RADIATIVE TRANSFER EFFECTS IN NATURAL CONVECTION ABOVE FIRES—OPAQUE APPROXIMATION*

BY M. P. MURGAI AND R. K. VARMA
Defence Science Laboratory, Delhi, India

Abstract. In continuation of a previous paper on a similar subject, we examine here, in detail, the opaque approximation of the radiative transfer equation by retaining the vertical diffusivity term within the boundary layer theory. The results seem to indicate the presence of a layer very near the horizontal boundary of the plume where the radiation effects are dominant. The thickness of this layer tends to decrease with increasing absorption coefficient.

1. Introduction. In a recent publication Murgai [1] attempted the solution of the radiative transfer effects in columnar convection above fires. The problem was treated within the framework of turbulent boundary layer approximation for an axially symmetric coordinate system. Two asymptotic solutions of the radiative transfer equation, corresponding to transparent and opaque approximations, were tried. While the former admitted a solution, the latter reduced to the case of no radiative transfer according to the usual arguments of boundary layer theory. This was due to the fact that the net radiative heating in the opaque case is proportional to $\nabla^2 B$, B being the Planck function, and when one of the second derivatives of B along the flow direction was neglected, in order to be consistent with the boundary layer approximation, the same set of equations reduced to the one for no radiative transfer, the other second derivative of B being integrated out. That is so because in such problems, the temperature at the edges of the plume equals that of the atmosphere, thus making the temperature gradient across the plume equal to zero. Furthermore, since the medium is opaque there is no effective leakage of the radiation through the main body of the plume (except from a very thin layer on the circular boundary). Thus one loses, in this opaque case, the more dominant term in the usual boundary layer theory. However, it is well-known that in the dynamical problems of this kind, the radiative transfer effects are usually greater when the medium is opaque. We therefore thought of re-examining the whole system of equations. The solution of the complete transfer equation still remains a formidable task, all the more so when it is coupled to a non-linear set. One has, therefore, to look for some tractable cases. We have considered two such examples. One is a re-examination of the diffusion approximation mentioned above and the other a boundary layer version of the radiative transfer equation. While the first one is discussed here, the second will form the subject of a subsequent paper.

In what follows, we define a parameter $\beta = k^* b_0/\alpha$ which relates the mean free path $1/k^*$ to a characteristic length b_0, the plume width, α being the entrainment constant. The value of this parameter varying from magnitudes less than 1, passing through intermediate values and eventually tending to infinity respectively characterize the transparent, general and the opaque cases. In the last case, as mentioned above, if in the energy equation, H is replaced by $\nabla^2 B$, its final form will differ from the usual

*Received December 2, 1963. Revised manuscript received April 24, 1964.

one for the boundary layer flow only by the presence of the vertical diffusivity term $\partial^2 B/\partial x^2$. Normally this term is ignored within the boundary layer approximation. However, it is conceivable that under certain circumstances it may be retained when significant radiative transport exists in the vertical direction. In such cases there is, effectively, a large temperature gradient along the flow. An analysis of the order of the various terms in the energy equation justifies the retention of this term provided

$$\left(\frac{\partial B}{\partial x}\right)_{x=0} > \frac{\sigma}{\pi}\left(\frac{T^4 - T_\infty^4}{\delta_T}\right),$$

where σ is the Stefan-Boltzmann constant, T and T_∞ the temperatures inside and outside the plume respectively, and δ_T the thermal boundary layer thickness (see for example [2]).

In Section 2 the fundamental equations of the problem are given along with the expression for H in terms of B as derived from the radiative transfer equation. Section 3 deals with the integration of the boundary layer equations and Section 4 gives a brief discussion of the results obtained. In the Appendix an approximate method of solution of the differential equations is developed.

2. Fundamental equations. The equations of continuity, momentum, and energy for a turbulent axi-symmetric convection column with radiative transfer are respectively

$$\frac{\partial}{\partial x}(\gamma r u) + \frac{\partial}{\partial r}(\gamma r v) = 0, \tag{2.1}$$

$$r\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right] = r\left(\frac{\gamma_\infty - \gamma}{\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial r}(r\tau), \tag{2.2}$$

$$\rho r\left[u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial r} + ug\right] = \frac{\partial}{\partial r}(rq) + rH. \tag{2.3}$$

In these equations x, r, u and v are the coordinates and the velocities in the vertical and horizontal directions respectively. $\gamma = \rho g$ is the specific weight of the medium. ρ being the density and g the acceleration due to gravity. γ_∞ is the value of γ far removed from the plume. The flow being turbulent, the quantities above refer to the local mean values. Thus turbulent components are either averaged out or are included in other terms as shear stress τ or heat flux vector q which, in this case, are given by $-\rho\langle u'v'\rangle$ and $-\rho c_p\langle v'T'\rangle$ respectively, where u', v', T' are the fluctuating parts of the velocity components and temperature T (in absolute scale) respectively. c_p is the specific heat at constant pressure. $h = c_p T$ is the enthalpy per unit mass; H stands for heating rate per unit volume due to radiation and is related to the transfer equation, as explained later. In the above equations use has been made of the hydrostatic approximation with regard to the pressure. In accordance with the well-known Boussinesq approximation the density has been assumed to be constant everywhere except as it effects the buoyancy term. Later on in converting the density to temperature use has been made of the linear law of variation applicable to incompressible fluids for small temperature changes. Also the energy due to dissipation is neglected.

The radiative transfer equation is

$$\frac{dI}{ds} = k^*(B - I), \tag{2.4}$$

where I is the integrated intensity in the s-direction, k^* the grey absorption coefficient, and B the integrated Planck function. H in equation (2.3) is given by

$$H = -\int \frac{dI}{ds}\, d\omega, \tag{2.5}$$

$d\omega$ being an element of solid angle.

For an optically dense medium Eq. (2.4) may be approximated to yield a simple relation connecting H and B (see for example [3]). This is obtained by expanding B which occurs in the formal solution of the radiative transfer Eq. (2.4) about an arbitrary point in a series in the inverse power of k^*. Omitting terms of the order higher than $1/k^*$, one obtains

$$H = \frac{4\pi}{3k^*} \nabla^2 B. \tag{2.6}$$

In the energy Eq. (2.3) the full expression for H given by equation (2.6) is retained. Thus it differs from its usual boundary layer form in having the term $\partial^2 B/\partial x^2$. This, in effect, implies the existence of a large temperature gradient along the flow.

The integrated intensity I can, in general, be expressed in terms of the outgoing and incoming radiations I_+ and I_- respectively. We therefore have

$$I = I_+ + I_- . \tag{2.7}$$

The radiation I_-, received at the boundary, consists of that from the outside atmosphere as well as from the rest of the plume. Since the medium is opaque the radiation from the outside atmosphere is absorbed within a thin layer on the surface of the plume. The radiation from the other parts of the plume is also absorbed, for similar reasons, within a few successive layers so that the effective contribution due to I_- at $x = 0$ will only be from layers a few mean free paths above. This may be neglected as compared to I_+ . Also from the continuity of temperature at the boundary we have $I_+ = B$. The net radiative flux $\mathbf{F}$ is given by (2.8)

$$\mathbf{F} = \int I \cos\theta\, d\omega. \tag{2.8}$$

Also for an opaque medium [3], we have

$$\mathbf{F} = -\frac{4\pi}{3k^*} \nabla B. \tag{2.9}$$

From Eqs. (2.7), (2.8) and (2.9) we have

$$I_+ + I_- = -\frac{4}{3k^*} \frac{\partial B}{\partial x}. \tag{2.10}$$

In view of the above discussions this reduces to

$$\frac{\partial B}{\partial x} = -\frac{3k^*}{4} B, \tag{2.11}$$

which provides a boundary condition.

3. Integration of the equations. Having expressed the temperature and density in terms of the corresponding potential quantities, we integrate the conservation equations with respect to r between the limits zero and infinity [4]. Thus,

$$\frac{d}{dx}(b_m^2 u_m) = \alpha b_m u_m , \tag{3.1}$$

$$\frac{d}{dx}(b_m^2 u_m^2) = g b_m^2 \theta_m , \tag{3.2}$$

$$\frac{d}{dx}(b_m^2 u_m \theta_m) = D[b_m^2(1 + \theta_m)^4], \tag{3.3}$$

where

$$u_m = \frac{\int_0^\infty r u^2 \, dr}{\int_0^\infty r u \, dr}, \qquad b_m^2 = \frac{2}{u_m}\int_0^\infty r u \, dr,$$

$$\theta_m = \frac{2}{b_m^2}\int_0^\infty r\left(\frac{\Delta\gamma_0}{\gamma_0}\right) dr.$$

The quantities u_m, b_m and θ_m may be considered as defining the equivalent top hat profiles as given in [4]. The operator D is given as

$$D = \frac{4\sigma T_{\infty 0}^3 P^{2\nu-1}}{3k^* \rho_0 c_p}\left[P^{2\nu}\frac{d^2}{dx^2} - \frac{8\rho_{\infty 0} g \nu P^\nu}{p_0}\frac{d}{dx} + \frac{12\rho_{\infty 0} g \nu^2}{p_0}\right],$$

in which

$$P = \frac{p}{p_0}, \qquad \nu = \frac{k-1}{k},$$

k being the ratio of the specific heats and p_0 the standard pressure.

In obtaining Eq. (3.3) we have assumed that the ambient atmospheric lapse rate is zero. The boundary conditions are

$$b_m = b_0 , \qquad u_m = u_{m0} , \qquad \theta_m = \delta, \qquad \frac{d}{dx}[b_m^2(1 + \theta_m)^4] = (1 + \delta)^4 \tag{3.4}$$

at $x = 0$.

We define the following set of non-dimensional variables

$$x' = \frac{\alpha x}{b_0}, \qquad u' = \frac{u_m}{(g b_0 \delta/\alpha)^{1/2}}, \qquad b' = \frac{b_m}{b_0},$$

$$\lambda = \frac{\theta_m}{\delta}, \qquad \beta = \frac{k^* b_0}{\alpha}, \qquad u_0 = \frac{u_{m0}}{(g b_0 \delta/\alpha)^{1/2}}$$

$$\chi = \frac{4\sigma T_{\infty 0}^3 P^{4\nu-1}}{3\beta \rho_0 c_p \delta^{3/2}(g b_0/\alpha)^{1/2}}, \qquad \chi^* = \frac{\rho_{\infty 0} g b_0 P^{-\nu}}{\alpha k p_0}$$

$$\chi_1 = 8k\nu\chi^*\chi, \qquad \chi_2 = 12(k\nu\chi^*)^2\chi, \qquad \epsilon = \frac{\chi\delta}{u_0}. \tag{3.5}$$

In terms of these non-dimensional variables the Eqs. (3.1) to (3.4), after removing the primes reduce to

$$\frac{d}{dx}(b^2 u) = bu, \tag{3.6}$$

$$\frac{d}{dx}(b^2 u^2) = b^2\lambda, \tag{3.7}$$

$$\frac{d}{dx}(b^2u\lambda) = \left(\chi \frac{d^2}{dx^2} - \chi_1 \frac{d}{dx} + \chi_2\right)[b^2(1 + \delta\lambda)^4] \tag{3.8}$$

and

$$b = \lambda = 1, \qquad u = u_0, \qquad \frac{d}{dx}[b^2(1 + \delta\lambda)^4] = -\frac{3\beta}{4}(1 + \delta)^4, \tag{3.9}$$

at $x = 0$.

The parameter u_0 except for α is the Froude number. To within a constant factor the quantity ϵ is a measure of the opacity of the medium, and therefore forms a suitable expansion parameter, which will be used later. The parameters χ_1 and χ_2 are expected to be small, except for very large fires and very large heights. In fact, they are proportional to χ^*—a parameter defined in [4], p. 623. Here χ_1 occurs as a coefficient of the gradient term. It is therefore difficult to assess its contribution before the problem is solved. We therefore solved this system of equations on an IBM 1620 firstly by retaining χ_1, χ_2 and then without these parameters for values of δ equal to 1 and 5, and $\epsilon = 10^{-5}$. The difference between the two solutions nowhere exceeded 3%. For this reason the equations were solved for a wider range of parameters $\delta = 1, 2, 3, 4, 5$; $u_0 = 1$; $\epsilon = 10^{-5}, 10^{-6}, 10^{-7}$ by excluding terms containing χ_1 and χ_2. The system thus obtained admits an approximate analytical solution the details of which are given in the Appendix. This solution is also compared with that obtained by numerical integration (without χ_1, χ_2). These two results agree within 1%.

TABLE

δ	T^*	b_0	χ	A 10^5x	h	ϕ	x	h
		10^3	10^{-5}	101.5	10.15	0.30	0.14	1400
1.0	327	10^4	10^{-6}	10.10	10.10	—	—	—
		10^5	10^{-7}	1.010	10.10	—	—	—
		10^3	5×10^{-6}	235.8	23.58	0.15	0.106	1060
2.0	627	10^4	5×10^{-7}	23.47	23.47	—	—	—
		10^5	5×10^{-8}	2.346	23.46	—	—	—
		10^3	3.3×10^{-6}	390.0	39.00	0.10	0.060	600
3.0	927	10^4	3.3×10^{-7}	38.98	38.98	—	—	—
		10^5	3.3×10^{-8}	3.896	38.96	—	—	—
		10^3	2.5×10^{-6}	543.5	54.35	0.075	0.049	490
4.0	1227	10^4	2.5×10^{-7}	54.21	54.21	—		
		10^5	2.5×10^{-8}	5.420	54.20	—		
		10^3	2×10^{-6}	681.6	68.16	0.06	0.035	350
5.0	1527	10^4	2×10^{-7}	67.85	67.85	—	—	—
		10^5	2×10^{-8}	6.782	67.82	—	—	—

Fifty per cent heights h for bouyancy for the opaque approximation (called approximation A) as well as the transparent approximation dealt in [1] (and called approximation B here), for certain relevant values ϕ, χ, δ and $k^* = 0.01\text{cm}^{-1}$ for typical fire sizes b_0. T^* is in °C while h and b_0 are in cms. x is the dimensionless height $\alpha = 0.1$, $T_{\infty 0} = 300°$

4. Discussion. A comparison of the results obtained here with those of [1] indicates a significant difference in fifty per cent heights for buoyancy. This is due to the retention of the term $\partial^2 B/\partial x^2$ in the energy equation. The above solutions seem to establish the existence of a layer near $x = 0$ where the radiation is dominant. They also show that for increasing values of $\beta(\epsilon \to 0)$ $\lambda \to 0$ in lesser and lesser intervals of x and in the limit $\beta \to \infty$, λ becomes zero on the boundary $x = 0$ itself. Thus the radiation boundary layer shrinks, in the limit, to the plane $x = 0$. This amounts to an infinite temperature gradient at $x = 0$ and a discontinuity in λ. This situation may be likened to the velocity discontinuity in a vortex sheet. The physical behavior of the plume in this case would be one of losing heat by radiation at an extremely large rate, the buoyancy becoming zero immediately, the velocity and plume size remaining almost constant. These, however, change later when the plume rises like a pure jet. Thus the effect of β assuming larger values, is to augment the process of heat loss due to radiation, a trend also exhibited in [1]. In that case, however, this effect is not so predominant as here, because the radiation parameter $\phi[\phi = (\text{const.})\beta]$ cannot be increased indefinitely because it is bounded by the transparent approximation.

In this context one may define a radiation Prandtl number (corresponding to radiative diffusivity) which is very much less than 1. Therefore, in regions where radiation is important the thickness of the thermal boundary layer is no longer small and confined to the edges. In fact, very near $x = 0$ it penetrates deep inside the plume and manifests itself as the radiation boundary layer mentioned above.

These results also indicate that the fifty per cent heights for buoyancy are independent of the fire size at the boundary $x = 0$ for plumes having the same absorption coefficient. This seemingly strange result, is in fact, just the way it should be. Phenomenologically speaking the opaque case reduces to a conduction-like problem, for which the radial part of the conduction term does not contribute to the radiative loss. The results obtained from such an analysis will therefore be independent of the original fire size.

The heights involved turn out to be very small and, suggesting that one could perhaps work in terms of ordinary rather than potential quantities. In fact, neglecting χ_1 , χ_2 in Eq. (3.8), amounts to doing this provided the pressure ratio P occurring in χ is taken to be 1.

The above analysis has been carried out for a constant value of the parameter $u_0 = 1$. This parameter is a measure of the change in the velocity of the plume at the boundary. It is expected that in the investigation of the effect of radiation on the dynamics of the plume, variation in u_0 will not make a substantial contribution to the nature of the results.

A few words about the nature of the differential equations, occurring here, would not be out of place. We observe that the coefficient of the highest derivative is a small parameter χ (or ϵ) which is characteristic of singular perturbation problems. In such cases, under certain conditions, the solutions of the full differential equations ($\chi \neq 0$) tend continuously to those of the degenerate system in which χ is replaced by zero, which amounts, here to the case of no radiative transfer. However, the above results show that when $\chi \to 0$, λ also tends to zero and hence, in the limit, the equations obtained are not those of the degenerate system ($\lambda \neq 0$, $\chi \neq 0$) but one of pure jet. In fact, the solutions for the transparent approximation, contained in [1] tend smoothly to those of no radiative transfer when $\phi \to 0$.

The results, obtained here, would be true only within the radiative boundary layer for small x and finite values of β, the case in which we are primarily interested. The solutions for the rest of the region is obtainable by a suitable matching with those of the pure jet as explained in the Appendix.

Appendix

To obtain an approximate solution of equations (3.6) to (3.9), we stretch the coordinate x near the boundary and transform the variables as shown below:

$$x = \epsilon y, \qquad \lambda = \frac{1}{\delta}\left(\frac{q}{Q} - 1\right), \qquad u = u_0 U, \qquad \xi = \chi\beta, \tag{A.1}$$

$$Q = 1/(1 + \delta).$$

Substituting these in the differential equations and integrating equation (3.8) we obtain

$$\frac{d}{dy}(b^2 U) = \epsilon b U, \tag{A.2}$$

$$\frac{d}{dy}(b^2 U^2) = \frac{\epsilon}{Q\delta u_0^2} b^2(q - Q), \tag{A.3}$$

$$\frac{d}{dy}(b^2 q^4) = Q^3 b^2 U(q - Q) - \delta\left(Q^4 + \frac{3\xi}{4u_0}\right), \tag{A.4}$$

with the boundary conditions

$$y = 0 : b = U = q = 1. \tag{A.5}$$

We now set

$$\begin{aligned} q &= q_0(y) + \epsilon q_1(y) + \epsilon^2 q_2(y) + \cdots, \\ b &= b_0(y) + \epsilon b_1(y) + \epsilon^2 b_2(y) + \cdots, \\ U &= U_0(y) + \epsilon U_1(y) + \epsilon^2 U_2(y) + \cdots. \end{aligned} \tag{A.6}$$

Substituting these in the equations (A.2), (A.3), (A.4) and (A.5) and comparing coefficients of the various powers of ϵ, we obtain differential relations for these coefficients.

The boundary conditions (A.5) reduce to

$$b_n = U_n = q_n = \begin{matrix} 1 & \text{for} & n = 0, \\ 0 & \text{for} & n > 0, \end{matrix} \qquad (n = 0, 1, 2, \cdots). \tag{A.7}$$

For the zeroth order approximations we have

$$\frac{d}{dy}(b_0^2 U_0) = 0, \tag{A.8}$$

$$\frac{d}{dy}(b_0^2 U_0^2) = 0, \tag{A.9}$$

$$\frac{d}{dy}(b_0^2 q_0^4) = Q^3 b_0^2 U_0(q_0 - Q) - \delta\left(Q^4 + \frac{3\xi}{4u_0}\right), \tag{A.10}$$

which with the boundary conditions (A.7) yield,

$$b_0 = U_0 = 1. \tag{A.11}$$

From (A.10) and (A.11) we have

$$y = \frac{4}{Q^3}\left[\tfrac{1}{3}(q_0^3 - 1) + \frac{C}{2}(q_0^2 - 1) + C^2(q_0 - 1) + C^3 \log\left(\frac{q_0 - C}{1 - C}\right)\right], \tag{A.12}$$

where

$$C = 1 + \frac{3\xi\delta}{4Q^3 u_0} \tag{A.13}$$

From the zeroth order solution, we note that in the neighborhood of the origin q_0 varies while b_0 and U_0 remain constant.

The relations for the first order approximation are

$$\frac{d}{dy}(U_1 + 2b_1) = 1, \tag{A.14}$$

$$\frac{d}{dy}(U_1 + b_1) = \frac{q_0 - Q}{2Q\delta u_0^2}, \tag{A.15}$$

$$\frac{d}{dy}[2q_0^3(2q_1 + b_1 q_0)] = Q^3[q_1 + (U_1 + 2b_1)(q_0 - Q)]. \tag{A.16}$$

Solving (A.14) and (A.15) we obtain

$$\begin{aligned} b_1 &= y - \frac{1}{2\delta u_0^2}\left[\frac{q_0^4 - 1}{Q^4} + \left(\frac{C}{Q} - 1\right)y\right], \\ U_1 &= y - 2b_1 . \end{aligned} \tag{A.17}$$

To solve (A.16), we eliminate b_1 with the help of (A.17) and differentiate q_1, with respect to q_0 as the independent variable. This reduces equation (A.16) to

$$\begin{aligned} \frac{dq_1}{dq_0} + \frac{2q_0 - 3C}{q_0(q_0 - C)} q_1 &= y\left[\frac{q_0 - Q}{q_0 - C} + \frac{C - Q}{Q\delta u_0^2} - 2\right] \\ &\quad + \frac{1}{Q^3}\left[q_0^4\left\{\frac{1}{Q\delta u_0^2} - \frac{2}{q_0 - C}\left(1 - \frac{q_0 - Q}{2Q\delta u_0^2}\right)\right\} - \frac{1}{Q\delta u_0^2}\right]. \end{aligned} \tag{A.18}$$

Integration then furnishes

$$\begin{aligned} \frac{(Qq_0)^3}{q_0 - C} q_1 &= Q^3 y\left[\frac{Q^2}{2}\left(\frac{C - Q}{4Q\delta u_0^2} - 1\right)y + \left(1 - \frac{C - Q}{q_0 - C}\right)q_0^3 - \frac{1}{4Q\delta u_0^2}\right. \\ &\quad \left. + 3(2C - Q)\left\{\frac{q_0^2}{2} + Cq_0 + C^2 \log(q_0 - C)\right\}\right] \\ &\quad - \frac{1}{Q\delta u_0^2}\left[\frac{q_0^8}{q_0 - C} - \left\{\frac{9}{7} + \frac{Q(1 + 2\delta u_0^2)}{q_0 - C}\right\}q_0^7\right. \\ &\quad \left. - \{9C - 7Q(1 + 2\delta u_0^2)\}\left\{\frac{q_0^6}{6} + \frac{Cq_0^5}{5} + \frac{C^2 q_0^4}{4} + \frac{C^3 q_0^3}{3} + \frac{C^4 q_0^2}{2} + C^5 q_0 + C^6 \log(q_0 - C)\right\}\right] \\ &\quad - 2(q_0 - C)^4\left[3C\left(5C - \frac{3Q}{4}\right) + \frac{(16C - Q)(q_0 - C)}{5} + \frac{(q_0 - C)^2}{3}\right] \\ &\quad - \frac{4C^2(q_0 - C)^3}{3}[59C - 12Q + (2C - Q)\{\log(q_0 - C)^3 - 1\}] \end{aligned}$$

$$- 2C^3(q_0 - C)^2\left[61C - 13Q + \frac{9}{2}(2C - Q)\{\log (q_0 - C)^2 - 1\}\right]$$
$$- 12C^4(q_0 - C)\left[10C - \frac{3Q}{2} + 3(2C - Q)\{\log (q_0 - C) - 1\}\right]$$
$$- 2C^5[8C + 3Q + 3(2C - Q) \log (q_0 - C)] \log (q_0 - C)$$
$$- 4C^6(C - Q)(q_0 - C)^{-1} + C_1 , \tag{A.19}$$

where the constant of integration c_1 is obtained by setting $y = 0$, $q_0 = 1$ and $q_1 = 0$.

The other differential relations are not solved in view of the fact that the parameter ϵ is very small, and it is to be expected that the contributions from them will be quite insignificant. The values of b, u and λ are obtained from the relations (A.1) and (A.6). These expressions are true for small values of y and hence of x.

From the numerical integration of the differential equations, it is noted that for increasing β, the quantity λ tends to zero in continuously smaller intervals of x. This, in the limit, reduces the problem to one of pure jet governed by the equations

$$\frac{d}{dx}(b^2u) = bu,$$
$$\frac{d}{dx}(b^2u^2) = 0, \tag{A.20}$$

with the boundary conditions

$$x = 0 : b = 1, \qquad u = u_0 .$$

For the case of finite β, the value of x when λ becomes zero, may be obtained from above. The remaining calculation is then carried out by means of the relations

$$b = x + \frac{k_2}{k_1}, \qquad u = \frac{k_1}{b}, \tag{A.21}$$

which are the solutions of the Eq. (A.20) for the case ($\lambda = 0$, $\epsilon \simeq 0$). The constants k_1 and k_2 are obtained by the values of b, u and x at which $\lambda = 0$.

Acknowledgements. Thanks are due to Dr. V. R. Thiruvenkatachar, Dr. C. D. Ghildyal and Mr. P. K. Khosla for many useful discussions and to the Research & Development Organisation, Ministry of Defence, Government of India for permission to publish this paper. Computational facilities (IBM 1620) provided by the D.R.D.L. Hyderabad are also thankfully acknowledged.

References

1. M. P. Murgai, *Radiative transfer effects in natural convection above fires,* J. Fluid Mech. **12**, (1962) 441
2. D. R. Chapman and M. W. Rubesin, *Temperature and velocity profiles in the compressible Laminar boundary layer with arbitrary distribution of surface temperature,* J. Aero. Sci. **16** (1949) 547
3. S. Chandrasekhar, *An introduction to the study of stellar structure,* University of Chicago Press, 1939
4. M. P. Murgai and H. W. Emmons, *Natural convection above fires,* J. Fluid Mech. **8** (1960) 611