permanents of doubly stochastic matrices, by M. Marcus and M. Newman—A search problem in the n-cube, by A. M. Gleason—Teaching combinatorial tricks to a computer, by D. H. Lehner—Isomorph rejection in exhaustive search techniques, by J. D. Swift—Some discrete variable computations, by Olga Taussky and J. Todd—Solving linear programming problems in integers, by R. E. Gomory—Combinatorial processes and dynamic programming, by R. Bellman—Solution of large-scale transportation problems, by M. Gerstenhaber—On some communication network problems, by R. Kalaba—Directed graphs and assembly schedules, by J. D. Foulkes—A problem in binary encoding, by E. N. Gilbert.

Tables of sines and cosines to ten decimal places at thousands of a degree. By H. E. Salzer and N. Levine. Pergamon Press, New York, 1962. xiv + 901 pp. $10.00.

The tables appear to be photo-offset reproductions of computer output. The sine and the cosine for each argument are tabulated side by side each entry being given with all its digits.

Fundamentals of celestial mechanics. By J. M. A. Danby. The Macmillan Co., New York, 1962. xiii + 348 pp. $8.00.

This text, based on courses given originally to undergraduate students at the University of Minnesota, is intended not only for astronomers, but also for mathematicians, physicists, engineers, and others who are not necessarily ready for analytical dynamics. However, the readers of this text should be familiar with elementary calculus, including elementary differential equations, and analytic geometry; a short review of properties of conics is included in the appendix.

In chapter 1 the author defines the basic vocabulary of astronomy and the systems of astronomical coordinates. Chapters 2 and 3 introduce vectors and vectorial mechanics. He uses vector notation whenever possible, and in these two chapters gives some idea how to use vector notation in mechanics and spherical trigonometry.

Chapter 4 concerns central orbits in analytical dynamics; some of the many problems are taken from other text books, such as Whittaker's Analytical Dynamics. In chapter 5 the author explains

some properties of solid bodies, starting from definitions of the center of mass and of moments of inertia and going on to the potential of homogeneous ellipsoids and ellipsoidal figures or rotating fluid masses.

Chapter 6 is concerned with the two-body system, and here actual problems in celestial mechanics come in. The author explains the properties of the two-body system and gives many problems, some of which are rather difficult. Formulas necessary for orbit determination are presented. The idea for determining orbits from observations is explained in chapter 7, without complicated formulas.

Chapter 8 treats the three-body system very briefly. The author explains Jacobi's integral of the restricted problem of three bodies, Tisserand's criterion for the identification of comets, zero-velocity curves, and equilibrium points and their stability. Chapter 9 defines the invariable plane and the force function of the solar system.

Before entering into perturbation theory, the author gives in chapter 10 the formulas of interpolation, numerical differentiation and integration.

The title of the next chapter is perturbations. The author first explains very briefly Cowell's and Encke's methods for perturbations and mentions the advantages and disadvantages of each. He derives formulas of variations of constants by geometrical or vectorial consideration and shows how to solve by an elementary method the equations for the motion of a satellite in the field of an oblate planet.

In chapter 12 the author treats the actual motion of the moon by solving the equations of motion by an elementary method.

In chapters 13 and 14 rotational motions of the earth and of the moon are treated, and precession, nutation, Cassini's laws, and the librations of the moon are explained.

Since approximately the first half of the text is devoted to preparation for the study of celestial mechanics, the subject itself covers only about 200 pages. However, the author is able to treat it very broadly, from orbit determination and two-body problems to the three-body problem and lunar motion, because he uses very elegant vector notations and avoids long and complicated formulas that occupy a major portion of the usual book on celestial mechanics. And when the reader has solved all the problems given in this text, he will have doubled the number of the pages of the book.

However, although the reader will become acquainted with techniques for solving the equations of motion and for determining orbits, he will not find stated in this text any new problems to take up in future. He may feel that every problem of celestial mechanics has been or can be solved. The book lacks, for example, an explanation of why we had to adopt perturbation methods to solve equations. The author should have mentioned that we cannot solve the equation by an exact analytical method, a goal that remains to be achieved. Complete explanations of gaps in the distributions of asteroids and in Saturnian gaps do not exist.

From the astronomer's point of view, an undergraduate course in celestial mechanics must stimulate the student's interest in celestial mechanics and staronomy. Some students may be interested in orbit determination or in perturbation theory. Other students may be attracted to the chapter on the three-body problem, which is purely mathematical. I feel that this text has an advantage in including a chapter on the three-body problem for this reason. The student may take up celestial mechanics as his major because he is attracted by such a mathematical problem, even though he is not interested in orbit determination or computing perturbations.

I would like to make one comment from the technical point of view. The reader may be confused when he reads the section on the motion of a satellite. There, secular terms are mentioned as long-periodic terms, without any further explanation. The readers must be careful to recognize the difference between planetary and satellite theories due to different time scales.

I can recommend this book to both undergraduate and graduate students who want to study astronomy or related subjects. I admire the author for writing a text on a complex subject in so precise and compact a way.

Y. KOZAI

Fundamental problems in turbulence and their relation to geophysics. Edited by F. N. Frenkiel. American Geophysical Union, Washington, D. C., 1962. 232 pp. $5.00.

This book contains 31 of 40 papers presented at an International Symposium on Fundamental Problems in Turbulence and their Relation to Geophysics held at Marseilles in September 1961. The symposium was arranged by a committee nominated by the International Union of Geodesy and Geophysics and the International Union of Theoretical and Applied Mechanics for the purpose of bring-

ing together fluid dynamicists and geophysicists whose activities overlap through an interest, voluntary or otherwise, in "turbulence". 80 scientists from 10 countries participated. The proceedings of this symposium are also contained in Journal of Geophysical Research, Volume 67, Number 8, July 1962.

No self-respecting fluid dynamicist will venture a definition out of context of "turbulence". "Geophysics" gives rise to worse difficulties. According to common usage it can mean "physics applied to geology", "physics of the solid Earth" or "physics of the whole Earth". To complicate matters, the *avant garde* would add to physics of the whole Earth, planetary physics and even astrophysics, this generalization having a parallel in the use of the word "geometry".

Represented at Marseilles were meteorology, upper atmospheric physics, oceanography and astrophysics. The range of topics covered is evident from the following list of papers, some by recognized experts in their field of specialization:- Some specific features of atmospheric turbulence, A. M. Obukhov; Structure of turbulence in stratified media, R. Bolgiano, Jr.; Recent broad-band spectral measurements of turbulence in the lower atmosphere, F. Pasquill; Laboratory measurements of turbulent diffusion in stratified flows, T. H. Ellison; A uniform gradient turbulent transport experiment, H. K. Wiskind; Turbulence in the presence of a vertical body force and temperature gradient, Robert G. Deissler; Thermal turbulence at very small Prandtl number, Edward A. Spiegel; Intensification of the Earth's magnetic field by turbulence in the ionosphere, H. K. Moffatt; Production of longitudinal vortices in the boundry layer along a concave wall, Itiro Tani; Some mathematical models generalizing the model of homogeneous and isotropic turbulence, A. M. Yaglom; On the Eulerian-Lagrangian transform in the statistical theory of turbulence, C. F. Wandel and O. Kofoed-Hansen; The vertical distribution of wind and turbulent exchange in a neutral atmosphere, Alfred K. Blackadar; Empirical data on turbulence in the surface layer of the atmosphere, A. S. Monin; Some suggestions for experimental studies of atmospheric turbulence, summary, Robert Betchov; Upper atmospheric turbulence determined by means of rockets, J. E. Blamont and C. de Jager; On the problem of formulating a realistic model of the general atmospheric circulation, D. R. Davies and M. B. Oakes; Convergence of horizontal flux of water vapor in the general circulation of the atmosphere, G. B. Tucker; Recent developments in the theory of wave generation by wind, O. M. Phillips; Energy transfer in an isotropic turbulent flow, Yoshimitsu Ogura; Perturbation analysis of the Navier-Stokes equations in Lagrangian form with selected linear solutions, Willard J. Pierson, Jr.; The budget of turbulent energy in the lowest 100 meters; H. A. Panofsky; Aerodynamic roughness of the sea, E. L. Deacon; Velocity measurements in the deep water of the Western North Atlantic, summary, J. Crease; Determination of the rate of dissipation of turbulent energy near the sea surface in the presence of waves, R. W. Stewart and H. L. Grant; Measurements of turbulence near the sea bed in a tidal current, K. F. Bowden; Integral diffusivity, J. C. Schönfeld; On the spectrum of the mean diffusion velocities in the ocean, J. Joseph and H. Sendner; Diffusion in the diabatic surface layer, F. A. Gifford; Studies of turbulent diffusion of dye patches in the ocean, Takashi Ichiye; On the interpretation of smoke diffusion and wind analysis data at Risø, O. Kofoed-Hansen; Some optical properties of turbulence in stratified flow near the ground, D. J. Portman, F. C. Elder, E. Ryznar and V. E. Noble.

This range is so wide that the reviewer is hardly competent to give a critical discussion of all the papers, and it would be invidious to single out a few for special mention. The proceedings have been ably put together by Dr. F. N. Frenkiel, who has been indefatigable in recent years in organizing symposia aimed at cross-fertilization by bringing fluid dynamicists into contact with scientists from other disciplines. The reviewer was disappointed that on this occasion it had been decided not to report the discussions which presumably followed the presentation of each paper.

R. HIDE

An introduction to mathematical machine theory. By Seymour Ginsburg. Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962. ix + 148 pp. $8.75.

This book is an introduction to selected topics in the theory of mathematical machines. Roughly speaking a mathematical machine is a finite state device whose output (if any and next state are uniquely determined by its input and present state. Three types of machines are studied. These are sequential machines, abstract machines, and tape recognition devices. Turing machines and the theory of computability are not discussed.

The first two chapters are devoted to complete and incomplete sequential machines respectively. These are considered from the point of view of terminal states; that is, input sequences versus output sequences. The topics of equivalent states, equivalent machines, reduction, and synthesis are treated in a thorough and orderly manner. Chapter 3 deals with abstract machines (which are a special case of "quasi-machines") from the same point of view but with somewhat less thoroughness. The fourth and final chapter discusses several types of tape recognition devices or automata and the "regular sets" of tapes which they "accept." However there is no explicit mention of Kleene's theorem, and the full significance of regular sets is left somewhat in doubt.

The book is written in an extremely formal style, consisting almost exclusively of definitions, lemmas, theorems, examples, and exercises. In addition it contains a number of logical faults, such as the intermingling of the definition of a set with the proof of its properties. Fortunately the effects of this combination are largely mitigated by the exercises and examples, which are plentiful, pertinent, and clear. The reader who makes full use of these will have little difficulty following the train of thought and will be well rewarded for his efforts.

W. S. BROWN

Creep in structures. Edited by N. J. Hoff. Academic Press, New York; Springer-Verlag, Berlin, 1962. viii + 375 pp. $15.00.

This volume contains the papers and some of the discussion presented at an international colloquium that was held at Stanford University under the sponsorship of the International Union of Theoretical and Applied Mechanics and the National Science Foundation. Grouped by subjects in the way they were presented at the colloquium, there are two papers on viscoelasticity (Stress distribution analysis for linear viscoelastic materials, by E. H. Lee and A. H. Corneliussen; Torsional response of linear viscoelastic plates subjected to thermal stresses, by H. H. Hilton and E. L. Walsh), two on creep stress distributions (On the creep stress analysis of some structures, by S. A. Patel and B. Venkatraman; The shear center in creep of beams of thin-walled open cross sections, by A. Phillips), three on creep laws (Mechanics of creep and combined stresses, by J. Marin; Lifetime of structures subjected to varying load and temperature, by S. Taira; The effect of incremental loading on creep behavior of metals, by E. T. Onat and T. T. Wange, two on large deformations (Applicability of the elastic analogue to creep problems of plates, membranes and shells, by F. K. G. Odqvist; Oil canning problems in creep, by J. Hult), three on disks and plates (Investigation of transient creep in thick-walled tubes under axially symmetric loading, by J. F. Besseling; A comparison of flow criteria applied to elevated temperature creep of rotating discs with consideration of the transient condition, by A. M. Wahl; Bending of a plate with non-linear strain-hardening creep, by T. H. Line, two on cylindrical shells (Effect of creep stresses on cylindrical shells, by H. Poritsky; On the creep of a wrinkle, by C. R. Calladine), three papers on columns (On the mechanics of column creep, by R. L. Carlson and W. W. Breindel; Comparison of ranges of applicability of predictions of creep buckling time, by B. M. Lempriere; Geometrically non-linear creep buckling of bars, by M. Zyczkowski), and three papers on miscellaneous subjects (Sur un modèle apte à traduire le fluage sous charge constante des structures, by R. Mazet; Some simple models for torsional creep buckling, by A. H. Chilver; Damping of the vibrations of a coiled spring due to creep, by N. J. Hoff).

W. PRAGER

Approximate calculation of integrals. By V. I. Krylov. Translated by A. H. Stroud. The Macmillan Co., New York, 1962. x + 357 pp. $12.50.

This new volume in the ACM Monograph Series is a welcome translation of a Russian text published in 1959. Except for a short section on the application of certain quadrature formulas for single integrals to the approximate evaluation of some classes of double and triple integrals, the book is exclusively concerned with single integrals. Part I (62 pp.) contains preparatory material on Bernoulli numbers and polynomials, orthogonals polynomials, interpolation, and linear normed spaces and linear operators. Part II (209 pp.) treats definite integrals; it contains chapters dealing with quadrature sums, interpolatory quadrature, quadratures of the highest algebraic degree of precision, minimization of

the remainder of quadrature formulas, quadrature formulas with preassigned nodes, quadrature formulas with equal coefficients, methods of increasing the precision of quadrature formulas, and convergence of the quadrature process. Part III (60 pp.) is devoted to indefinite integrals; in addition to an introductory chapter concerned with error, convergence and stability, it contains chapters on integration of functions given in tabular form, calculation of indefinite integrals using few values of the integrand, and methods using several previous values of the integrand. Three appendices contain tables for Gauss-Legendre, Gauss-Hermite, and Gauss-Laguerre quadrature.

W. PRAGER

Programming systems for electronic computers. By D. M. Chorafas. Butterworth Inc., Washington, D. C. and London, 1962. xvi + 188 pp. $9.25.

The section titles are "The programming problem", "Early programming media" (the author uses "media" as a synonym for "techniques"), "Procedure oriented languages" (in this section Fortran, Algol and Cobol are described), "Processing systems" and "Advanced programming concepts". The reader is introduced to a large number of interesting ideas in the advanced programming field. The book may well stimulate a casual reader to a greater interest in the programming art. However, the reviewer finds himself quite unable to recommend it, for the following reasons: The book was written in 1961 and published in 1962, yet in the section on Algol the author describes some details of the 1958 version which were rendered obsolete by the later version of 1960. Although the author claims that the book "reviews the evolutionary process of programming in considerable detail", knowledge of the existence of two versions of Algol is withheld from the reader. Whereas an experienced programmer is likely to be annoyed by the many omissions and misleading discussion scattered throughout the book, the beginner, while happily unaware of the former, will have considerable trouble in following the latter, for many terms peculiar to the jargon of programmers are used without explanation (for example, "floating" on p. 15, "upper-card loader", "BCD", and "core" on p. 16, "index register" on p. 54, "trap" on p.84 and "actual parameter" on p. 88. Comprehension is made even more difficult by the use of a very peculiar English style. As an example, selected from many, the reviewer was quite unable to understand the following sentence (from p. 125); "This can be expressed as follows: Is a file created with a printed record of the processing operation which could be integrated above what this particular program does, and how to run it?". One cannot help but wonder what the author is trying to say when he writes (or was it the typesetter?); "Arithmetic expressions are defined as a number, a variable (other than Boolean), or a function expressed in one of the forms: K, L, 285." Simple errors, such as the use of a Greek letter gamma as an example of an Algol expression, and the many printer's errors are relatively unimportant.

B. A. CHARTRES

A guide to ALGOL programming. By Daniel McCracken. John Wiley & Sons, Inc., New York, London, 1962. vii + 106 pp. $3.95.

The present book is the continuation of a series of books by the same author. The author's aim is to introduce the reader to preparing problems in science and engineering for solution by digital computers. Thus, he develops the notion of algorithms and the use of block diagrams by discussing a very simple example, and he raises the question of the algorithmic description acceptable to a computer. The rest of the book is devoted to the description of the essential features of ALGOL, the algorithmic language proposed for the international communication between scientists as well as for communication with a computer.

The testbook could do a very good job in making ALGOL more familiar to scientists or could be recommended for use in ALGOL courses if it did not contain numerous errors and misleading examples.

For example, in Fig. 2.2 the use of subscripted variables in normal mathematical formulas and the use of unsubscripted variables in the corresponding ALGOL statements are confusing without further explanation.

In section 3.2 *goto* L is said to be the general form of a goto statement, where L stands for an explicit label, although later on the author treats also conditional designational expressions and switches. An

analogous contradictory wording occurs in the discussion of the conditional statement: in the general form of the conditional statement the statement following *then* is at first allowed to be any statement and later on the author corrects himself saying that it must be unconditional.

Figure 3.3 is incomplete as there is no declaration for the variable two pi. Moreover, the Stop statement is not an ALGOL statement as correctly stated in the text, but it is not needed, by definition, in an ALGOL program. The fact that such Stop statements are sometimes given and sometimes omitted without motivation must be confusing to the reader.

Although the author defines quite precisely a for-statement and a for-clause, he several times uses the term statement referring to a for-clause. Furthermore, the explicit description of the operation of a while-construction in a for-clause (page 38) is incorrect insofar that S_2 is supposed to be a statement and thus it cannot appear in a goto-statement as label.

The examples of blocks given in figures 5.2 through 5.6 and all exercises given in chapter 5 together with the corresponding answers are misleading and make no sense, because some of the variables are only correctly declared but are not assigned actual values. Therefore, the program segments operate, on one hand, on undefined variables and, on the other hand, the computer values of some variables are lost upon exit from the block. The reviewer considers this to be the most serious error in the book, since the examples may give at first an entirely wrong idea of the block structure discussed in the subsequent chapter.

It is to be regetted that the author mixes the handy ALGOL notation for numbers with the restrictive notation of FORTRAN. Using two different notations for constants even in the same line without any reason is confusing, and seems to indicate that the author thinks in terms of FORTRAN while writing a book on ALGOL.

The general impression of a hastily written book is enhanced by many minor errors such as a missing multiplication sign in the last line of Fig. 7.3 or missing declarations for the variables w, x and y and the matching *end* in Fig. 7.5 and the absence of references to published papers in the field.

H. R. SCHWARZ

Advances in computers. Edited by Franz L. Alt and Morris Rubinoff. Volume 3. Academic Press, New York, London, 1962. xii + 361 pp. $12.00.

The present volume of this interesting series treats various modern computer applications, thus constituting a sample of today's computer science and technology. The contributions describe not only scientific problems, but deal also with problems in data processing, discuss appropriate methods of numerical analysis and of computer organization.

The first article by Samuel D. Conte which is entitled "The Computation of Satellite Orbit Trajectories" introduces the reader to the problems of artificial satellites and gives an extensive description of the methods of orbit prediction and determination on high speed computers. The various methods available for the numerical integration of the equations of a satellite's motion are discussed in detail and compared from the points of view of speed, over-all accuracy, flexibility, complexity of the formulas, and the storage requirements of the corresponding subroutines. The most important problem, the accumulation and propagation of round-off errors due to the finite number of digits held in a computer is discussed in an expository way for the methods of Runge-Kutta and predictor-corrector type as well as for perturbation methods. Several suggestions for studying the accuracy of a trajectory as computed by a certain mathematical formulation and by a given method of numerical integration are presented and their use is discussed. Whereas the first part of the article deals with methods of predicting the trajectory of a satellite to be launched, this very interesting article of great actuality terminates with a survey of the problems and methods for determining the actual orbit of a satellite that has been launched.

E. F. Codd's contribution entitled "Multiprogramming" deals with the computer-oriented question of how to organize a machine in order that it may be used in an optimal and most efficient way to minimize the execution time for a certain workload. With a STRETCH-like system in mind, the author discusses in full detail all the organizational problems arising when a set of programs or program segments is being executed in a concurrent manner sharing time and space of the computer.

"Recent Developments in Nonlinear Programming" by Philip Wolfe presents a valuable survey

of the majority of techniques that have been proposed for solving nonlinear programming problems during the last few years. The general mathematical problem is stated and discussed geometrically.

The objective function is assumed to be concave and the constraint functions are supposed to be convex, assumptions that give the nonlinear programming problem the important property that any local solution is the extremal solution. Geometrical considerations lead in a natural way to gradient methods which may be applied in the general case. Large-step gradient methods become possible in problems having a nonlinear objective function but linear constraints. The original Simplex method is shown to be a gradient method in a more liberal interpretation of the concept of gradient. The combination of the gradient and Simplex methods provides new approaches, such as the Simplex-corrected gradient and the projected-gradient procedures. The special case of "quadratic programming" is treated and it is shown how an exact solution may be obtained by linear methods. Then, two principal and to some extent dual types of linearization techniques are considered for the reduction of any programming problem to an approximating linear problem. Discussion of how to initialize one of the previous methods (determination of a feasible solution) concludes this very interesting contribution.

"Alternating Direction Implicit Methods" by Garrett Birkhoff, Richard S. Varga and David Young deals with the techniques for solving elliptic partial difference equations in the plane and gives a survey of present methods and their theoretical backgrounds. In a first part the Peaceman-Rachford and Douglas-Rachford methods are discussed in the special case of one single iteration parameter, and the question of convergence and of optimal choice of the parameter is treated. The next part studies the rate of convergence of the two methods in case of more than one parameter. The theory and analysis of choosing optimal values of the parameters, however, is restricted by theoretical reasons to the special case where the basic linear operators H, V, Σ of the vector equation $(H + V + \Sigma)\, u = k$ are all permutable. Another part surveys successive overrelaxation variants, compares their effectiveness with the alternating direction implicit methods and establishes some analogies between them. The authors performed a set of systematic numerical experiments, the results of which are given and discussed in detail.

The book contains two more contributions on computer applications, namely "Combined Analog-Digital Techniques in Simulation" by Harold K. Skranstad and "Information Technology and the Law" by Reed C. Lawlor.

H. R. SCHWARZ

Dynamics—systems, variational methods, and relativity. Volume II. By Robert L. Halfman. Addison-Wesley Publishing Co., Inc., Reading, Mass., Palo Alto, London, 1962. ix + 584 pp. $6.00.

This volume consists of Chapters 8 through 12 of a text which had it not been divided would have run to 584 pages. The jacket blurb states that Volume II can be used independently of Volume I, Particles, Rigid Bodies and Systems. The reviewer believes that a reader would be seriously handicapped if he did not have Volume I at his side while reading Volume II. The later chapters refer freely to figures, equations, discussions and examples from the earlier chapters. The first volume can be used alone without sacrifice but both volumes are required if the second volume is to be used efficiently. The decision to divide the text has resulted in a financial saving for those readers who desire only the early chapters but has increased the total cost for those primarily interested in the later chapters.

The subjects discussed in Volume II are an interesting selection of advanced topics in dynamics. In every case the author has provided the simplest possible presentation with a maximum of motivation. The level is suitable for a senior or first-year graduate course. Chapter 8, Signals and Their Transmission, treats Fourier series and integrals and their transmission through linear and nonlinear circuits. In Chapter 9 Laplace transforms, superposition integrals, autocorrelation functions and power spectra are introduced. In Chapter 9, Linear Systems, certain advanced techniques of linear dynamic theory are discussed. In studying system performance and stability, the author uses the Routh-Hurwitz criteria, the root-locus method and the Nyquist diagram. In discussing normal modes, both unconservative and unsymmetric systems are treated in addition to the classical symmetric conservative case. Chapters 10 and 11 are called Variational Calculus and Variational Dynamics respectively. The treatment is unusual in that the problems of Mayer and Bolza are introduced as well as the usual Lagrange formulation. The principal application is not to Hamilton's principle and Lagrange's equa-

tions (although these are briefly presented) but to performance optimization of aircraft and rockets. The final chapter, Relativity, is a lucid introduction to Lorentzian Kinematics, Einstein's "special relativity," Einstein's gravitational theory, and the twin paradox. Many of the concepts are developed in well chosen illustrative examples. Each chapter is followed by a collection of problems (without answers). There are short appendices on matrix notation and Laplace transforms.

Volume II (backed up by Volume I), would make an excellent textbook for an advanced dynamics course. Its deceptively simple style sets a high standard for the instructor.

STEPHEN H. CRANDALL

The Fourier integral and its applications. By Athanasios Papoulis. McGraw-Hill Book Co., Inc., New York, San Francisco, London, Toronto, 1962. ix + 318 pp. $10.75.

This textbook for first-year graduate electrical engineering students includes material on such matters as analysis of frequency characteristics of filters, Hilbert transforms, minimum phase-shift functions and causality conditions, Laplace transforms, characteristic functions and limit theorems, the Delta function, correlation, power spectra, analytic functions, generalized harmonic analysis, spectrum analyzers, sampling theory, Poisson's formula, the uncertainty principle, group, phase, and signal-front velocity, the method of stationary phase and saddle-point integration, echo theory, running spectra, the Paley-Wiener condition, numerical methods, and prolate spheroidal wave functions. With this breadth of coverage in such a short book, there is necessarily little rigor involved. The treatment is also short on intuitive arguments, presumably because the material may be motivated in other courses taken by the same students. The presentation is based on dozens of worked examples, hundreds of graphs and diagrams, and manipulations involving close to two thousand equations.

A. C. PIPKIN

Partial differential equations. By Bernard Epstein. McGraw-Hill Book Co., Inc., New York, San Francisco, London, Toronto, 1962. x + 273 pp. $9.50.

The title of this book is not an accurate description of its contents, and that is a very good thing. The book is on existence theorems for partial differential and integral equations. Since I had long ago resigned myself to the impossibility of understanding existence proofs, I would not have read the book if I had realized beforehand what it was about. Tricked into reading it by its innocuous title, I found myself understanding and even enjoying the theorems.

The explanation is that Professor Epstein has written a textbook, complete with exercises for the student, and not a monograph or a catalog of existence theorems. He generally gives ample intuitive motivation for the theorems to be proved, and outlines the strategy of the proof before going into detail. The treatment is introductory in the sense that the ideas and methods used in proving existence theorems are discussed and explained rather than taken for granted.

The material covered includes first-order equations, the Cauchy problem, integral equations, potential theory, the Dirichlet problem, the heat equation, and Green's functions. There is also an especially good preliminary chapter which explains such matters as equicontinuous families of functions and the Weierstrass approximation theorem.

Although methods of obtaining approximate solutions are implicit in the existence proofs, the reader who is seeking an introduction to formal techniques of solution would do well to look elsewhere. To any more advanced student of applied mathematics, the book is strongly recommended.

A. C. PIPKIN

Tables of random permutations. By L. E. Moses and R. V. Oakford. Stanford University Press, Stanford, Cal., 1963. 233 pp. $7.00

The volume contains tables of random permutations of 9, 16, 20, 30, 50, 100, 200, 500, and 1000 integers. An Introduction of 8 pages presents typical uses of the tables, discusses the method used to generate the permutations, and reports on the results of various tests for randomness.

Multivariate procedures for the behavioral sciences. By W. W. Cooley and P. R. Lohnes. Wiley & Sons, Inc., New York, 1962. x + 211 pp. $6.75.

This is primarily a collection of tested flow charts and FORTRAN-coded programs for procedures of multivariate analysis. The following list of chapter headings indicates the scope of the book: An Introduction to Multivariate Analysis and Computer Programming; Computing the Basic Matrices and Vectors; Multiple and Canonical Correlation; Multivariate Analysis of Variance and Covariance; Analysis of Variance and Covariance for Two- and Three-Way Designs; Multiple-Discriminant Analysis; Classification Procedures; Factor Analysis; Utility subroutines for Numerical Analysis. The first chapter has more the nature of a brief review than an introduction. Each of the following chapters briefly describes a particular procedure, illustrates it by the computation for a small-scale example, and gives a flow chart as well as a FORTRAN program. The last chapter contains subroutines for matrix inversion and the determination of eigenvalues and-vectors of real matrices.

W. PRAGER

Non-linear problems. Edited by R. E. Langer. The University of Wisconsin Press. Madison, Wisconsin, 1963. xiii + 321 pp. $7.50.

The volume contains the complete texts of fifteen invited lectures and the abstracts of twenty-six contributed papers presented at a symposium conducted by the Mathematics Research Center, United States Army, at the University of Wisconsin, on April 30–May 2, 1962. The titles and authors of the invited lectures are: Nonlinear hyperbolic systems of conservation (P. D. Lax); The extant nonlinear mathematical theory of plasma (C. L. Dolph); Some considerations on radiation magnetohydrodynamics (S. I. Pai); The initial value problem for the Navier-Stokes equations (J. Serrin); On the Stokes paradox and related questions (R. Finn); Some nonlinear eigenvalue problems (H. H. Schaefer); Stability and nonlinear character of ordinary differential equations (J. Moser); Boundary value problems for non-linear elliptic equations (D. Gilbarg); I. On van der Pol's equation with large k; and II. Celestial mechanics over very long time (J. E. Littlewood); Rigidity of a class of closed surfaces (L. Nirenberg); The present state of Lyapunov's direct method (W. Hahn); Almost periodic solutions of equations with periodic coefficients (M. L. Cartwright); On uniqueness and differentiability of ordinary differential equations (P. Hartman); Some remarks on critical point theory in Hilbert space (E. H. Rothe); Elastic stability and postbuckling behavior (W. T. Koiter).

Studies in mathematical analysis and related topics. Essays in honor of George Pólya. Edited by G. Szegö, C. Loewner, S. Bergman, M. M. Schiffer, J. Neyman, D. Gilbarg, and H. Solomon. Stanford University Press, Stanford, Cal., 1962. xxi + 447 pp. $10.00.

In addition to a portrait of George Pólya and a comprehensive list of his publications, the volume contains sixty papers. Since a complete list of their titles and authors would exceed the space available for this review, only those obviously falling within the scope of this Quarterly are listed here: On the approximation to a stable probability distribution (H. Cramér); A result on non-linear Volterra integral equations (A. Erdélyi); Proof of uniqueness by symmetrization (P. R. Garabedian); Contribution to the method of interior parallels applied to vibrating membranes (J. Hersch); Probabilities of rankings for two widely separated normal distributions (J. L. Hodges, Jr. and E. L. Lehmann); Statistical mechanics of some one-dimensional systems (M. Kac); An integral approach to the calculus of variations (C. Lanczos); A procedure for obtaining upper bounds for the eigenvalues of a Hermitian Symmetric operator (P. D. Lax); Remarques sur un problème relatif aux lois stables (P. Lévy); On the principal frequency of a membrane and the torsional rigidity of a beam (E. Makai); On some aspects of non-linear oscillations (N. Minorsky); Some isoperimetric inequalities in the torsion problem for multiply connected regions (L. E. Payne); Le probleme d'itération posé par certaines lois d'impots (M. Plancherel); Convergence de distributions dont les dérivées convergent (L. Schwartz); An inequality for Jacobi polynomials (G. Szegö); On an eigenvalue problem occurring in quantum mechanics (E. C.

Titchmarsh); Symmetrization in uniformly elliptic problems (H. F. Weinberger); A necessary and sufficient condition in the maximum-minimum theory of eigenvalues (A. Weinstein).

Aerodinamica Transonica. By C. Ferrari and F. G. Tricomi. Edizioni Cremonese, Roma, 1962. xv + 632 pp. $14.62.

The book is one of the mathematical monographs of the Consiglio Nazionale Delle Ricerche (National Research Council is a Government Institute). It is a unified and coordinated presentation of many of the known problems and of additional original work in transonic flow aerodynamics. The presentation of all of the transonic problems is well organized and given in a systematic way. A special effort has been made to give a consistent presentation of the different approximations used in transonic flow analyses, and special emphasis is given to basic fluid dynamic and mathematical concepts. Therefore, the book must be considered an important contribution to the understanding of transonic aerodynamics.

The book is divided into six chapters, five of which have been written by Professor Ferrari and one by Professor Tricomi. Professor Tricomi also contributed the Appendix on Hypergeometric Functions. The first chapter is a general introduction to the basic equations of compressible flow and of shock waves. Chapter II is a discussion of the mathematical problems and of the approximations of transonic flow analyses. Chapter III is a discussion of mathematical properties of transonic flow solutions and of mixed type equations. Chapters IV, V, VI are related to discussions of applications of transonic flow analysis to specific problems. In this part new material is presented. Chapter IV and V consider the inverse problem. Chapter IV is related to flow inside nozzles. In Chapter V a detailed discussion of the flow field around two-dimensional profiles for subsonic, sonic, and supersonic free stream velocity is presented. Chapter VI considers the "direct problem" and includes two- and three-dimensional flows. Again, subsonic, sonic, and supersonic free stream velocities are considered. In this chapter, approximate solutions related to area rule and law of equivalence are also presented.

The book is a good reference book for aerodynamics and is a good basic book for specialists in transonic flow. The material is clearly presented and well organized. The only comment I have is in relation to the use of the symbols; the same symbols are often used to indicate different quantities and they are not defined in some way in a list of symbols. This requires the careful attention on the part of the reader for interpretation of the equations.

ANTONIO FERRI

Renewal theory. By D. R. Cox. John Wiley & Sons, Inc., New York, and Methuen & Co. Ltd., London, 1962. ix + 142 pp. $4.50.

Renewal theory began with the problem of evaluating the number of times one need replace defective parts during some given time if replacements have statistically independent survival times. The theory presently deals with a more general class of problems with similar mathematical structure. The present monograph gives a survey of the theory with particular emphasis on the formal aspects and applications rather than the abstract theorems. It maintains the high quality of previous books in this series of monographs and treats a subject that has received little attention in other books.

G. F. NEWELL

Lineare Entscheidungsmodelle. By W. Kromphardt, Rudolf Henn and Karl Forstner. Springer-Verlag, Berlin, Gottingen, Heidelberg, 1962. 465 pp. $17.00.

This volume is an introduction to a number of aspects of contemporary mathematical economics and operations research. It covers linear programming in thorough style, with considerable emphasis upon the geometric background of the simplex technique, discusses the intimate connections with game theory, and considers stochastic processes, with particular attention to Markov processes. A final chapter is devoted to a brief description of dynamic programming with examples of both deterministic and stochastic multistage decision processes.

The merit of the book lies in the fact that it covers a lot of important territory in its 450 pages. The defects are first that this coverage is uneven in places, and secondly that there is no discussion of how the various methods can be used in conjunction with a digital computer to provide numerical solutions. Nevertheless, the advantages outweigh the disadvantages and the book represents a contribution to the literature in the field.

RICHARD BELLMAN

An introduction to computational methods. By K. A. Redish. John Wiley & Sons, Inc., New York, 1962. xii + 211 pp. $5.75.

The author has set out to provide an introductory text in computational methods suitable for university students (in England) who have had little more than a high school education in mathematics. An elementary knowledge of calculus and ordinary differential equations is assumed. The student is assumed to have heard of matrices only insofar as they are used as a short hand notation for simultaneous equations. The subjects treated are: Simultaneous linear equations, non-linear equations, polynomial interpolation, differentiation and integration, ordinary differential equations, interpolation in functions of two variables, and partial differential equations. Matrix inversion and eigenvalue calculation are each dismissed within a single page.

The author states that the book is intended for "the occasional computer" and he has selected for presentation methods that are easy to understand, to apply, and to remember. There is an emphasis on graphical methods and the use of tables of finite differences. A large part of the detail of the exposition is concerned with the careful tabulation of each step of a calculation. Methods suitable for use with digital computers, whether desk or electronic, are not presented. The book is therefore unlikely to be popular in this country as a text for the customary course in "digital computation." However, it should be remembered that the research worker, confronted with a problem requiring a numerical solution, normally attempts first to obtain an estimate of the solution by a graphical method or a rough interpolation, then turns to a digital computer, in the "library" of which he expects to find all the standard numerical methods already programmed. Consequently, the methods described in this book, although seeming so out of date to a computer enthusiast, still have their rightful place in any course on numerical methods.

The reviewer was amused by the author's interest in the suitability of various methods for examination purposes. Thus we find in the preface; "Similarly, the undergraduate student ··· will frequently be less interested in the amount of arithmetic involved, since the differences in various methods are generally slight where an examination question lasting 15–20 minutes is concerned," while on page 133 we are told that; "For examination questions a simple method with a small interval is probably best ··· [whereas] for practical purposes the interval should be as large as the [human] computer can handle."

B. A. CHARTRES

The language of computers. By Bernard A. Galler. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1962. viii + 244 pp. $8.75.

This book is an introduction to solving problems on computers through the medium of a high level programming language, in this case MAD (the Michigan Algorithm Decoder which is available on the IBM 7090 and certain other machines). It is intended for students finishing high school and starting at University. The main features of the language are explained through a series of examples, including sorting, integration, and the solution of linear simultaneous equations, spread over 10 chapters. There is sufficient detail so that it should be understandable with the minimum of supervision, but nowhere is there any account of what a computer actually is. The author suggests a couple of books on hardware which the reader might wish to look at *after* reading this book but my feeling is that the absolute beginner will benefit by some kind of general introduction beforehand, unless he is prepared to accept the idea of the language purely as a communication language for algorithms.

Although there is no mention of machine language (except briefly in Chapter 12) there is a chapter entitled "A Program to Produce MAD Programs" which in effect explains the principles of a compiler

with a nicely chosen example. Finally there is a brief comparison of the MAD language with FORTRAN and ALGOL. To summarize the book is an excellent tutorial manual but it is not a comparative study of computer languages as its title might suggest.

R. A. BROOKER

Selected translations in mathematical statistics and probability. Volume 2. Authors: A. A. Borovkov, I. I. Gihman, J. Hajek, M. Jirina, A. N. Kolmogorov, J. Linnik, V. Rhiter, Y. I. Rivkind, E. L. Rvaceva, O. V. Savamanov, V. P. Skitovoic, K. Urbank, V. Varadarayan, F. Zitek. American Mathematical Society, Providence, R. I., 1962. iv + 251 pp. $4.80.

This book contains translations of nineteen papers selected mostly from the Russian literature. Except for some earlier papers by Kolmogorov (1949) and Linnik (1947) the papers were originally published during the period 1953–1959.

G. F. NEWELL

Theory of elasticity. By V. V. Novozhilov. Translated by J. K. Lusher from the original Russian edition (Leningrad, 1958). Pergamon Press, New York, Oxford, London, Paris, 1961. xii + 448 pp. $12.50.

As stated in the preface, this book is a further development of the author's monograph entitled "Foundations of the Nonlinear Theory of Elasticity" (English translation published by Graylock Press, 1953), and he promises a sequel to the present volume at a later time. In writing the present volume, Professor Novozhilov's goal has been to "··· bring all the many problems in the modern theory of elasticity within the reach of as large a number of readers as possible, and the presentation therefore must be sufficiently simple ··· ," which may explain his choice of nontensorial notation.

The contents of the first four chapters, which comprise about one half of the book, are within the framework of the general nonlinear theory of elasticity and deal, respectively, with deformations, equilibrium, and stress-strain relations, together with energetic and related theorems. The remainder of the book is confined to the linear theory of elasticity. In Chapter 5 is collected an account (71 pages) of the linear theory and the resulting general theorems, and the next two chapters deal with plane problems and Saint-Venant's torsion and flexure problems.

There is no subject or author index, and the translation is reproduced in smaller than usual print by non-letter press which the publisher justifies in view of otherwise delay in appearance of the book and an increase in price. It is pertinent to note here that another English translation of this same volume (translated by J. J. Shorr-Kon), much lower in price and more handsomely reproduced, was published simultaneously by the Israel Program for Scientific Translations, Jerusalem, 1961.

P. M. NAGHDI

Numerical solution of ordinary and partial differential equations. By L. Fox. Pergamon Press, Ltd., Oxford, 1962. ix + 509 pp. $10.50.

The book is divided into four parts, of which the first three give a survey of various known methods for the solution of ordinary differential equations, integral equations and partial differential equations of quasi-linear type. This survey is more expository and references are given for a more detailed description of the methods, whereas the fourth part deals in detail with the solution of practical problems in partial differential equations.

Part I on ordinary differential equations starts with the notations used for finite differences proceeding to finite-difference and integration formulae. The next few chapters are devoted to methods for initial-value problems, for which the computation proceeds in a step-by-step manner. Chapter 2 deals with methods of Runge-Kutta type, whose advantages and disadvantages are discussed. Next, methods of predictor-corrector type are treated leading to iterative processes whose convergence is investigated.

Likewise some possibilities of starting the integration are outlined. In the following chapter the numerical stability of the step-by-step methods is investigated with respect to both truncation and rounding errors. The different types and origins of instability are indicated, such as inherent, partial and strong instability.

Chapter 5 leads over to boundary-value problems. The two methods for solving them, namely initial-value processes and boundary-value techniques are explained and compared from the point of view of time and convenience with respect to computers. The problem of accuracy, however, is treated in chapter 8, where an extensive analysis of the error sources is given for all methods explained in the book for both initial-value and boundary-value problems. The discussion of boundary-value problems proceeds in a natural way to eigenvalue problems. Chapter 6 mentions some of the well known theorems on eigenvalues and eigenfunctions and discusses moreover some of the computational methods as well as properties of eigenvalues and eigenfunctions in case of singularities of the boundary conditions or of infinite ranges. The one-dimensional Schrödinger equation is treated in detail for discussion of the various methods.

Presentation of well known facts concerning Chebyshev polynomials and of properties of Chebyshev approximations prepares for the solution of ordinary differential equations, which are however supposed to be linear and have polynomial coefficients. The methods of Lanczos and Clenshaw are presented to produce Chebyshev series for the solutions of this type of ordinary differential equations.

In part II the integral equations of Fredholm and Volterra type are considered and appropriate numerical methods and their difficulties in exceptional cases are discussed. This general review is followed by a chapter in which some of the problems of singularity and non-linearity together with the appropriate and partly new computational techniques are indicated. This part of the book is terminated by a description of how some of the numerical methods presented so far can be applied to a particular class of integro-differential equations arising in nuclear collision problems.

Part III giving an introduction to partial differential equations is divided by natural reasons into the discussion of hyperbolic, parabolic and elliptic equations of quasi-linear type. A first look at partial differential equations of first order scans the phenomena to be expected in general. The notion of characteristics is introduced, the problem of uniqueness of the solution and the propagation of discontinuities are investigated. After a discussion of the classification of quasi-linear partial differential equations of second order, the general solution of hyperbolic equations by integrating along characteristics is presented. Beside the customary process for solving hyperbolic equations, explicit and implicit finite-difference methods are examined and their stability and convergence behavior are studied. Discontinuities, however, are hard to treat by finite-difference equations.

The following two chapters are devoted to the discussion of parabolic differential equations in two dimensions. Stability and convergence are the most important problems of the processes as well as the questions of reducing the local truncation error or the extension of the methods to non-linear equations and boundary conditions.

Finally, elliptic equations are treated, some methods for producing finite-difference equations are discussed and the difficulties in case of irregular boundaries are mentioned. The set of linear equations thus produced may be solved by direct methods which take advantage of the special band form of the equations. Thus the solution by triangularization of the band-matrix as well as the method of partitioned matrices are presented. Some of the iterative methods, extremely suitable for the solution of the finite-difference equations, are discussed such as the methods of Jacobi and Gauss-Seidel, the successive over-relaxation whose theoretical foundation however exists only for matrices with "Property A," block-relaxation methods and finally the attempt to accelerate convergence by the use of Chebyshev polynomials.

The chapters of Part IV on practical problems in partial differential equations have been contributed by scientists with special experience in these problems. Thus this last part uses the techniques described earlier and much of it is very new and therefore extremely valuable for everyone who is faced with the practical solution of similar problems. The chapters cover the solution of elliptic equations as they arise in studying nuclear reactors, the characteristic method for the one-dimensional unsteady flow, and the solution of the transport equation governing the flow of the neutral particles in a scattering medium. A new element is introduced by the discussion of Monte Carlo methods for the solution of neutronics problems. The presentation of computational techniques applied in plasma physics and numerical weather prediction terminates this interesting book.

H. R. SCHWARZ

High-speed analog computers. By R. Tomovic and W. J. Karplus. John Wiley & Sons, Inc., New York, London, 1962. xi + 255 pp. $9.95.

As opposed to one-shot or slow-speed analog computers which usually require several seconds to complete a solution and on which initial conditions must be manually reset before the computation may be repeated, the repetitive or high-speed computer is designed to reset itself automatically and repeat the computation many times each second. Outputs of such computers may be viewed continually and the effect of varying parameters thereby becomes immediately obvious.

This book provides an advanced and, in places, a rather technical introduction to the field of repetitive electronic analog computers. In addition, it contains a survey of some very recent applications of repetitive computers to the solution of problems in applied mathematics. In Part III, entitled *Applications*, may be found examples from the literature of the application of high-speed computers to the solution of ordinary, both linear and nonlinear, and partial differential equations, integral equations, conformal mapping and statistical analysis.

In Part I, *Theory*, there is a chapter devoted to error analysis. The approaches discussed are presented clearly and in a very general way. However, the lack of concrete examples may cause this chapter to receive less appreciation than it merits. Part II, *Equipment*, surveys the important components of repetitive analog computers. The treatment here varies from lightly descriptive to highly technical, and certain sections will be of interest primarily to workers concerned with the design of analog computers. For example, a complete circuit schematic for a commercial function-of-two-variables generator is included.

Certainly one of the most valuable aspects of this book lies in its bibliographies, in which a significant representation is enjoyed by very recent European and Russian contributions. In places the book suffers from excessive terseness and reads more like a technical paper than a text, but this is perhaps characteristic of survey writing in general.

S. P. Sutera

Statistical management of inventory systems. By Harvey M. Wagner. Operations Research Society of America, Publications in Operations Research No. 6, and John Wiley & Sons, Inc., New York, London, 1962. xiv + 235 pp. $8.95.

The problem considered here is how to control a system which has been put into operation to, in turn, control the levels inventories carried. The author characterizes his work as essentially a "theory of implementation" for some inventory control system. It is assumed that ideal rules for controlling inventory have already been derived. There exists a great deal of literature on this subject. In particular, it is assumed here that an (s, S) inventory policy has been adopted for each of the many items involved in the firm's operations, where s is the level of inventory below which reorders are placed and S is the level to which inventory is replenished when reorders are made. It is further assumed that there are two levels of management in the firm, lower management who is responsible for detailed operations of the (s, S) rules and upper management who wants to make sure that lower management obeys the (s, S) rules set up (i.e., they want to control the inventory control systems). Upper management is looking for a control index scheme which, without having to incur the expense of detailed audits of lower management's activities, will encourage lower management to obey the rules and will signal to upper management when the rules are not being obeyed. For the most part, it is assumed that the demands for the items stocked are uncertain but follow known and stationary probability distributions.

Chapter 2 is concerned with a detailed investigation of the (s, S) inventory control schemes. The main purpose is to derive probability distributions of the major variables characterizing the status of the inventory systems at each moment of time. These distributions are the basis upon which control schemes are built in the succeeding two chapters. The primary variables needed are the distributions for the amount of inventory on hand and on order and the number and frequency of orders placed. Distributions for other variables can easily be derived from these. The entire process of the inventory system moving from period to period is envisioned as a Markov chain, with the distribution equations stated in terms of the appropriate transition matrices. Several computational schemes are suggested (iteration of a Markov chain, a recursive algorithm, Monte Carlo simulation), some quite simple,

for these distributions, with a number of numerical examples given to illustrate and compare the methods. Several different demand distributions and variations of (s, S) policies are included.

Although this work is clearly preliminary to the main purpose of deriving aggregate control schemes, it takes up almost half of the book and contains the majority of concrete results reached. This portion may, by itself, be useful in at least two areas. First, it provides useful additional results for (s, S) systems and gives a framework for additional work on these types of systems. Second, it suggests a way to compute the necessary distributions from any inventory control scheme to use for constructing aggregate control indexes. This second area is broader in scope and more directly related to the main issue of the book, but it is also the area where one can have more doubts about the progress made. The work presented is excellent, but restriction to particular types of (s, S) policies may not provide a general framework within which other types of inventory schemes and other types of aggregate control schemes can be investigated. Further work is indicated to resolve this point.

The main problem of finding a suitable control index is considered in Chapter 3. The search is for an index number and method of using it which will be consistent. Consistency here means a control scheme which positively motivates lower management to follow the prescribed (s, S) rules and indicates to upper management violations of this desire. Two types of control schemes are considered, barometer and quota types. Derivation of both types draw on the results in Chapter 2. The former consists of rewarding or punishing lower management in proportion to how far the observed control index number is away from some set target value of the index. The index used is closely associated with an imputed cost function for running the inventory system, including costs of shortages, storage, and ordering. It is shown how to compute such an index so that the minimum expected value of the index is obtained when the prescribed (s, S) rules are being followed exactly. Setting the target at this minimum expected value will then bring losses to lower management whenever they deviate in any way from the (s, S) policy set-up; they can break-even only by following the rules. The general results are that computing the index number as suggested and using a barometer type of scheme will yield a consistent control of the inventory control scheme.

Quota controls consist of setting some upper limit to the index number, and auditing lower management activities whenever the index number gets above that value under the assumption that they have been violating the (s, S) rules. The index number needed for this purpose must have the greatest probability of being below the limiting value when and only when the rules are being exactly followed. Unfortunately, quota schemes cannot, in general, be consistent. The problem lies in the fact that both the mean and the variance of the index number is involved here, similar to statistical hypothesis testing. It is generally possible for lower management to find non-ideal (s, S) rules (entailing higher operating costs to the firm) which yield a higher expected value for the index number but a smaller variance, thus leaving open the possibility that the index has a larger probability of being below the limit even though the system is not in control.

While consistent schemes are to be preferred, the author argues at the end of this chapter that we may have to settle for much less. It may be very difficult to institute barometer type schemes where punishing lower management has any real meaning or effect on their activities. In fact, barometer schemes may work like quotas, since lower management may feel that any meaningful amount of upper management's wrath will be upon them only when the paper losses get so great that a full scale audit is undertaken. It is finally concluded that inconsistent quota controls may be better than no controls at all. This suggests that the author envisions little practical use for his development of consistent control schemes. This apparent restriction must not be taken too seriously, however. It seems evident that work undertaken to evolve any aggregate control scheme at all must take into account the methods and issues raised here. Full scale development of procedures for investigating and comparing alternative less-than-optimal policies is, however, only suggested here and not carried out.

Chapter 4 contains a brief attempt to drop the known and stationary assumptions for demand and allow for changing demand probability distributions. Very few results are developed. Some consistent schemes for controlling internal demand can be developed, but nothing consistent can be worked out for controlling a complete (s, S) inventory control policy. A major problem remains in trying to ascertain the probability distribution of demand using only a few empirical observations. Most of the regions of the distribution may not be observed before the whole distribution shifts again. Yet all the aggregate control indexes discussed utilize this demand distribution in their computations (and so will the initial (s, S) policies). There is little guide for management here except, as noted above, a realization of the problems involved.

One might contrast the reasonable completeness and rigor of the discussion of the (s, S) policies in Chapter 2 with the decreasing ability to derive concrete and useful results as the field of inquiry is expanded and assumptions relaxed in Chapters 3 and 4. A recognition of the problems and a framework of analysis has been given, but only a start has been made. Perhaps these methods will be the basis for adding some solid content to the previously rather empty field of so-called "organizational theory." Effective organization must mean that management can efficiently control their own optimizing policies. The last sentence in the book may best reveal the tenor of progress made here: "Our discussion, which highlights the nature of the problems embodied within any control system, should suggest the critical points of comparison that are to be encountered in any evaluation process [of alternative aggregate control schemes]."

MARK B. SCHUPACK

The rational mechanics of flexible or elastic bodies, 1638-1788. Introduction to Leonhardi Euleri Opera Omnia, Vol. X et XI Seriei Secundae. By C. Truesdell. The Swiss Society of Natural Sciences, 1960. 435 pp. $25.00.

The volume under review, beautifully printed and handsomely bound, contains an historical account of the initial developments of the theory of elasticity and an extensive record of the achievements in the subject during the period 1638–1788. This is not just a light history or a mere reproduction of the early researches in elasticity. It is a profound and magnificent exposition, the study of which every serious student of mechanics will find highly rewarding.

In a cursory review of this kind, it is not possible to do justice to the contents of this treatise, nor is it possible to reflect the enormous effort that must have gone into it. An idea of the scope of this work can be gained, however, from the Foreword wherein the author writes: "As will appear, the giants of our subject are James Bernoulli and Euler. Here, for the first time, may be read the story of what these men really did for the theories of flexible or elastic bodies. Modern theories of materials are set chiefly upon the foundations laid down by Cauchy from 1822 to 1845. Thus our account serves as a preface to his researches." All mathematical developments and formulas are given in a uniform modern notation, thus making the reader's task much easier. Professor Truesdell's own welcomed comments and interpretations which occur frequently are placed in square brackets. No undertaking of this magnitude on the history of elasticity has been available previously.

P. M. NAGHDI