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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos \dfrac{\pi x}{2b}}{\cos \dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEWS—

Extensions of minimal transformation groups. By I. U. Bronstein. Sijthoff and Noordhoff, Alphen aan de Rijn, Netherlands, 1979. viii + 319 pp.

Important relationships exist between nonautonomous differential equations and the concept of extension of dynamical systems. This book gives the theory of minimal transformation groups and minimal extensions together with applications to nonautonomous differential equations. It is well written and should be a stimulus for persons in the qualitative theory of differential equations as well as those in abstract topological dynamics.

JACK K. HALE (*Providence*)

Qualitative methods in elasticity. By Piero Villaggio. Sijthoff & Noordhoff, Alphen aan den Rijn, 1977. 694 pp.

The unifying theme underlying this book is that of obtaining *a priori* estimates for problems of classical linear elasticity in terms of geometric and boundary data. Thus the author is concerned with "finding certain properties of the solutions to the problems of elasticity without solving the equations explicitly". Acknowledging that the merits of this approach have long been recognised by theoreticians, the author also wishes to introduce engineers to the "enlightened use of *a priori* estimation".

The tone for the book is set in the opening words of the Preface with the statement that "the classical theory of elasticity constitutes a highly satisfactory branch of mathematical physics, in that mathematical rigour and physical fact are combined with surprisingly rich results". The author describes the work as "not in fact a treatise on the reformulation of the fundamental mechanics of the theory, which has already been decanted to a certain level, but rather of setting out afresh methods for solving the various problems in a unified form". He continues: "Those interested in the theory will be able to find many questions raised here that do not usually appear in the classical formulation, like those of local bounds for the solution, of its behavior at infinity, of stress concentrations, or lastly of the effects of geometrical deformations. Those interested in applications will be able to note that the so-called abstract methods also embody in them some potential instruments for describing solutions quantitatively".

The book is in seven chapters: 1. *The linear theory of elasticity*, 62 pages; 2. *The generalized theory of the boundary value problems in elastostatics*, 45 pages; 3. *Strong formulation and the Gauss–Green formula*, 37 pages; 4. *Mean value properties and elementary inequalities*, 69 pages; 5. *Two-sided energy bounds*, 106 pages; 6. *Pointwise bounds*, 161 pages; 7. *Behavioural properties of solutions*, 141 pages.

Following a brief Introduction which outlines the contents and scope of the work and introduces some notation, Chapter 1 provides a self-contained summary of the basic theory of linear elasticity (including some beam and rod theory) derived from the principles of continuum mechanics. As the author acknowledges, the treatment (and notation) follows that of Gurtin [1].

Chapter 2 is concerned with summarizing basic concepts of functional analysis needed in the sequel, with emphasis on weak solutions and regularity theory for elliptic partial differential equations. Existence theory for weak solutions in linear elastostatics is discussed. Further application of standard results from the modern theory of elliptic partial differential equations to elasticity (e.g. fundamental solutions, Green's functions, maximum principles) is carried out in Chapter 3.

The remaining four chapters constitute the essence of this book. Chapter 4 provides a useful summary of the various mean-value theorems of elastostatics. This is followed by a detailed treatment of a method due to Signorini for finding lower bounds for stresses and displacements. Numerous examples (e.g. beam and plate problems) are discussed to illustrate the theory.

In Chapter 5, an extensive discussion is presented of techniques based on the variational principles of elastostatics for finding upper and lower bounds for quadratic ("energy") functionals in terms of geometric and boundary data. The results are illustrated with the aid of several examples from structural mechanics.

The longest chapter in the book, Chapter 6, is devoted to the important and analytically more difficult problem of obtaining *pointwise a priori* bounds for stresses and displacements. Here the pioneering work of

Diaz, Greenberg, Synge, Bramble, Payne, Weinberger and others is thoroughly discussed. The essential role played by the Korn inequalities is demonstrated.

The final chapter, Chapter 7, is concerned with a number of different topics classified by the author as "behavioral properties". The first of these deals with domain perturbation and symmetrization. Then the work of Gurtin and Sternberg on the behavior of elasticity solutions at infinity is summarized. The third topic concerns Saint-Venant's Principle. Here the treatment paraphrases that of Gurtin's summary [1] of the fundamental contributions of Sternberg, Toupin and Knowles. The fourth item concerns the work of Sternberg and co-workers on concentrated loads in elastostatics. A brief discussion of stress concentration problems is given. The final topic considered is that of stress diffusion, the treatment being based on the work of Muki and Sternberg.

Features of the book include a set of problems (with solutions) and examples at the end of each section (these are designed to elucidate the formal results and fill in some technical details); a bibliography, arranged in chronological order; and name and subject index. It seems unfortunate that the bibliography is current only up to 1972. Thus, in addition to the tendency of this book to overlap, perhaps too often, with Gurtin's article [1], no significant up-dated bibliographic information is provided over that contained in [1].

These reservations aside, the reviewer feels that the author has succeeded admirably in presenting a fresh viewpoint of classical linear elasticity theory for both student and researcher alike. The material presented in this book constitutes an invaluable synthesis of varied techniques and results widely scattered throughout the literature. The unifying theme of *a priori* estimation is still undergoing rapid development from both theoretical and practical considerations. Thus the work should serve to attract the attention of researchers in partial differential equations to the field of elasticity, while rekindling the affections of elasticians for the mother subject.

[1] Gurtin, M. E., The linear theory of elasticity, in *Handbuch der Physik*, pp. 1–295, Vol. 6a, Part 2 (edited by S. Flügge and C. Truesdell). Springer-Verlag, Berlin (1972).

CORNELIUS O. HORGAN (*East Lansing, MI*)

—BOOKS RECEIVED—

A primer in probability. (Statistics: Textbooks and Monographs Series, Volume 28.) By Kathleen Subrahmanian. Marcel Dekker, New York, 1979. viii + 325 pp. $12.50.

This is a textbook for a first course, presupposing only highschool algebra.

Solitons in action. Edited by K. Lonngren and A. Scott. Academic Press, Inc., New York, 1978. xiii + 300 pp. $15.00.

These are the proceedings of a workshop, sponsored by the Mathematics Division, Army Research Office, held at Redstone Arsenal in October 1976. There are eleven papers, by R. M. Miura, H. E. Moses, R. Hermann, A. R. Bishop, H. C. Yuen and B. M. Lake, K. E. Lonngren, H. Ikezi, R. D. Parmentier, D. W. McLaughlin and A. C. Scott, J. H. Batteh and J. D. Powell, and G. S. Deem and N. J. Zabusky.

Computational geometry for design and manufacture. (Mathematics and its applications series.) By I. D. Faux and M. J. Pratt. Ellis Horwood Ltd., Chichester, England, 1979. 329 pp. $21.50. (Halsted Press, New York, distributor in USA.)

The purpose of this book is to outline the principal developments of computational geometry—the computer representation, analysis and synthesis of shape information. The relevant branches of geometry are reviewed first and used in the treatment of curve and surface representations. The book's origin is in courses given to practicing engineers and industrial mathematicians engaged in computer-aided design or manufacture. The nine chapters are headed: 1. Plane coordinate geometry; 2. Three-dimensional geometry and vector algebra; 3. Coordinate transformation; 4. Three-dimensional curve and surface geometry; 5. Curve and surface design; 6. Composite curves and splines; 7. Composite surface; 8. Cross-sectional designs; 9. Computing methods for surface design and manufacture. Five appendices.

The boundary element method for engineers. By C. A. Brebbia. John Wiley & Sons, New York, 1978. 189 pp. $27.50.

The term "boundary elements" is used to indicate the method whereby the external surface of a domain is divided into a series of elements over which the functions under consideration can vary in different ways, rather as in the finite element method, over which it is said to have the advantage of generating smaller systems of equations. This book presents the boundary element method, using FORTRAN programs. Applications to potential and elasticity problems are discussed in detail.

Mathematics: an introduction to its spirit and use. (Readings from *Scientific American.*) Selected and introduced by Morris Kline. W. H. Freeman and Co., San Francisco, 1978. 256 pp. $12.00 cloth, $6.00 paper.

This is a selection of articles on mathematics and some few applications, previously published in *Scientific American.* There are forty papers, arranged in groups: 1. History; 2. Number and algebra; 3. Geometry; 4. Statistics and probability; 5. Symbolic logic and computers; 6. Applications. The emphasis is on the history and spirit of pure mathematics, with the applications confined to game theory, some operations research, computer logic and music—hardly an exhaustive or even representative list. Essentially, the volume presents mathematics as one of the humanities with which every educated person ought to be acquainted at some level—and as such the collection succeeds very well.

Nonlinear programming 3. Edited by Olvi L. Mangasarian, Robert R. Meyer and Stephen M. Robinson. Academic Press, New York, 1978. ix + 478 pp. $29.50.

This volume contains seventeen of the papers prepared for the Nonlinear Programming Symposium, No. 3 held at Madison in July, 1977. The first eight papers describe some of the most effective methods available today for solving linearly and nonlinearly constrained optimization problems. The ninth paper gives algorithms for the solution of nonlinear equations together with computational experience. The other papers give some modern applications of optimization in operations research, propose a measurement procedure for optimization of algorithm efficiency, give methods for solving large quadratic programs, describe algorithms for solving stationary and fixed point problems and the minimization of certain types of nondifferentiable functions and present a new type of Newton method.

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Thermodynamic formalism: the mathematical structures of classical equilibrium statistical mechanics. By Davis Ruelle. Addison-Wesley, Reading, Mass., 1978. xix + 183 pages. $21.50.

This is volume 5 of the Encyclopedia of Mathematics and its Applications, edited by Gian-Carlo Rota, in the section on Statistical Mechanics which is edited by Giovanni Gallavotti. According to the editors, thermodynamics is still, as it always was, at the center of physics, and this monograph is perhaps the first entirely rigorous account of the foundations of thermodynamics. It is based on lectures given in the mathematics departments of Berkeley and Orsay and its aim is to describe the mathematical structures underlying the thermodynamic formalism of equilibrium statistical mechanics in the simplest case of classical lattice spin systems. The volume is aimed at mathematicians interested in such topics as ergodic theory, topological dynamics, constructive quantum field theory, the study of certain differentiable dynamical systems, notably Anosov diffeomorphisms and flows. It is also of interest to theoretical physicists concerned with the conceptual basis of equilibrium statistical mechanics. The level of the presentation is generally an advanced one, with the objective of providing an efficient research tool and a text for use in graduate teaching. Background material has been collected in the appendices to help the reader, who is assumed to be familiar with basic facts of functional analysis, but does not necessarily have a knowledge of physics. Extra material is given in the form of exercises; open problems are collected at the end of the book. Chapter headings are: 0. Introduction; 1, Theory of Gibbs states; 2. Gibbs states: complements; 3. Translation invariance. Theory of equilibrium states; 4. Connection between Gibbs states and equilibrium states; 5. One-dimensional systems; 6. Extension of the thermodynamic formalism; 7. Statistical mechanics on Smale spaces; A1. Miscellaneous definitions and results; A2. Topological dynamics; A3. Convexity; A4. Measures and abstract dynamical systems; A5. Integral representations on convex compact sets; B. Open problems; C. Flows.

Permanents. By Henryk Minc. Addison-Wesley, Reading, Mass., 1978. xviii + 205 pages. $21.50.

If $A = (a_{ij})$ is an $m \times n$ matrix over any commutative ring, $m \leq n$, then the permanent of A is defined as Per $A = \sum_{\alpha} a_{1\alpha(1)} a_{2\alpha(2)} \cdots a_{m\alpha(m)}$, where the summation extends over all one-to-one functions from $\{1, \ldots, m\}$ to $\{1, \ldots, n\}$. The product $a_{1\sigma(1)} \ldots a_{m\sigma(m)}$ is called a diagonal product of A and the permanent of A is thus the sum of all diagonal products of A.

The purpose of the book is to give a complete account of the theory of permanents, their history and applications, in a form accessible not only to mathematicians but also to workers in various applied fields, and to students of pure and applied mathematics. This volume is the first complete account of the theory of permanents. It is a survey in the style of MacDuffee's "The Theory of Matrices" and of "A Survey of Matrix Theory and Matrix Inequalities," by Marcus and Minc. However, it differs from both works in several respects. The style is more leisurely, the proportion of theorems proved in detail is much higher, and the scope is wider—the book covers virtually the whole of the subject, a feature that no survey of the theory of matrices can even attempt. The work also contains many results stated without formal proofs.

Every chapter ends with a set of problems of varying difficulty. Thus, it can be used as a textbook at the advanced undergraduate or graduate level. The only prerequisites are a standard undergraduate course in the theory of matrices and a measure of mathematical maturity.

A special feature of the monograph, and in fact the foundation on which the book is built, is the bibliography which contains every paper and book on permanents known to the author and published before the end of 1977 or awaiting publication at that time. Each of the 300 entries in the bibliography is accompanied by a short abstract of the paper or the book.

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Advanced engineering mathematics. 4th edition. By Erwin Kreyszig. John Wiley & Sons, Inc., New York, 1979. xvii + 939 + A-76 + I-16 pages $24.95.

This book is intended to introduce students of engineering and physics to those areas of mathematics which seem to be the most important in connection with practical problems, assuming a course in elementary calculus as the only prerequisite. The chapter headings are: 1. Ordinary differential equations of the first order; 2. Ordinary linear differential equations; 3. Systems of differential equations, phase-plane, stability; 4. Power series solutions of differential equations; 5. Laplace transformation; 6. Linear algebra Part I: Vectors; 7. Linear algebra Part II: Matrices and determinants; 8. Vector differential calculus. Vector fields; 9. Line and surface integrals. Integral theorems; 10. Fourier series and integrals; 11. Partial differential equations; 12. Complex numbers. Complex analytic functions; 13. Conformal mapping; 14. Complex integrals; 15. Sequences and series; 16. Power series, Taylor series, Laurent series; 17. Integration by the method of residues; 18. Complex analytic functions and potential theory; 19. Numerical analysis; 20. Probability and statistics. There are many problems, with answers to odd-numbered ones.

A practical guide to splines. By Carl de Boor. Springer-Verlag, New York, 1978. xxiv + 392 pages.

This is volume 27 in the series Applied Mathematical Sciences. It stresses the representation of splines as linear combinations of basis-splines (kth order differences of the truncated power function), provides proofs for only some of the results stated but offers many FORTRAN programs, and presents only those parts of spline theory the author has found useful in calculations.

New frontiers in high-energy physics. Edited by A. Perlmutter and L. F. Scott. Plenum Publishing Corp., New York, 1978. ix + 670 pages. $59.50.

This is Volume 14 of *Studies in the Natural Sciences*, a series from the Center of Theoretical Studies, University of Miami. It surveys the most important recent advances in high-energy physics. Included among the list of contributors are P. A. M. Dirac, Behram Kursunoglu, Alan Krisch, and Howard Georgi. As a whole, the volume stresses results in the study of strong interactions, and previews several areas that will prove especially significant in the coming years. Specific topics include polarization phenomena and spin effects at very high energies, viewed from both theoretical and experimental perspectives; supersymmetry and supergravity; flavor interactions and neutrino and lepton interactions; and hadron phenomenology, including charged particle decays and very-high-energy scattering. In addition, attempts to solve problems in quantum chromodynamics are discussed, with emphasis on quark confinement, scaling, and the role of instantons and solitons.

The physics of vibrations. Volume I, containing Part I: the simple chemical vibrator. By Sir Brian Pippard. Cambridge University Press, New York, 1978. ix + 431 pages. $54.00.

The purpose of this book—which will comprise two volumes—is to bridge the gap between the normal undergraduate curriculum and detailed topics at the research level. The emphasis is on the physical processes, and the mathematical treatment is kept at a fairly elementary level, with particular emphasis on the graphical treatment of topics. Simple harmonic oscillations and resonance are central to a course on vibrations and the book builds on this foundation, showing by use of the impulse response function the underlying unity of all linear vibrating systems; it proceeds from this to develop the topic more widely, with emphasis on nonlinear vibrations. Chapter 2 is a revision course in the physics required; by chapter 6 a grounding in wave-motion and the general character of such special forms as acoustic and electromagnetic waves is taken for granted; and chapter 8 will only be understood by someone with a background in solid-state physics and quantum mechanics. The chapter headings of this volume are: 1. General introduction; 2. The free vibrator; 3. Applications of complex variables to linear systems; 4. Fourier series and integral; 5. Spectrum analysis; 6. The driven harmonic vibrator; 7. Waves and resonators; 8. Velocity-dependent forces; 9. The driven anharmonic vibrator; subharmonics; stability; 10. Parametric excitation; 11. Maintained oscillators; 12. Coupled vibrators (which contains a section on the theory of the Josephson function). This is a highly personal and original work which can be read with pleasure and profit by undergraduate students, research scientists, and all in between.