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## OF

# APPLIED MATHEMATICS



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## SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter $O$, between the numeral one (1), the letter $l$ and the prime ($'$), between alpha and $a$, kappa and $k$, mu and $u$, nu and $v$, eta and $n$.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$\exp\,[(a^2 + b^2)^{1/2}]$ is preferable to $e^{(a^2+b^2)^{1/2}}$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

# BOOK REVIEWS

*Introduction to functional analysis for scientists and technologists.* Pergamon Press Ltd., London, and Addison-Wesley Publishing Co., Inc., Reading, Mass., 1963. xi + 404 pp. $10.00.

The book was published in Moscow (Fizmatgiz) in 1958. The present translation has been edited by Ian N. Sneddon.

*Mathematical apparatus of the theory of angular momentum.* By A. P. Yutsis, I. B. Levinson, and V. V. Vanagas. Israel Program for Scientific Translations Ltd., Jerusalem, 1962. xiii + 158 pp. $6.00.

This has been translated from the original Russian edition (1960 by A. Sen and R. N. Sen. Aside from occasional misspellings of names that were transliterated from the Russian (e.g. Gordan for Jordan), the translation has been carefully and competently done.

*Mathematical theories of planetary motions.* By Otto Dziobek. Translated by Mark W. Harrington and William J. Hussey. Dover Publications, Inc., New York, 1962. vi + 294 pp. $2.00.

This new edition is an unabridged and corrected republication of the work first published in 1892 by the Register Publishing Company.

PHILIP DAVIS

*Tables des coefficients du binôme et des factorielles.* By F. Giannesini and J. P. Rouits. Dunod, Paris, 1963. 120 pp. $2.46.

This table gives values of $C_n^p$ for $n$ from 1 to 100 to 10 significant figures and values of $n!$ for $n$ from 1 to 1775 to 20 significant figures.

*Introduction to mathematical logic.* By Elliott Mendelson. Van Nostrand Co., Inc., Princeton, 1964. x + 300 pp. $7.25.

The field of applied mathematics has made some astonishing acquisitions among which mathematical logic (through computer design, processing languages, theory of computability, etc.) is one of the more important. It was, however, a difficult task to advise mathematicians not primarily interested in foundations as to what textbook they would best consult as a concise introduction to mathematical logic. For this purpose the book under review is ideally suited. The author has succeeded in presenting the skeleton of the subject in a readable manner: Propositional Calculus, Quantification Theory (including some topics for the theory of models), Formal Number Theory (treating the notion of recursiveness, proof of the deep theorems of Gödel, Church and Tarski on the internal limitations of formal systems, incompleteness and undecidability), Axiomatic Set Theory (including a discussion of the axiom of choice), and the Theory of Computability (discussing various, equivalent, precise definitions of the intuitive concept of computability: Markov algorithms, Turing machines, recursive functions). There is also a valuable appendix with a proof (after Schütte) of the consistency of formal number theory.

E. ENGELER

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# BOOK REVIEWS

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*Methods in computational physics. Volume* 2: *Quantum mechanics.* Edited by B. Adler, S. Fernbach, and M. Rotenberg. Academic Press, New York, 1963. xi + 271 pp. $10.00.

The volume contains the following articles: The Gaussian Function in Calculations of Statistical Mechanics and Quantum Mechanics, by I. Shavitt; Atomic Self-Consistent Field Calculations by the Expansion Method, by C. C. Roothan and P. S. Bagus; The Evaluation of Molecular Integrals by the Zeta-Function Expansion, by M. P. Barnett; Integrals for Diatomic Molecular Calculations, by F. J. Corbató and A. C. Switendick; Nonseparable Theory of Electron-Hydrogen Scattering, by A. Temkin and D. E. Hoover; Estimating Convergence Rates of Variational Calculations, by C. Schwartz.

*Nonlinear mathematics.* By Thomas L. Saaty and Joseph Bram. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1964. xv + 381 pp. $12.50.

This book consists of six chapters on various parts of analysis of contemporary interest where nonlinear functions and functionals enter in an essential fashion: Linear and nonlinear transformations; nonlinear algebraic and transcendental functions; nonlinear optimization, nonlinear programming and systems of inequalities; nonlinear ordinary differential equations; introduction to automatic control; linear and nonlinear prediction theory.

Of these chapters, only that on nonlinear optimization may be considered to be well enough organized and to contain enough material to represent a contribution to mathematical literature. The others show lack of understanding of the basic ideas and methods, lack of organization, or both. This is particularly true of the chapters on control theory and nonlinear differential equations.

The book is definitely not recommended for either students or teachers.

RICHARD BELLMAN

*Selected translations in mathematical statistics and probability.* Volume 4. American Mathematical Society, Providence, 1963. vi + 387 pp. $5.30.

This volume contains translations of 33 papers in the general area of probability theory, into English, from Chinese, Hungarian, Rumanian, Russian and the other slavic languages. Twelve of the papers are short notes from Dokl. Akad. Nauk SSSR and similar transactions. The remaining, longer articles are listed below, followed by the number of the appropriate Mathematical Reviews article (M.R.) when available.

Ambarcumjan, G. A.—Entropy of Markov chains, Izv. Akad. Nauk Armjan SSR Ser. Fiz.-Mat. Nauk 11 (1958), no. 2, 31–40 (M.R. 21–1641)

Chang Li-chien—On the ratio of an empirical distribution function to the theoretical distribution functions, Acta. Math. Sinica 5 (1955), 347–368 (not reviewed)

Dupac, Vaclav—On the Kiefer-Wolfowitz approximation method, Casopis Pest. Mat. 82 (1957), 47–75 (M.R. 19–693)

Fabian, Vaclav—Some modifications of interval estimation and choice of number of observations in a special case of a binomial random variable, Apl. Mat. 4 (1959), 35–52 (not reviewed)

Firescu, D.—Estimation functions for the transition probabilities of a Markov chain, An. Univ. "C. I. Parhom" Bucuresti Ser. Sti. Nat. 7 (1958), no. 20, 37–47 (not reviewed)

Takacs, Lajos—A probability method for the treatment of the secondary electron emission, Magyar Tud. Akad. Mat. Fiz. Oszt. Kosl. 6 (1956), 199–211 (M.R. 20–6740)

Mamai, L. V.—On the theory of characteristic functions, Vectnik Leningrad. Univ. 15 (1960), no. 1, 85–99 (not reviewed)

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# BOOK REVIEWS

*(Continued from p. 326)*

Mandl, Petr—On the asymptotic behavior of probabilities in groups of states of a homogeneous Markov chain, Casopis Pest. Mat. 84 (1959), 140–149 (M.R. 21–7566)

Petrov, V. V.—On the law of the iterated logarithm, Uspehi. Mat. Nauk (N.S.) 15 (1960), no. 2 (92), 189–194 (M.R. 24–A568)

Prekopa, A., Renyi, A. and Urbanik, K.—On the limiting distribution of sums of independent random variables in bicompact commutative topological groups, Acta. Math. Acad. Sci. Hungar. 7 (1956), 11–16; Zentralblatt 71, 347–348 (not reviewed)

Renyi, Alfred—On a one-dimensional problem concerning random space filling, Magyar Tud. Akad. Mat. Kutato Int. Kozl. 3 (1953), no. 1/2, 109–127 (not reviewed)

Renyi, Alfred—On the distribution function, Magyar Tud. Akad. Mat. Kutato Int. Kozl. 2 (1957), 43–50 (M.R. 20–6148)

Rihter, V.—A more precise form of an inequality of S. N. Bernstein for large deviations, Vestnik Leningrad. Univ. 14 (1959), no. 1, 24–29 (not reviewed)

Salaevskii, O. V.—Stability in Raikov's theorem, Vestnik Leningrad. Univ. 14 (1959), no. 7, 41–49 (M.R. 21–4468)

Saramanov, O. V.—Investigation of stationary Markov processes by the method of eigenfunction expansion, Trudy Mat. Inst. Steklov. 60 (1961) 238–261 (M.R. 26–1935)

Smidov, F. I.—An application of metric contingencies, Uspehi Mat. Nauk (N.S.) 15 (1960), no. 6 (96), 169–174 (M.R. 24–A804)

Svesnikov, A. A.—Determination of the probabilistic characteristics of three-dimensional sea-waves, Izv. Akad. Nauk SSSR Otd. Tehn. Nauk Meh. Masinostr. 1959, no. 3, 32–41, (M.R. 21–3086)

Theiler, G.—A statement from the non-parametric statistical theory, Acad. R. P. Romine. Stud. Cerc. Mat. 9 (1958), 481–490 (not reviewed)

Wang Shou-jen—A remark on interpolation in a homogeneous random field on the lattice points in $R_k$, Advancement in Math. Volume 2 (1957), 257–262 (not reviewed)

Jaglom, A. M.—On a problem of linear interpolation of stationary random sequences and processes, Uspehi Mat. Nauk (N.S.) 4 (1949), no. 4 (32), 173–178 (not reviewed)

Jaglom, A. M.—Extrapolation, interpolation and filtration of stationary random processes with rational spectral density, Trudy Moskov. Mat. Obsc. 4 (1955), 333–374 (not reviewed)

The translations are quite variable in quality. The first of the above papers by A. Renyi, is translated so poorly that the statement of the problem is rendered unintelligible.

FRANK SPITZER

*Basic matrix algebra and transistor circuits.* By G. Zelinger. The Macmillan Co., New York, 1963. xv + 116 pp. $6.50.

About half of this slim book is devoted to basic matrix algebra. The remainder consists of applications of matrix algebra to the analysis of transistor circuits. The mathematics is elementary; the matrices are all $2 \times 2$, the most involved idea is that of an inverse matrix, which is described in terms of determinants. The book might be useful to a practicing engineer working with transistor circuits but I doubt if an applied mathematician would find it to be of more than passing interest.

B. HAZELTINE