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CONTENTS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

—BOOK REVIEW SECTION—

Linear sequential switching circuits—selected technical papers. By D. A. Huffman, Bernard Elspas, Juris Hartmanis, T. E. Stern, B. Friedland, Martian Cohn, Neal Zierler, and C. V. Srinivasan. Edited by William H. Kautz. Holden-Day ,Inc., San Francisco, London, Amsterdam, 1965. v + 234 pp. $6.75.

As the title indicates this book is a collection of the basic papers having to do with linear sequential circuits, that is, digital circuits made up of delay elements, modulo adders, and modulo multipliers. The basic reason why such a circuit is of interest is that its output is a linear function of the present input and past inputs. Because the function is linear the mathematics describing the behavior of the circuits is reasonably tractable. The papers included are *The Synthesis of Linear Sequential Coding Networks* by D. A. Huffman, *The Theory of Antonomous Linear Sequential Networks* by B. Elspas, *Linear Multivalued Sequential Coding Networks* by J. Hartmanis, *The Linear Modular Sequential Coding Circuit Generalized* by T. E. Stern and B. Friedland, *Controllability of Linear Sequential Machines* by M. Cohn, *Linear Recurring Sequences* by N. Zierler, *State Diagrams of Linear Sequential Machines* by C. V. Srinivasan, *Several Binary-Sequence Generators* by Neal Zierler, *A Linear Circuit Viewpoint of Error Correcting Codes* by D. A. Huffman, and *Application of Modular Sequential Circuits to Single Error-Correcting P-Nary Codes* by T. E. Stern and B. Friedland. The general mathematical level of these papers is reasonably high, most of the papers using techniques and ideas of linear algebra freely. Applications are mentioned only in passing. The several authors use arguments of various stages of mathematical sophistication, thus almost every interested reader, from the novice to the specialist, and from the circuit designer to the mathematician, will find something of interest. Because of this range in objectives the book will probably be more useful as a source book than as a textbook. As a source it will be invaluable and the editor is to be congratulated on its preparation.

CORRECTION: On page 9 of the book there is a reference to "page 89"; the reference should be page 13.

B. Hazeltine

The elements of computational mathematics. Edited by S. B. Norkin. Translated from the Russian by G. J. Tee and English translation edited by A. D. Booth. Pergamon Press, Inc., New York, 1965. xiii + 192 pp. $6.00.

This is a translation of a first book in numerical analysis intended for correspondence course students. The original dates from 1960. The individual chapters were written by different authors. The book hardly betrays a sign that there are such things in the universe as electronic computers. No, this isn't quite right; in the introduction the assumption is made that the student will put in a hitch in the salt mine world of desk computers because "The use of fast machines for the comparatively small calculations most often arising in engineering and industry is not advantageous, and sometimes it is actually inconvenient. That is the reason why future engineers are recommended to train themselves on desk calculators and to use them for performing problems of mathematical practice and subsequently for work in the special disciplines of their future industrial activity." True or false in 1966?

At any rate, the student presumably provides himself with a copy of "Five Figure Mathematical Tables" by Segal and Semendyaev and starts punching away.

This book falls in the category of well-written Russian books which there was little reason to translate. It is a little bit like making the works of TSCHARLS DIKENS available in English.

P. J. Davis

Markov processes. By E. B. Dynkin. Translated with the authorization and assistance of the author by J. Fabius, V. Greenberg, A. Maitra, and G. Majone. Springer-Verlag, Berlin, Göttingen, Heidelberg, and Academic Press Inc., New York, 1965. xii + 365 pp. (Volume I) $12.00. viii + 274 pp. (Volume II) $12.00.

This two volume work is the authorized translation of the Russian edition (1964), which followed the author's Foundations of the Theory of Markov Processes (1959). In contrast to that book which gave only the necessary definitions and the most basic theorems asserting the existence of Markov processes possessing desirable continuity properties, the present work delves immediately into the mathematically interesting and probabilistically significant theorems of the subject. The results needed from the "Foundations" are summarized at the places they are first used.

The book consists of five principal parts. The first five chapters emphasize the semigroup aspects of the theory of Markov chains. This part begins with a lucid treatment of the Hille–Yosida theorem and ends with a study of the characteristic operator (the author's important extension on the infinitesimal generator, in which the time parameter t is replaced by the expectations of the absorption times for a family of regions which shrink to a fixed point). The next six chapters (six to eleven) form the core of the book, being concerned with the theory of additive functionals of a Markov process, which has been by far the most active area in recent work on general Markov processes. The basic idea is to transfer to the potential theory of general Markov processes the classical theorem of F. Riesz dealing with the representation of superharmonic functions as potentials of measures, and the solution is furnished by the potentials of the additive functionals (a generalization of the occupation times of a subset of the state space). As applications of this theory the author presents recent results concerning transformations of Markov processes, the complete classification of the additive functionals of Brownian motion, and Itô's theory of stochastic integrals and integral equations. The next two chapters (twelve and thirteen) deal with ideas from classical potential theory, presented here in the general context of continuous Markov processes (diffusions). The main results are the probabilistic solution of the Dirichlet problem and the connection between functionals $\int_0^t V(x_\tau)\, d\tau$ and the equation $\Delta f - Vf = g$. In the fourth part of the book (chapter 14) the results of the previous chapters are applied to n-dimensional Brownian motion and its transformations. The results complement those in the recent monograph of Itô and McKean (Diffusion processes and their sample paths, Springer 1965). The last part (chapters 15 and 16) treats the construction of the most general one-dimensional diffusion (strong Markov process with continuous real-valued trajectories), but here the above mentioned treatment of Itô and McKean goes somewhat farther in deriving the transformation of an arbitrary diffusion into Brownian motion by means of a random time change.

The book as a whole gives a most impressive picture of the present state of Markov processes and their relation to semigroups, differential equations, and potential theory. It goes without saying that there must be omissions in a work of such vast scope; thus one misses Martingale theory, not only because of its dominant role in the study of superharmonic functions, but also as a technical tool whose systematic use might have simplified a number of demonstrations.

The book is virtually self-contained, and the reader is aided by an appendix concerning difficult results from measure theory, by lucid and useful historical remarks, and by the considerate device of labeling theorems whose statements are important, whereas their proofs are of purely technical interest. Nevertheless, the non-specialist will find the book difficult primarily because the setting is often sufficiently general and complicated to permit inclusion of recent results which are still incomplete or known only under unsatisfactory conditions. And the most intuitive chapters, i.e., those concerning Brownian motion, can only be read after at least partial mastery of the general theory which they illustrate and further elaborate. For the specialist however, or the serious student of stochastic processes, this book is indispensable.

F. Spitzer

Methods in computational physics—advances in research and applications. Edited by B. Alder, S. Fernbach, M. Rotenberg. Volume 4. Academic Press, New York and London, 1965. xi + 385 pp. $14.00.

This volume is a collection of articles on the solution of fluid mechanical problems by high speed digital computers. An overall view of the scope of the book is best obtained by merely listing the titles (and contributors) of the articles: "Numerical Simulation of the Earth's Atmosphere" (C. E. Leith), "Nonlinear Effects in the Theory of a Wind-Driven Ocean Circulation" (K. Bryan), "Analytic Continuation Using Numerical Methods" (G. E. Lewis), "Numerical Solution of the Complete Krook–Boltzmann Equation for Strong Shock Waves" (M. T. Chahine), "The Solution of Two Molecular Flow Problems by the Monte Carlo Method" (J. K. Haviland), "Computer Experiments for Molecular Dynamics Problems" (R. A. Gentry, F. H. Harlow, and R. E. Martin), "Computation of the Stability of the Laminar Compressible Boundary Layer" (L. M. Mack), "Some Computational Aspects of Propeller Design" (W. B. Morgan and J. W. Wrench, Jr.), "Methods of the Automatic Computation of Stellar Evolution" (L. G. Henyey and R. D. Levée), "Computations Pertaining to the Problem of Propagation of a Seismic Pulse in a Layered Solid" (F. Abramovici and Z. Alterman).

The reader is assumed to be thoroughly familiar with finite difference schemes for integrating differential equations so that emphasis is placed on convergence and computational subtleties. Most of the papers give a physical introduction sufficient to acquaint the uninitiated reader with the problem. However, there is much variability from paper to paper in both the amount of physical discussion and computational detail.

IRA M. COHEN

Inventory systems. By Eliezer Naddor. John Wiley & Sons, Inc., New York, London, Sydney, 1966. xiv + 341 pp. $11.50.

This is the most comprehensive treatment of inventory problems from a system point of view, and an excellent introduction to the field. Following an exposition of the basic concepts, part two treats deterministic systems, starting out with the lot-size approach and contrasting and then linking it with the order-level-system. There is a brief treatment of variable demand. The problems of stochastic demand are divided rather naturally into "scheduling period systems" (part three) and "reorder point systems" (part four).

The reader is introduced to the standard techniques and to some interesting new variants. The dynamic programming approach, whose introduction by Arrow, Harris, and Marschak revolutionized the theory of inventory control, is explained in Chapter 16. But the s, S policy is assumed, and the fertile "principle of optimality" (Bellman) is not discussed. The important results of Scarf on the optimality of the s, S policy are not even mentioned. Although the title might suggest this to some students, there is no treatment of multi-stage inventory problems.

Within its self-imposed limitations the book gives an excellent account of the fantastic variety of systems that arise in inventory control. Much thought has obviously gone into the taxonomy and the selection of the cases that are presented. No work of manageable size sould conceivably exhaust all possibilities. The exposition is elementary enough to make this book readable by a larger audience. To the management scientist and operations researcher it is a must.

MARTIN J. BECKMAN

Non-linear transformations of stochastic processes. Edited by P. I. Kuznetsov, R. L. Stratonovich and V. I. Tikhonov. Translation edited by J. Wise and D. C. Cooper. Pergamon Press, New York, 1965. xii + 484 pp. $20.00.

This volume comprises a collection of 39 papers on the application of stochastic processes, principally to the field of radio-engineering. The papers are grouped under 5 chapters: 1. Some problems of the theory of stochastic processes. 2. The effect of noise on certain non-linear elements. 3. The effect of fluctuations on oscillator operation. 4. Random function excursions. 5. Optimum filtration. The first chapter contains papers which develop the mathematical theory of stochastic processes and the methods of linear and non-linear transformation. The next three chapters are devoted to the solution of problems in radio and control engineering relating to the effect of stochastic processes on non-linear systems and devices. The last chapter contains a selection of articles on the synthesis of optimum systems.

Error in digital computation. Edited by Louis B. Rall. Volume I. John Wiley & Sons., Inc., New York, London, Sydney, 1965. ix + 324 pp. $6.75.

This volume is the proceedings of an advanced seminar conducted by the Mathematics Research Center, United States Army, held October 5–7, 1965. It contains the following five papers: 1) The problem of error in digital computations, by John Todd, in which the general problem of error in digital computation is discussed, realistic standards for error estimates are defined and practical techniques for testing numerical methods for accuracy are demonstrated. 2) Techniques for automatic error monitoring and control by Robert L. Ashenhurst, in which significance arithmetic and its application to automatic error estimation is treated. 3) The automatic analysis and control of error in digital computing based on the use of interval numbers by Raymon E. Moore, in which interval arithmetic is used to obtain new and important results in error estimation and control in the numerical solution of ordinary differential equations. 4) Error in digital solution of linear problems by Ernest L. Albasiny, where a comprehensive survey of error estimates is given for the solution of linear algebraic problems in both fixed and floating point arithmetic. 5) Error in digital integration of ordinary differential equations by Peter Henrici, in which the error estimation problem for the numerical solution of ordinary differential equations by a number of methods is surveyed.

A very comprehensive 117 page bibliography of the literature on error estimation that has appeared in the last 25 years, concludes the volume.

Digital computation and numerical methods. By R. W. Southworth and S. L. Deleeuw. McGraw-Hill Book Co., New York, 1965. xiv + 508 pp. $9.75.

This book has been developed from material used in a freshman engineering course offered at Yale University. It is divided into two main sections: programming (Chapters 2 through 4, 143 pages) and numerical methods (Chapters 5 through 12, 345 pages). The programming language employed is FORTRAN IV, but a synthetic machine language is used when compilation of a FORTRAN program is explained. The second portion of the book includes: errors in computation, solution of equations, interpolation, numerical differentiation and integration, and the fitting of experimental data. All topics are presented in detail and clear style on an introductory level, followed in some instances, such as matrices, by development to considerable depth.

The authors have succeeded in all respects in fulfilling their aims and the resulting book is one of the very best of its kind.

An introduction to the theory of mechanics. By K. E. Bullen. Seventh Edition. Cambridge University Press, New York, 1965. xvi + 365 pp. $6.50.

This is the seventh edition, but first substantial revision, of a standard introductory textbook to mechanics as taught to first year under-graduates at British (and Australian) Universities. The emphasis is on dynamics but there are short chapters on statics and hydrostatics. The book assumes no prior knowledge of mechanics or calculus, but it assumes that the student is starting to learn the calculus. In the author's words: "Special efforts have been made to present the subject of dynamics as applied mathematics—i.e. as a branch of science in which mathematics is auxiliary to the central purpose of describing a body of observational evidence about the natural word." This aim is achieved with an admirable sense of proportion and much skill of exposition.

Invariant imbedding and time-dependent transport processes. By R. E. Bellman, H. H. Kagiwada, R. E. Kalaba and M. C. Prestrud. American Elsevier Publishing Co., Inc., New York, 1964. x + 256 pp. $7.50.

This volume extends the results of the authors' earlier book *Invariant Imbedding and Radiative Transfer in Slabs of Finite Thickness* by R. E. Bellman, R. E. Kalaba and M. C. Prestrud, American

Elsevier Publishing Co., New York, 1963, on the invariant imbedding technique to certain time-dependent problems, in particular one-dimensional neutron multiplication in a rod and diffuse reflection of incident radiation from a slab. The basic approach is to apply a Laplace transformation with respect to the time variable, which has the effect of replacing time derivatives by absorption terms, so that the transformed equations can formally be thought of as representing a steady state situation.

The small amount of textual material (there are only about fifty pages) is devoted mainly to the numerical inversion of Laplace transforms, with special reference to computer methods. The remainder of the book consists of appendices which tabulate results for the diffuse reflection problem and also various quantities arising in connection with the Gaussian quadrature techniques used to invert Laplace transforms. Fortran programs for a number of calculations are also given.

On the whole, this is a highly specialized book, and will be of use mainly to readers interested in the particular physical situations discussed by the authors.

Stephen Prager

Stochastic processes in mathematical physics and engineering. Volume XVI. Edited by Richard Bellman. Proceedings of Symposia in Applied Mathematics. American Mathematical Society, Providence, R. I., 1964. viii + 318 pp. $7.60.

This volume is composed of fifteen papers presented at the Sixteenth Symposium in Applied Mathematics in New York in April, 1963, under the sponsorship of the American Mathematical Society and the Society for Industrial and Applied Mathematics. In what is frequently at least a semi-expository style, the authors range over a wide variety of topics related to the use of probabilistic concepts in various problems of applied science. There is no particular effort to make the treatment unified or reasonably complete; rather, this is an excellent introduction to several areas in which active research is under way.

Some idea of the scope of the book may be obtained from a listing of the papers appearing in its Table of Contents: G. Adomian, *Stohastic Green's Functions;* A. T. Bharucha-Reid, *On the Theory of Random Equations;* G. C. Rota, *Reynolds Operators;* Victor Twersky, *On Propagation in Random Media of Discrete Scatters;* W. C. Hoffman, *Wave Propagation in a General Random Continuous Medium;* J. B. Keller, *Stochastic Equations and Wave Propagation in Random Media;* Richard Bellman, *Stochastic Transformations and Functional Equations;* K. B. Gray, *The Application of Stochastic Approximation to the Optimization of Random Circuits;* E. W. Montroll, *Random Walks on Lattices;* Emanuel Parzen, *On Statistical Spectral Analysis;* W. L. Root, *Stability in Single Detection Problems;* Eugene Wong, *The Construction of a Class of Stationary Markov Processes;* David Blackwell, *Probability Bounds via Dynamic Programming;* Cyrus Derman, *Markovian Sequential Decision Processes;* J. M. Richardson, *Application of Truncated Hierarchy Techniques.*

Seven of the papers (the first six and the last one), constituting approximately 60% of the book, are concerned mainly with various aspects of stochastic differential equations, both ordinary and partial. These range from fairly concrete problems in wave propagation to more abstract and general papers on the theory of stochastic equations.

The remaining papers seem to defy a similar classification, either by subject or methodology. Neither are they altogether unrelated to those mentioned above since several involve differential equations in one way or another. Among the topics touched upon more than once are stochastic iteration, decision and control problems, and dynamic programming.

In most volumes of this sort the intensity of a reader's interest is likely to vary widely from one paper to another, and this was the reviewer's experience here. However, because of the range of topics considered and the authoritative knowledge of the authors, almost anyone involved in the application of probability to physical problems will find many things here worthy of his attention.

W. E. Boyce

The design of structures of least weight. By H. L. Cox. Pergamon Press, New York and London, 1965. vii + 134 pp. $6.00.

This book is primarily addressed to engineers, who will find in it a veritable mine of information on the best way to conduct detailed design, as well as a stimulating account of the problem of the optimum choice of layout. It is this last aspect which will appeal most to applied mathematicians.

A. G. M. Michell gave, in 1904, sufficient conditions for a pin-jointed structure, which safely equilibrates given forces, to have minimum weight. Little attention was paid to this important result at the time, but more recently thanks to a generalisation to cover stiff-jointed frames by J. D. Foulkes, to an extension to continua by D. C. Drucker and R. T. Shield and to many ingenious applications of Michell's methods by H. L. Cox, W. Prager and others, the subject is now in a fairly rapid state of development.

The problems involved are not easy and present a fascinating challenge to mathematicians. One might mention the problem of generalisation to cover alternative loading systems. This is of extreme practical importance, since it is rare that one loading case completely designs a structure and while a knowledge of optimum structures for each of several loading systems may well be of value to a designer, it would be much more valuable to him if guidance could be given on the best method of compromise. Another problem of great practical interest is the optimum design of shells. Michell gave the solution for torsion, which is in fact a sphere, but hardly any other "Michell Shells" are known. Mathematically this last problem reduces to finding the surfaces which can be generated by the lines of principal strain in three dimensional strain systems, whose principal strains are constant. Optimum design of structures can be carried out, at least approximately, using the digital computer. The problems which arise are those of "linear programming" and the difficulties experienced in applying the standard techniques, stem from the large size of the problems involved in tackling even the simplest design questions. The need is thus for more powerful techniques at least for the special forms of linear programming involved.

H. L. Cox's book presents the basic Michell theory together with recent developments and applications in mainly framework design. It poses as many questions as it answers, but this of course makes it invaluable to the research worker in this field as a source of inspiration. It is an up to date account of our knowledge and our ignorance too and like everything the author writes and says is well worth the maximum of attention from every engineer and mathematician, who is interested in least weight structures.

W. S. Hemp

Elementary statistics. By Paul G. Hoel. (Second Edition). John Wiley & Sons, Inc., New York, London, Sydney, 1966. ix + 351 pp. $7.25.

An author who undertakes to treat probability, discrete and continuous distributions, estimation and testing, correlation, regression (including multiple and non-linear regression) and analysis of variance on the frail basis of high school algebra places himself in a well-known cleft stick. Professor Hoel chooses the most popular escape: an anecodotal approach to useful, classical material. This second edition of his successful book differs from the first mainly in the format and typography, which has been much improved.

Frank A. Haight

Electromagnetic diffraction and propagation problems. By V. A. Fock. Pergamon Press, New York and London, 1965. ix + 414 pp. $25.00.

This volume, in English, is a collection of seventeen papers by Academician Fock and his co-workers on the theory of diffraction and propagation of electromagnetic waves. The original papers, mostly in Russian, were with one exception published between 1945 and 1958. Their collection in this volume is useful to research workers in diffraction and wave propagation although many of the papers have been available in English translation for some time. In fact, English translations of all but four of the seventeen papers appear in *Diffraction Refraction, and Reflection of Radio Waves*, edited by N. A. Logan, Air Force Cambridge Research Center, AFCRC-TN-57-102, (1957). This report can be requested from the Armed Services Technical Information Agency, Document Service Center, Dayton 2, Ohio, as ASTIA Document No. AD 117276. The translations in the Pergamon volume appear to be practically identical to those in the 1957 report.

The volume is in two parts. Part I, on the asymptotic theory of diffraction, contains papers which have had an important influence on some aspects of the theory of diffraction of high-frequency waves. In particular, for diffraction by smooth convex objects, a successful method is developed for treating the difficult problem of representing the scattered field near the shadow boundary. The theory is developed partly on the basis of an exact analysis of diffraction of a plane wave by a paraboloid of revolution.

Then the insight gained from the study of the solution of this problem is used to develop a representation for the field near the surface of a general smooth convex body. The scattered wave in the whole space is then obtained from the surface field by a well-known integral formula.

Part II deals with radio wave propagation in homogeneous and stratified media. The method developed in part I is applied to the propagation of electromagnetic waves around the earth. Both diffraction and refraction are treated. The volume includes an appendix on the definition and properties of the Airy functions and a table of their values.

ROBERT M. LEWIS

Representation theory of finite groups. By Martin Burrow. Academic Press, New York and London, 1965. ix + 185 pp. $6.50.

The algebraic background of representation theory is well laid in Chapters I and II, the pattern of development resembling that of Curtis and Reiner's treatise with a similar title. Ordinary character theory follows in Chapter III, though the language of Frobenius and Schur has all but disappeared. The reason is of course that the discussion must lead up to the development of the modular theory given in Chapter VI. Chapter IV introduces algebraic numbers and proves the classical theorem that the degree of an irreducible representation of G must divide the order g of G, concluding with two theorems of Burnside and Frobenius which indicate the significance of characters for the abstract theory.

In Chapter V we have the author's own contribution to the theory which was published in the Canadian Journal of Mathematics in 1954. In brief, he generalizes Young's definition of a primitive idempotent of S_n to apply to an arbitrary finite group G, with an associated character formula given in the Appendix. The rest of the chapter is devoted to constructing character tables and actual matrices for the representations of S_4, A_4, and A_5. This reviewer must take exception to the introductory paragraph of §3 on p. 106 as ignoring Young's Fundamental Theorem which yields all representative matrices for S_n and A_n immediately (cf. Puttaswamaiah and Robinson, C.J.M.(16)1964, 587–601). Of course the author is illustrating his own version of Young's method. Is it too much to hope that this may some day be extended to provide a generalization of the Fundamental Theorem?

The final chapter on the modular theory is useful so far as it goes, but for those who are able and willing to follow it in detail this is just the beginning.

In his introduction the author suggests that ".. the book might prove useful as supplementary reading for a course in group theory or in the applications of representation theory to physics." Having had some experience in giving courses in both these areas this reviewer would heartily agree with the first suggestion, particularly if the students had a good grounding in abstract algebra. With regard to the second suggestion, he has always fostered the hope that physicists might extend the really crucial practical role now played by ordinary irreducible representations, particularly of S_n, to the further reduction allowed by the modular theory, but so far he has been disappointed. This is a stimulating little book and a welcome addition to the literature of the subject.

G. DE B. ROBINSON

Linearized analysis of one-dimensional magnetohydrodynamic flows. By Roy M. Gunderson. Springer-Verlag, New York, 1964. viii + 119 pp. $5.50.

The author states that "the primary purpose of this monograph is the presentation of a method for discussing weakly non-isentropic quasi-one-dimensional flows of an ideal, inviscid, perfectly conducting compressible fluid subjected to a transverse magnetic field."

The topics that are discussed include shock propagation in non-uniform ducts, piston-driven shock waves, flows with heat addition, simple wave flows, formation and decay of shock waves and a brief discussion on the effects due to an oblique applied magnetic field. Most of the material is derived from the author's own publications.

The governing unsteady equations are linearized about a known isentropic, uniform state and expressed in characteristic coordinates. The concept of Riemann invariants is modified so as to apply to magnetohydrodynamic flows. Additional source terms appear which modify the compatability conditions along the waves. Extensions of the non-conducting results for the aforementioned topics then become tractable.

The author has made no mention of the pertinent boundary conditions on the magnetic field, nor has he stipulated the conductivity or permeability of the channel walls. Consequently, there is no ad hoc justification for his neglect of the transverse gradient of the axial magnetic field in Ampère's law for the current density. In particular, this term is dominant in many practical geometries, even within the scope of the quasi-one-dimensional approximation.

It is not clear to this reviewer why the author, prior to his formal linearization, made use of certain constant area, homentropic relationships, *e.g.*, $B/c^{2/\gamma-1}$ = constant instead of $B/Ac^{2/\gamma-1}$ = constant, among others, while retaining similar area and entropy perturbations in the continuity and momentum equations. For this reason the modified Riemann invariants are also suspect.

Finally, the author states that in the non-isentropic perturbation of a centered simple wave flow the solution does not reduce to that for the non-conducting fluid in the limit of vanishing magnetic field. Similar discrepancies occur in the theory of piston-driven shock waves. While a significant number of results are presented in this monograph, it is felt that in certain instances the physics has been obscured by the mathematics.

S. G. Rubin

Excursions in number theory. By C. Stanley Ogilvy and John T. Anderson. Oxford University Press, New York, 1966. $5.00.

This book is a collection of number theoretic curiosities expounded at the high school level of mathematics. It includes latest data (1965) on factorization of Mersenne primes and Fermat primes as well as Diophantine equations of historical interest. The use of an electronic computer (apparently just a giant slide rule) is frequently cited, but the admiring section on prodigies almost cancels out any missionary effect these citations might have. The authors get around to very recent theoretical ploys such as Lucky numbers, but most of the book is on material familiar to high school mathematics clubs and very attractively presented.

Harvey Cohn

The theory of finitely generated commutative semigroups. By László Rédei. Translation edited by N. Reilly. Pergamon Press, New York and London, 1965. $13.50.

A set with a binary operation which is associative is called a semigroup. When the discussion is limited to finitely generated commutative semigroups, (*f.g.c.s.*), the outsider may be forgiven for thinking that there is nothing worth saying on the subject, or ar least, that whatever is to be said has been said long ago. Professor Rédei demonstrates that this is not so: his work, really a paper of book length, develops the subject from scratch, and in the course of 350 pages is able to give a very detailed picture while making it clear that the last word of the subject has not been said.

The prototype of the object to be studied is the set N of non-negative integers, with addition as operation. Every *f.g.c.s.* is a homomorphic image of a direct product of a number of copies of N, say N^k. This semigroup N^k has a natural embedding in the free abelian group Z^k of rank k, and Chapter I shows how to describe each congruence C on N^k (giving rise to a homomorphic image) by a certain function on Z^k, the *kernel function* of C. In Chapter II the attention shifts to kernel functions: some rather technical results about them are proved, but no interpretation in terms of congruences is given; this is followed by a closer study of the axioms for a kernel function. Chapter III presents the ideal theory of N^k and contains the solitary reference to other published work (by Dickson); the properties found are used in Chapter IV to make a more detailed study of kernel functions. The most interesting result of this chapter is the theorem that every *f.g.c.s.* is finitely definable, *i.e.* it can be defined by a finite number of equations. The final chapter deals with the important problem when two kernel functions give rise to isomorphic semigroups; this is not solved in any sense, but a prescription is given for deriving, from a given kernel function, all others equivalent to it.

The theory of *f.g.c.s.* presents an immense amount of detail, much of it interacting with number theory and lattice theory, and it is the author's merit to have drawn attention to it, and to have made a very promising attempt at introducting order into this chaos. He has also uncovered a rich source of problems, many non-trivial without being impossibly hard, but here as so often in pure mathematics the chief difficulty confronting the student is to pick the problems which can be answered in a significant and interesting way. The author himself has not entirely resolved this difficulty; the majority of theorems in his book are not about semigroups, but about the auxiliary notions developed to deal with them. The preoccupation with kernel functions is such that even when there are tangible conclusions for semigroups to be drawn, these are not spelt out. For example, to obtain the well-known classification of one-generator semigroups one may apply Theorem 94, a technical result on kernel functions whose statement takes up over half a page, but the explicit description of these semigroups is not even mentioned.

One may also regret the fact that there has been no mention of any applications (to information theory, automata theory etc.), but within the self-imposed limitations the author has provided a most stimulating account of a hitherto neglected branch of semigroup theory, which will surely do much to make the subject known in wider circles.

P. M. Cohn

Complex analysis—an introduction to the theory of analytic functions of one complex variable. By Lars V. Ahlfors. (Second Edition). McGraw-Hill Book Co., New York, London, Sydney, 1966. xiii + 317 pp. $8.95.

This is a second edition of Ahlfors' well known and much admired introduction to the theory of analytic functions of one complex variable. In this review, we need, therefore, only list the changes. In the words of the author,

1. The exponential and trigonometric functions are now defined by means of power series. In order to do so, it was necessary to introduce an early elementary section on complex power series, a procedure that is not without didactic value in itself.

2. The introduction to point set topology has been rewritten. It now includes the fundamental properties of metric spaces and a more detailed discussion of compactness.

3. Normal families are approached in a more direct manner, and the connection with compactness is emphasized.

4. The Riemann mapping theorem has been combined with a section on the Schwarz-Christoffel formula.

5. A short chapter on elliptic functions has been added. It is deliberately very concentracted in an effort to spare the reader from the customary maze of notations that are needed only by specialists.

6. The exercise sections have been enlarged, and some starred exercises with generous hints for their solution have been included. The latter are to be regarded as part of the test, and students should be encouraged to composed complete proofs.

This reviewer is particularly pleased to have 2. and 3.

P. J. Davis

Some topics in complex analysis. By E. G. Phillips. Pergamon Press, New York, London, 1966. viii + 141 pp. $5.50.

This slim volume is a sequel to the author's "Functions of a Complex Variable." The topics include elliptic functions, the Jacobi elliptic functions, conformal transformations, Schlicht functions, the maximum modulus principle, integral functions, and special functions via contour integrals. The author sips, rather than drinks, at each of these springs, but the exposition is a pleasure and the book can comfortably be made the basis of a semester's course in intermediate complex variable. There are exercises at the end of each chapter.

P. J. Davis

Grenzschicht-Theorie. By Hermann Schlichting. Verlag G. Braun, Karlsruhe, Germany, 1965. xvi + 736 pp.

In the preface to this fifth edition of the well-known, much used, and much appreciated book, the author states that he intentionally avoided adding to the content of earlier editions the treatment of "material which seems still in the state of development or under discussion." Reviewer agrees that updating a book is a delicate task for each author, but thinks that there are advantages in indicating to the student or other reader at least that certain new developments are caused by the changing state of engineering.

Also, the representation of the "classical material" might be improved at some points. In the derivation of the Navier-Stokes equations there appears only the first viscosity coefficient μ. The influence of the second (bulk) viscosity coefficient is not discussed. (Compare *e.g.* C. E. Brown's derivation in Chapter 68 of the Handbook of Engineering Mechanics). The vorticity transport equation is derived only for the two-dimensional case; a short note seems desirable that for three-dimensional motion the equation written in vector form is more complicated than the reader may conclude from the given formula for two-dimensions.

Reviewer wonders why in the more than 20 years since Holstein and Bohlen's idea for a simple numerical integration of the boundary-layer momentum equation was published, the author has not found it worthwhile to change the table of auxiliary functions (P. 191) in such a way that the form-parameter κ varies in simple steps. Each user knows how inconvenient the interpolation in the κ column is; but still the no longer used λ has the nice steps.

The section XIII on the laminar compressible layer has been enlarged and updated, and a new section on turbulent compressible layer XXIII has been added.

With all appreciation of the enlargement and presentation of Chapter XIII, reviewer regrets that still the Gruschwitz integral method is chosen as sole representative of all integral procedures, in spite of the fact that it is known and even stated in a footnote that its approximations can lead to unrealistic temperature distributions. The well-developed finite difference methods for compressible layers are not mentioned [they are mentioned in Chapter IX (incompressible flow), in the brief discussion of step by step methods]. There is also no statement that supersonic flow may occur with relatively low Reynolds numbers and that one may have to reexamine Prandtl's theory, which is a first-order theory. M. Van Dyke's work which is intimately connected with second-order theory is mentioned in only one sentence on P. 123.

In spite of these critical remarks the reviewer believes that the book offers so much valuable material in good presentation that it will attract many users, as it has done before.

I. Flügge-Lotz

Fortran IV programming and computing. By James T. Golden. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965. 270 pp. $4.50.

The author's intention was to write an introductory college-level programming text whose objectives are "to develop the reader's ability to generate algorithms and to guide him in creating strategies for problem solving on a digital computer." He has fulfilled his intention employing Fortran IV as the computer language. The author uses techniques which make this a readable text. For example he introduces many new language concepts and instructions as logical extensions of already learned concepts. In addition he has been liberal in the use of flowcharts and short, simple examples to illustrate algorithms and longer numerical non-trivial complete programs.

The scope of the book can best be illustrated by listing chapter headings. Since some chapter headings are in themselves sufficiently descriptive, comments will be made on only a few which need comment.

Introduction: A short history of the development of computers and the stored program. An example of a simple program using eight operation codes illustrates a basic stored program. Introduction to Fortran IV: A simple subset of the Fortran IV language including the arithmetic statement, cookbook input/output, simple control statements and flowcharting; Subscripted variables: Input-Output operations: Includes a description of input/output for magnetic disk, magnetic drum and magnetic tape, as well as the reader and printer; Subroutines: Complex Numbers, Boolean Algebra, Simulation: Practices and Pitfalls in Computing.

The style of writing is logical and lucid and so makes an excellent as well as interesting self-study guide for the scientifically oriented individual. In summary this book is recommended for engineers, scientists and numerical analysts who do not necessarily have prior knowledge of computers and who would like to learn a sufficient amount of programming techniques to employ in their fields. It can also serve as a programming supplement to a numerical analysis course.

E. CERUTTI

Calculus of variations and partial differential equations of the first order—Part I. By C. Carathéodory. Translated by Robert B. Dean and Julius J. Brandstatter. Holden-Day, Inc., San Francisco, London, Amsterdam, 1965. xvi + 171 pp. $8.50.

This is a translation of Carathéodory's classic treatise of thirty years ago. It contains a systematic discussion of the theory of characteristics, Hamilton-Jacobi equations, Poisson brackets and related topics. Although the presentation has, from the point of view of pure mathematics, been made obsolete by the work of modern differential geometers, the book will be of great interest and delight to applied mathematicians, physicists and engineers to whom the modern discoveries are, as yet, of little relevance.

WALTER FREIBERGER

An introduction to Fourier series and integrals. By Robert T. Seeley. W. A. Benjamin, Inc., New York and Amsterdam, 1966. x + 104 pp. $7.00.

This introduction constitutes part of a sophomore-to-senior level course on calculus or advanced calculus. The interplay between physics and mathematics is emphasized and numerous problems are included.

Introduction to finite mathematics. By John G. Kemeny, J. Laurie Snell, Gerald L. Thompson. Second Edition. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1966. xiv + 465 pp. $8.95.

The second edition of this well-known and successful book differs little from the first. There are changes in the sections on logical possibilities and counting techniques; Bayes theorem replaces geometric probabilities; the section on the law of large numbers and central limit theorem has been split in two and those on linear equations and linear programming have been rewritten; there is more on Markov chains and there are new sections on equivalence classes in communication networks and on computer simulation.

Introduction to matrices and linear transformations. By Daniel T. Finkbeiner, II. Second Edition. W. H. Freeman and Co., San Francisco and London, 1966. xi + 297 pp. $7.75.

This book provides a lucid and unified introduction to linear algebra. Although suitable for able undergraduates, there is much in it of interest to research workers: equivalence relations and canonical forms, the duality of geometry and algebra, metric concepts. Computational aspects are not ignored.

Major changes in the second edition include an earlier introduction of the dual space; the use of cyclic subspaces in the derivations of the Cayley–Hamilton theorem and the Jordan canonical form; an expanded discussion of combinatorial equivalence and its application to linear programming.

The book has considerable individuality and flair; its style is stimulating and the selection of topics propitious.

The programming language LISP: its operation and applications. By Edmund C. Berkeley and Daniel G. Bobrow, Editors. The M.I.T. Press, Cambridge, Mass. and London, 1966. ix + 382 pp. $5.00.

This is a collection of nineteen articles by different authors on the best known of the list-processing languages. The book is designed to prepare the reader for the LISP manual and succeeds admirably; the many applications—to the checking of mathematical proofs, to symbolic differentiation and integration, and many others—will give the reader an appreciation of LISP'S power.

Fundamental phenomena in hypersonic flow. Edited by J. Gordon Hall. Cornell University Press, New York, 1965. xiii + 354 pp. $12.50.

Conference proceedings have become more widespread, more dilute in technical content, and more expensive to the purchaser in recent years. This volume, in commemoration of Cornell University's centennial is a happy exception. Acknowledged leaders in the various aspects of hypersonic flow were invited to contribute a paper in their field of expertise, and almost without fail the quality was high. The invited comments and commentary from the floor added perspective as well. Space permits only a mere listing of the contributions with just a capsule comment.

"The Aerodynamic Heating of Atmosphere Entry Vehicles—A Review," H. J. Allen (excellent survey). "Entry Heat Transfer at Superorbital Speeds," J. A. Fay (heat transfer estimates with many parameter plots). "The Blunt-Body Problem Revisited," M. D. Van Dyke (delightful as usual). "Asymptotic Theory in Hypersonic Flow," J.-P. Guiraud (elegant shock layer asymptotics). "Viscous Hypersonic Blunt-Body Problems and the Newtonian Theory," H. K. Cheng (useful survey with emphasis on work at frontiers). "The Diffusion of Species in Frozen Hypersonic Boundary Layers," N. C. Freeman (Mellin transform solution). "Temperature Measurement and Relaxation Processes in Shock Tubes," A. G. Gaydon (discussion of spectroscopic techniques). "Chemical Kinetics and Hypersonic Flow," H. B. Palmer (chemical reactions in shock tubes). "CO_2 Relaxation Processes in Shock Waves," M. Camac (electron beam and IR measurements). "A Milne–Eddington Model in Heat and Mass Transfer," R. Goulard (interpolation formulae). "A Simplified Description of Rarefied-Gas Flows by Means of the Hydrodynamic Equations," (an ultra-simple kinetic model for transition flows). "Rarefied-Flow Transition at a Leading Edge," Y. S. Pan and R. F. Probstein (competent solution of an "impossible" problem). "Stability of Laminar Boundary Layers and Wakes at Hypersonic Speeds," L. Lees and H. Gold.

The Symposium was held June 25–26, 1964.

IRA M. COHEN

Introduction to the theory of differential equations with deviating arguments. By L. E. El'sgol'ts. Translated by R. J. McLaughlin. Holden-Day, Inc., San Francisco, London, Amsterdam, 1966. ii + 109 pp. $6.95.

This short monograph is a good introduction to some problems and techniques of the theory of differential equations with deviating arguments. There is no claim to generality and usually the ideas are presented by means of simple examples with extensive supplementary references for the interested reader. This theory is far from complete and, as a result, the presentation does not have the coherence of usual mathematical theories. On the other hand, the careful reader can become acquainted with the subject very quickly and in many cases will discover problems that require further investigation.

JACK K. HALE

Mathematics and statistics—for students of chemistry, chemical engineering, chemical technology and allied subjects. By C. J. Brookes, I. G. Betteley, S. M. Loxston. John Wiley and Sons Ltd., London, New York, Sydney, 1966. vii + 418 pp. $10.00.

This book covers the mathematics and statistics required of students of chemistry, chemical engineering and chemical technology in most undergraduate courses. It should also prove useful for engineers and technologists in industry, since the emphasis is on applications, which are illustrated with many worked examples.

Basic statistics with business applications. By R. C. Clelland, F. E. Brown, J. S. deCani and D. S. Murray. John Wiley & Sons Inc., New York, London, Sydney, 1966. xvi + 708 pp. $11.50.

This is a book on the calculus level, dealing with business problems and the economic applications of statistics. The authors are all members of the Wharton School of Finance and Commerce at the University of Pennsylvania, and their experience has guided them well in selecting topics and choosing a mode of presentation appropriate for undergraduate business students.

Wave propagation in dissipative materials—a reprint of five memoirs. By B. D. Coleman, M. E. Gurtin, I. Herrera R. and C. Truesdell. Springer-Verlag New York Inc., 1965. v + 338 pp. $4.50.

This is a reprint of five papers from the Archive for Rational Mechanics and Analysis. Truesdell's paper, *General and Exact Theory of Waves in Finite Elastic Strain*, sets the stage for the four parts of *Waves in Materials with Memory*, of which Part I is by Coleman, Gurtin, and Herrera, and the remaining parts are by Coleman and Gurtin. These papers discuss, in precise mathematical terms, the propagation of singular surfaces in materials which behave elastically under instantaneous finite changes in strain. In Part I, *The Velocity of One-Dimensional Shock and Acceleration Waves*, it is shown that the expressions for wave velocities in terms of instantaneous elastic moduli are of the same form as for purely elastic materials. The principal result of Part II, *On the Growth and Decay of One-Dimensional Acceleration Waves*, is a simple and explicit formula for the time-dependence of the strength of a wave propagating into a medium at rest. A corollary of this result is that there is in general a critical initial amplitude below which all waves decay, and above which either compressive or expansive waves blow up in finite time. In Parts I and II, the theory is purely mechanical. Part III, *Thermodynamic Influences on the Growth and Decay of Acceleration Waves*, modifies or generalizes earlier results by taking into account heat conduction and temperature history. In Part IV, *Thermodynamics and the Velocity of General Acceleration Waves*, three-dimensional situations are considered. It is shown that the analogy with elastic materials still holds, and in particular that the instantaneous acoustic tensors for homothermal and homentropic waves are symmetric.

A. C. Pipkin

Viscometric flows of non-Newtonian fluids theory and experiment. By B. D. Coleman, H. Markowitz and W. Noll. Springer-Verlag New York Inc., 1966. x + 130 pp. $5.50.

Modern engineering practice has made it necessary to understand and deal with a wide variety of materials whose behavior is sufficiently complex that the classical constitutive equations of linear elasticity or Navier–Stokes fluid dynamics are not useful descriptions. The diverse possibilities of nonlinear behavior make it doubtful that any comparatively simple constitutive equation can do more than describe limited aspects of the properties of materials. For this reason, some investigators are turning to the analysis of specific classes of motions for which a complete description of all material properties is neither necessary nor even desirable.

The monograph at hand represents an outstandingly successful example of this approach. In steady viscometric flows, which are flows equivalent in a certain sense to steady simple shearing motions, the state of stress of a material element can depend on the material properties only through three functions of the shear-rate and the temperature, if the fluid is incompressible. These three functions, the viscometric functions, form the basis for the exact analysis of Poiseuille and Couette flows and a class of rectilinear and helical motions between coaxial cylinders, as well as the approximate (low Reynolds number) analysis of torsional flow, cone and plate flow, and certain free boundary problems, to mention only those covered in the present book.

The presentation here is a short but complete, detailed, and self-contained description of both the theoretical and the experimental aspects of the subject, written by leading contributors to it. Specific experimentally determined forms of the viscometric functions for a variety of materials are provided as illustrations. There is an extensive bibliography and a short history of the experimental and theoretical development of the subject. The book will presumably become the standard reference on the subject, and I strongly recommend it to anyone with even a peripheral interest in this field.

A. C. Pipkin

Topics in higher analysis. By Harold K. Crowder and S. W. McCuskey. The Macmillan Co., New York, and Collier-Macmillan Ltd., London, 1964. xi + 545 pp. $10.00.

This is an advanced calculus text, with chapters on vector analysis, line and surface integrals, functions of a complex variable, improper integrals and infinite series, infinite series in the complex plane and the theory of residues, Fourier series and integrals, the Laplace transformation, special functions, partial differential equations, conformal mappings, and calculus of variations. Problems, some with answers, are provided. Proofs of most theorems are given, and applications are illustrated.

A. C. Pipkin

Gasdynamik. By Ernst Becker. B. G. Teubner Verlagsgesellschaft, Stuttgart, 1965. 248 pp. $9.92.

The book gives an excellent introduction to gasdynamics, i.e. the dynamics and thermodynamics of gases. Under restriction to simple mathematical tools it gives a good survey of the problems encountered in two-dimensional stationary and one-dimensional nonstationary flow of compressible fluids. The investigations are not limited to ideal gases; deviations from the laws of ideal gases and from non-inhibited thermodynamical equilibrium are also considered, whereas flows of highly rarefied gases, which can no longer be approximated as continuum flows, and effects of magnetogasdynamics (ionization, electromagnetic forces), are not covered.

Chapter I contains the basic thermodynamic laws: first and second law of thermodynamics, equations of state, conditions of equilibrium, chemical potentials, law of massaction, dissociation. Chapter II deals with the basic laws of continuum mechanics: kinematics of fluids, equations of momentum and energy, vortex laws of Thomson and Crocco. The largest part is taken by chapter III which covers non-viscous flow phenomena: one-dimensional non-stationary flows (rarefaction-and compression waves, formation of normal shock waves, wave propagation in a gas with relaxation effects), two-dimensional stationary supersonic flows (Mach-waves, oblique shock waves, two-dimensional simple waves, Prandtl–Meyer flow), linear theory of small perturbations (flow past an undulating wall, subsonic flow past slender profiles), limitations of linear theory in the transonic and hypersonic range, relaxation effects in stationary flows. Chapter IV finally presents a short description of phenomena of viscous flow: transport quantities, flow through a normal shock wave, flows containing viscous boundary layers.

The book, presumably, is intended mainly for engineers; therefore the author justly cares much about conspicuous physical and geometrical interpretations. Investigations requiring more profound mathematical tools like nonlinear problems of stationary subsonic flows or problems in more than two independent variables (three-dimensional stationary and non-stationary gas flows) are not treated in the book. For engineers the book can be highly recommended.

R. Sauer

Quantum mechanics and path integrals. By R. P. Feynman and A. R. Hibbs. McGraw-Hill Book Co., New York, St. Louis, San Francisco, Toronto, London, Sydney, 1965. xiv + 365 pp. $12.50.

This book is an important contribution to the pedagogy of elementary quantum mechanics. It makes available an extended account of the treatment of elementary quantum mechanics by means of path integrals. The considerable insights to be obtained by this approach are well known if for no other reason than the virtuoso exploits of one of the authors using it in quantum electrodynamics and solid state physics. As the authors themselves indicate (page ix) the approach using path integrals does not supersede other more traditional approaches to quantum mechanics, but it is so useful that everyone who teaches the subject is likely to want to recommend this book at least as auxiliary reading.

The flavor of the book is indicated by the title of its chapters "Developing the concepts with special examples." There is no fancy pseudo-general theory. Instead the nature and power of the method are made clear by a careful, elementary, and detailed account of a wide variety of physical problems ranging from the diffraction of a particle passing through slits to some questions of statistical mechanics and quantum electrodynamics.

The book can be recommended to mathematicians as rich in mathematical ideas and challenges, though it does not pretend to offer any mathematically precise formulation of the methods it discusses. (This recommendation is given in spite of somewhat curious attitude toward mathematics that the authors display at times. See e.g. page 34 . . . "the concept of the sum over all paths, like the concept of an ordinary integral, is independent of a special definition and valid in spite of the failure of such definitions.")

A. S. Wightman

Generators and relations for discrete groups. By H. S. M. Coxeter and W. O. J. Moser. Second Edition. Springer-Verlag New York Inc., 1965. ix + 161 pp. $8.00.

This is the second edition of a monograph that appeared first in 1957. The new edition differs from the first one by minor corrections and additions and by a newly inserted brief account of the use of electronic computers for enumerating cosets in a finite abstract group.

The appearance of a second edition is a testimonial to the great achievement of the authors who managed to organize what, at first sight, looks like a complete and nearly infinite chaos. Their success is due in part to the use of geometrical ideas as a guiding principle. To the applied mathematician, the chapters on abstract crystallography and on groups generated by reflections are likely to be the most useful ones. The chapters on the symmetric, alternating, modular, and linear fractional groups provide an abundance of technical information on these important groups. Nine pages of tables and fourteen pages of references facilitate orientation and access to additional information.

Wilhelm Magnus

Gaussian quadrature formulas. By A. H. Stroud and Don Secrest. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966. ix + 374 pp. $14.95.

This book is a useful addition to the numerical integration literature, particularly from the practical point of view. Chapter 1 is concerned with various properties of the orthogonal polynomials related to the Gaussian quadratures and of the quadratures themselves. The main parts of Chapter 2 deal with the calculation of the Gaussian weights and base points. The approximations, as well as the relevant Fortran programs, are stated. Various uses of the Gaussian formulas are given in Chapter 3. Of particular interest is the section on multiple numerical integration. Cross product rules are formed from one-dimensional Gaussian formulas in order to integrate over n-dimensional cubes. Appropriate changes of variable yield formulas for certain other n-dimensional regions, for which cross product error terms can be stated.

Chapter 4 is concerned with error estimates and the computations related to them seem especially useful. Two types of error bounds are considered. The first type has been called, variously, the "Peano representation," the "method of influence functions," etc. It amounts to applying the error functional R to the appropriate Taylor expansion with the integral form of the remainder term. If N is the number of points in the formula and r is the order of the function $f(r \leqq 2N)$, then the function space considered is $\{g: g^{(r)}$ is piecewise continuous and is uniformly bounded, in the sup norm$\}$. The upper bound thus found is "best" in the sense of Sard and is the product of two terms, one being the uniform bound and the other being independent of the function to be integrated. The latter term is calculated for a variety of cases. The second type of error bound was developed by Davis and Rabinowitz, for functions analytic in a certain region of the plane containing the interval of integration. This method amounts to using the upper bound $|R(f)| \leqq ||R|| \cdot ||f||$, where $||R||$ is computed using the Riesz Representation Theorem for Hilbert space. The upper bound on $|R(f)|$ derived from this method is again the product of a factor that depends on the function and of a factor that is independent of the function being integrated. Both factors depend on the region of analyticity of f. The reviewer has extended the method of Davis and Rabinowitz and, as a check on certain computer programs, calculated $||R||$ for some of the cases given. The numerical results agree with those of Stroud and Secrest.

The main part of the book is the extensive collection of tables, calculated to thirty digits. These tables are carefully laid out and hence easy to read. Mainly, these tables contain Gaussian weights, base points and error terms for various weight functions and intervals of integration.

Robert E. Barnhill

The logic of decision. By R. C. Jeffrey. McGraw-Hill Book Co., New York, 1965. xiv + 201 pp. $7.95.

Various authors (notably Ramsey, Savage and von Neumann) have discussed the problem of how a man should reach decisions in a consistent manner. Briefly, their conclusions are that consistency can only be achieved by assigning probabilities and utilities, and maximizing expected utility. The probabilities are assigned to the unknown states of nature, and the utilities to the consequences of a decision in a particular state of nature. The discussion introduces probability by means of a gamble. Professor Jeffrey's method of reaching the same conclusion that one should maximize expected utility is different. He attaches probabilities and utilities alike to propositions, and the combinations he considers are not gambles but the usual logical operations on propositions. In the usual approach it is possible to show that the assignment of probabilities is unique for a given preference structure of decisions, and that the utility is similarly unique up to a linear transformation. Jeffrey's proves a similar theorem which allows a rather more general class of transformations. The formulation of the decision problem in terms of propositions and the proof of this theorem (at a very elementary mathematical level) constitute the bulk of the book. This development is preceded by an exposition of Bayesian decision theory, and followed by a discussion of points that arise, mainly of a philosophical nature.

There seem to me to be two objections to the new approach. First, the earlier authors cited above demonstrated the *existence* of probabilities and utilities. That is, they started from certain modest and generally acceptable axioms about consistent preferences between acts, and deduced from the axioms the result about maximizing expected utility. There is no such existence theorem in the present development. The basic axioms (pp. 69, 70) are the existence of probabilities and an axiom for desirability (the author's terminology for utility) which incorporates the expectation notion. Certainly there is some arbitrariness about the choice of one's axioms but these seem totally unacceptable to me because they are so complicated and embracing. It is rather like developing Greek geometry by starting with Morley's trisection theorem as an axiom. My second objection probably stems from a misunderstanding of the author's case; but it would appear from the general argument in the body of the book and from the simple examples at the beginning that in a finite decision problem Jeffrey would assign probabilities and utilities (desirabilities) to each ordered pair (state of nature, decision) thereby having two matrices of elements p_{ij}, u_{ij} (i denoting the state, j the decision). In the usual approach only a utility matrix and a probability vector of elements p_i are required. Indeed only the product $p_i u_{ij}$ is strictly needed but it is convenient to separate them because of the simple rules (Bayes's theorem in particular) for the manipulation of the p_i. The new approach is therefore more complicated than the old. It is a pity that the major development of the book is not illustrated by a single decision-making example. In the early examples, though not later, the probabilities are all conditional on the decision: a complexity which can easily be removed by the usual (p_i, u_{ij}) structure.

The exposition is not always as clear and precise as one expects of a philosopher. (Thus, Example 4.2 appears to ignore the possibility of rain and snow; on several occasions the need for the propositions to be exclusive before using the addition law of probabilities is omitted; and, as just mentioned, the examples are not always apposite). There is a good collection of simple exercises. Someone should be taken to task for the ugly fount used in the printing—the book gives the impression of having been typewritten. (Why are American printers anonymous?) It is good that philosophers are becoming interested in the results of mathematicians and statisticians, and Jeffrey's book is to be welcomed by all interested in the theory of decision-making. It deserves careful consideration.

D. V. Lindley

Symmetry groups in nuclear and particle physics. By F. J. Dyson. W. A. Benjamin, Inc., Amsterdam, 1966. xii + 320 pp. $9.00.

This book is one of a series of lecture notes and reprint volumes devoted to various aspects of modern physics. It and its predecessor *The Eightfold Way* M. Gell–Mann and Y. Ne'eman give convenient access to the original papers on the application of group theory to the physics of elementary particles. For the serious student of the subject they are indispensable unless he has already made a private notebook covering the same material. The volume under review has some charm also for the general reader who is sufficiently earnest to want to go beyond the level of the Scientific American.

The book opens with three elegant lectures by Dyson which develop an analogy between the present state of our knowledge of the structure of atomic nuclei and of elementary particles. The analogy is illustrated on the side of nuclear physics with the Elliott model and the Wigner supermultiplet model and on the side of particle physics with Gell–Mann's SU(3) model and recently developed SU(6) models.

The remainder of the book consists of reprints of original papers. They are divided into eight groups. The first "The Classics" contains a paper of Wigner on the supermultiplet model, another on the unitary representations of the inhomogeneous Lorentz group and a paper of Bargmann and Wigner on relativistic wave equations. Group two "Symmetry in Nuclear Physcis" begins with a paper by Flowers in which representations of the symplectic group are used to classify nuclear and atomic energy states. There follows the basic paper on the Elliott model and a paper by Goshen and Lipkin showing how a non-compact group can be used to describe collective motions in a model system. The last paper of this group is a note by Franzini and Radicati comparing the supermultiplet model with experiments on the ground state energies of nuclei.

The remaining six groups of papers are a judicious selection from the paper explosion on SU(6) and its relativistic generalizations which took place between the summers of 1964 and 1965. They are divided into: group three, Successes of SU(6) in particle physics; group four, Attempts at relativistic SU(6) theories; group five, Experimental difficulties of SU(3) and SU(6); group six, Theoretical difficulties of SU(6); group seven, Rigorous theorems forbidding the mixing of internal and space-time symmetries; and group eight, One way out of the impasse; the exploration of groups which are not symmetry groups.

It is an untidy and exciting bundle of ideas, and, whatever its ultimate fate, it is what's been happening.

A. S. Wightman

Differential equations—a modern approach. By Harry Hochstadt. Holt, Rinehart and Winston, New York, Chicago, San Francisco, Toronto, London, 1964. viii + 249 pp. $6.50.

This book has many commendable attributes. It is an attempt to present ordinary differential equations at an elementary level as an interesting coherent subject rather than merely tricks to solve particular equations. The author introduces the student to many of the basic ideas of both linear and nonlinear differential equations, always stressing throughout the general underlying principles followed by applications to specific problems.

The table of contents by chapters is: 1. Linear Algebra (36 pages), 2. Linear Differential Equations

(48 pages), 3. Series Solutions for Linear Differential Equations (39 pages), 4. Boundary Value Problems (67 pages), 5. Linear Differential Equations with Periodic Coefficients (13 pages), 6. Nonlinear Differential Equations (52 pages), 7. Two-dimensional Systems (34 pages).

Linear problems are presented in terms of systems of first order differential equations with the specialization to nth order scalar equations following as an application of the general theory. In the chapter on boundary value problems, there is a discussion of integral equations which initiates the student to some simple aspects of operators in function spaces. The most challenging part for the author must have been the writing of chapters 6 and 7 where in 86 pages, he exposes the student to existence theorems, series solutions, stability theory, Liapunov's method, periodic solutions, limit cycles, the Poincaré index, and perturbation theory for periodic solutions of second order equations. The reviewer feels that some amplification of these chapters would have been desirable since such rapid exposure to so many topics may leave the student in a daze.

In the hands of a capable instructor, this book should serve as an adequate text for good students who have just completed calculus or less mature students who have previously been exposed to differential equations.

JACK HALE

Homotopy theory and duality. By Peter Hilton. Gordon & Breach, New York, London, Paris, 1965. x + 224 pp. $11.00.

This book follows very closely the lecture notes of an advanced graduate course given by the author at Cornell University during the academic year 1958–59. The course in turn was mainly an exposition of joint work of the author and Professor Beno Eckmann of Zurich, Switzerland.

Homotopy theory has been recognized as one of the standard branches of algebraic topology for several generations. There can be no doubt as to the beauty and importance of the contributions of Eckmann and Hilton to this subject. It will be impossible for future expositors to give a balanced account of this area of topology without including much of the material of the volume under review.

Up to the present, algebraic topology has had little influence on applied mathematics. What the future will bring in this regard, nobody can predict. However, this reviewer would hazard the guest that homotopy theory and duality will be one of the *last* areas of algebraic topolopy to find physical applications.

W. S. MASSEY

Théorie de La Valeur—analyse axiomatique de l'équilibre économique. By G. Debreu. Dunod, Paris, 1966. xii + 121 pp. $3.95.

This is an excellent French translation of the well known book: G. Debreu: Theory of Value. Cowles Foundation Monograph 17, New York, Wiley 1959. It is fitting that this important monograph, one of the highlights of the application of modern mathematics to the problem of economic equilibrium, whose author is a Frenchman educated in the great economic tradition of the French technical universities, should finally be translated into French. Both the English original and the French translation can be warmly recommended to mathematicians who are interested in the somewhat novel applications of mathematics to economic theory, which are certainly different from applications to physics and engineering with which most mathematicians are more familiar.

The mathematical part of the book, summarized in Chapter one, deals with sets, functions and correspondence, preorderings, real numbers, limits, continuous functions, vectors and fixed points. The presentation is short, but well illustrated by examples and in the French tradition of N. Bourbaki. The topics are of course selected with a view to the applications to problems of economic equilibrium, which are treated in the remainder of the book: Commodities and prices, producers, consumers, equilibrium, optimum, uncertainty. There is also an excellent bibliography, summarizing the literature in many languages.

G. TINTNER