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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

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The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

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Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed, the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\left[(a^2 + b^2)^{1/2}\right] \text{ is preferable to } e^{[a^2+b^2]^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \dfrac{x}{2b}}{\cos \dfrac{a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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NEW BOOKS

Lecture notes in pure and applied mathematics. Marcel Dekker, Inc., New York and Basel, 1982.

Volume 66: *Regressive sets and the theory of isols.* By Thomas G. McLaughlin. vi + 371 pp. $49.50.

This volume integrates the scattered literature in the field, providing a unified presentation of recursion-theoretic, algebraic, and set-theoretic results and methods, as well as several theorems never previously published.

Volume 68: *Commutative algebra: analytic methods.* Edited by Richard N. Draper. vi + 371 pp. $38.50.

This volume consists of papers contributed to the NSF-CBMS Regional Research Conference held at George Mason University, August 6–10, 1979. The ten keynote lectures by Melvin Hochster on the subject of the conference will be published separately.

Volume 70: *Classical mechanics and dynamical systems.* Edited by Robert L. Devaney and Zbigniew H. Nitecki. x + 237 pp. $35.00

The papers collected in this volume were written by participants in the NSF-CBMS REgional Conference, "Recent Developments in Celestial Mechanics," held at Tufts University, August 13–17, 1979. The theme of the conference was the appliction of the geometric methods associated with the global theory of dynamical systems to mathematical problems in mechanics. The main focus of the conference was the recent progress on the n-problem which has been achieved by geometric methods. The principal lecturer, R. McGehee, delivered ten lectures on this subject, which will be published separately in the CBMS Regional Conference series. The supplementary talks, by D. Saari, R. Broucke, J. Palmore, R. Easton, and A. Dèprit, ranged more widely among various problems in classical mechanics. Some of these are reproduced here. The participants in the conference included mathematicians with interests in topology, topological and smooth dynamical systems, differential equations, and classical mechanics, as well as astronomers and physicists. The papers in this volume treat relations between geometric thinking about flows and diffeomorphisms and various mathematical problems of classical and statistical mechanics.

Volume 71: *Places and valuations in noncommutative ring theory.* By Jan Van Geel. ix + 112 pp.

There are three chapters: 1. The general theory, 2. Value functions related to primes, 3. Primes related to the arithmetic of central simple algebras.

Volume 72: *Injective modules and injective quotient rings.* By Carl Faith. viii + 105 pp. $19.50.

This book offers readers the key concepts and methods used in both noncommutative and commutative ring theory.

Volume 73: *Mathematical programming with data perturbations I.* Edited by Anthony V. Fiacco. x + 237 pp. $34.50.

The articles in this book are based on the first Symposium on Mathematical Programming with Data Perturbations held on May 34–25, 1979, at the George Washington University. It reports the implementation of recent results in linear, nonlinear, integer, and stochastic classes of programming and applications.

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Differential equations: Classical to controlled. By Dahlard L. Lukes. Academic Press, New York, 1982. xiii + 322 pp. $37.50.

This book deals with the classical theory of differential equations, showing some of the impact that computers and control theory are having on the subject. The historical development of differential equations and control theory is sketched in Chapter 1, in which motivation and an overview of the subjects are also provided. Chapter 2 contains a brief review of matrix theory and notions of linear algebra. In Chapter 3 the computational aspects of linear constant-coefficient systems are introduced on the basis of a heuristic matrix approach, called the ABC method. The theoretical underpinning and the move to progressively more general equations are found in succeeding chapters. The ABC approach developed in this book reflects the author's efforts to reduce the problem of generating the analytical, closed-form solutions to large systems of linear equations and to compute eigenvalues, and hence to exploit the great progress made in this field in recent years. The study of the variable-coefficients systems which appears in Chapters 7 and 8 (commutative linear equations and periodic linear equations, respectively) grew out of this renewed interest in computing. In chapters 9–12 many of the standard topics dealing with nonlinear equations are studied: 9. Local existence and uniqueness theory; 10. Global solutions; 11. The general solution—dependence of solutions on parameters; 12. Limit properties of solutions. Chapter 13 is intended to provide only an introduction to control theory, but the feedback theory presented here goes beyond the root locus and transfer function methods. Special attention is devoted to the computational aspect of the decoupling problem.

Secure communications and asymmetric cryptosystems. Edited by Gustavus J. Simmons. Westview Press, Inc., Boulder, CO, 1982. x + 338 pp. $30.00.

This is a volume in the AAAS Selected Symposia Series. It reports on the enormous amount of work that has been done in the past three years on the new concept of "asymmetric" cryptography. In asymmetric systems, the information held by the transmitter and by the receiver is not only different, but also is related by a "computationally complex" problem that makes it infeasible to derive one key from the other. The eleven chapters are divided into three parts: The Contemporary (1981) Scene, The Origins of the Subject, and The Future. There is an introduction by the editor, and the author are: Hugh C. Williams, Whitfield Diffie, Ralph C. Merkle, Gustavus J. Simmons, Martin E. Hellman, Ronald L. Rivest, Adi Shamir and Leonard M. Adleman.

Group analysis of differential equations. By L. V. Ovsiannikov. Translation edited by W. F. Ames. Academic Press, New York, 1982. xvi + 416 pp. $54.00.

This book gives a systematic exposition of the theory of Lie groups and Lie algebras and its application to creating algorithms for solving the problems arising in the group analysis of differential equations. One of the principal problems is to study the action of the group admitted by the given equation, or system of equations, in a set of solutions to this equation or system. An essential task there is that of finding the widest group admitted by the equation or system. Another theoretically and practically important problem consists in using the group analysis technique for the group classification of differential equations. Chapter headings: 1. One-Parameter Transformation Groups. 2. Groups Admitted by Differential Equations. 3. Full Groups of Concrete Systems of Equations. 4. The Lie Theory. 5. Invariant Solutions. 6. Partial Invariance. 7. Differential Invariants. 8. Special Problems. There is an appendix with tables of full groups or group classification of equations (equations of Euler, Einstein, Maxwell, Dirac, Hopf, Lamé; heat convection, boundary layer, viscoplastic equations, and others).

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Lecture notes in pure and applied mathematics. Marcel Dekker, Inc., New York and Basel, 1982.

Volume 74: *Algebraic structures and applications.* Edited by Phillip Schultz, Cheryl E. Praeger and Robert P. Sullivan. ix + 168 pp. $29.50.

This collection of papers on a wide variety of topics emphasizes the relevance of algebraic structures and applications to many other disciplines. Topics include the groups of actions defined by Rubik's Cube, statistical applications, generalized and extended properties of Abelian groups to wider classes of algebras, and algebraic structures of various types.

Volume 75: *Rings, modules and preradicals.* By L. Bican, T. Kepka and P. Nemec. ix + 241 pp. $35.00.

This volume offers the first systematic introduction to the use of general theory of preradicals in ring theory. The authors show how the language of preradicals can unify the numerous aspects of ring and module theory, emphasize the interplay between the properties of preradicals categorized as modules and the structure of the underlying ring, and provide systematic studies of the basic properties of preradicals, methods generating filters and radical filters, and various aspects of splitting.

Volume 76: *Convexity and related combinatorial geometry.* Edited by David C. Kay and Marilyn Breen. viii + 243 pp. $29.75.

This book presents twenty original, self-contained papers on convexity and combinatorics. Highlights included an article by Victor Klee on the famous d-step conjecture which is pertinent to linear programming, an article by Carl Lee on the recently solved problem of characterizing the f-vectors of a simplicial polytope, an article by G. D. Chakerian that provides an introduction to the field of mixed volumes and Quermassintegrals, an article by Robert Jamison-Waldner exploring new connections of abstract convexity with a variety of diverse areas of mathematics, including number theory, a bibliography for billiards in the article by Philip Turner on convex caustics, an extensive list of research problems on the subject of Radon's Theorem by John Reay, a new approach to the study of projective planes of order 10 by Andrew Sobczyk and the problems posed at an AMS special session on convexity and combinatorics held at Davis, California.

Volume 77: *Quasi-uniform spaces.* By Peter Fletcher and William F. Lindgren. viii + 216 pp. $29.75.

This work brings together the widely scattered literature in the field, providing a unified resource on these important structures. By utilizing the theory of quasi-proximities in conjunction with the theory of quasi-uniformities, it offers fundamental material and the latest results, encouraging the use of quasi-uniformities in general topology.

Elements of group theory for physicists. By A. W. Joshi. John Wiley & Sons, New York, 1982. xiii + 334 pp. $16.95.

This is the third corrected edition of an introduction first published in 1973.

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Classification problems in ergodic theory. By William Parry and Selim Tuncel. Cambridge University Press, London, 1982. iii + 101 pp. $14.95.

This is volume 67 in the London Mathematical Society Lecture Note Series. These notes grew out of M.Sc. lecture courses given at the University of Warwick and the material presented here is concerned only with those parts of the course which are not found in well-known texts. Chapter headings: 1. Introduction. 2. The Information Cocycle. 3. Finitary Isomorphisms. 4. Block-Codes. 5. Classifications of Topological Markov Chains.

Cartesian tensors: With applications to mechanics, fluid mechanics and elasticity. By A. M. Goodbody. John Wiley & Sons, New York, 1982. 298 pp. $75.00.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. There are seven chapters: 1. Introductory Topics—Scalars and Vectors. 2. Second-Order Cartesian Tensors. 3. Third-Order Cartesian Tensors. 4. Fourth-Order Cartesian Tensors. 5. The Inertia Tensor. 6. The Application of Cartesian Tensors to Fluid Mechanics. 7. The Application of Cartesian Tensors to Elasticity.

Handbook of applicable mathematics, volume IV: Analysis. Edited by Walter Ledermann and Steven Vajda. John Wiley & Sons, New York, 1982. xxiii + 865 pp. $85.00.

This is the fourth core volume of the Handbook (the first three were Algebra, Probability, and Numerical Methods, and two volumes, on Geometry and Combinatorics, and Statistics, are still to come). There are twenty-one chapters, each written by an authority in the field: 1. Sequences and Series. 2. Functions of one (Real) Variable. 3. Differential Calculus. 4. Integral Calculus. 5. Functions of Several (Real) Variables. 6. Multiple Integrals. 7. Ordinary Differential Equations. 8. Partial Differential Equations. 9. Functions of a Complex Variable. 10. Special Functions. 11. Metric Spaces. 12. Calculus of Variations. 13. Integral Transforms. 14. Mathematical Modelling. 15. Non-Linear Programming. 16. Dynamic Programming. 17. Classical Mechanics. 18. Stochastic Differential Equations. 19. Functional Analysis. 20. Fourier Series. 21. Inequalities.

Kernel discriminant analysis. By D. J. Hand. Research Studies Press (a division of John Wiley & Sons), New York, 1982. x + 253 pp. $37.95.

This is volume 2 in the Pattern Recognition and Image Processing Research Studies Series. It surveys the important area of kernel estimation in discriminant analysis, with exhaustive references to the literature. Chapter headings: 1. Introduction. 2. Kernel Estimators. 3. Choice of Smoothing Parameters and Kernel for Continuous Variables. 4. Categorical Variables. 5. Loss Estimation and Variable Selection. 6. Other Topics. 7. Kernel Methods Versus the Rest.

Hyperbolic boundary value problems. By Reiko Sakamoto. Cambridge University Press, London, 1982. viii + 207 pp. $34.50.

This book is designed to introduce the reader to the general theory of the existence and uniqueness of solutions of hyperbolic initial value problems in partial differential equations of higher order. It is divided into three chapters: Chapter 1 is a collection of facts later used in the main body of the text, which are basic to the theory of second-order partial differential equations. Chapter 2 discusses hyperbolic boundary value problems with constant coefficients by the method of Fourier-Laplace transforms. Chapter 3 deals with hyperbolic boundary value problems with variable coefficients by the method of energy inequalities. The reader is assumed to have some prior acquaintance with the elementary theories of a complex variable and Lebesgue integrals.

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Asymptotic methods in nonlinear wave theory. By A. Jeffrey and T. Kawahara. Pitman advanced publishing program, Boston, MA, 1982. x + 256 pp. $39.95.

The authors present the fundamental ideas underlying a number of recently developed asymptotic perturbation methods. Their aim is more to identify and discuss some of the basic concepts rather than to present a complete survey. They intentionally offer only a formal development, even when rigor would have been possible, in order to keep the presentation at a heuristic level. There are seven chapters, and an introduction, divided into three parts: I. Fundamental singular perturbation methods and nonlinear oscillations; II. Various asymptotic methods useful in nonlinear wave theory (asymptotic equations and perturbation methods, the Lagrange formalism, geometrical optics-ray theory); III. Special topics in nonlinear wave theory (two-timing and related methods, inhomogeneous media and higher approximations, methods of exact solution).

Advanced engineering analysis. By J. N. Reddy and M. L. Rasmussen. John Wiley & Sons, New York, 1982. xiv + 488 pp. $39.95.

The objective of this book is to present for engineers and applied scientists the basic mathematical concepts of vector and tensor analysis, the extension of these concepts into abstract function spaces, and the unification of these subjects with the variational calculus and associated methods of numerical approximation. The text is divided into three parts: 1. Elements of Vector and Tensor Analysis, 2. Elements of Functional Analysis, and 3. Calculus of Variations and Variational Methods. Numerous examples, most of which are applications of the concepts to problems in various fields of engineering, are provided throughout the book.

Mathematical techniques of optimalization, control and decision. Edited by J. P. Aubin, A. Bensoussan and I. Ekeland. Birkhauser Boston, Inc., Cambridge, MA, 1981. viii + 212 pp. $24.95.

This is a volume in the series Annals of the CEREMADE (Centre de Recherches de Mathematiques de la Decision). The ten contributions were initiated as colloquium talks at the Centre during 1979/80. There is a general overview of J. B. Aubin and the other papers concern convex analysis, partial differential equations and control, ordinary differentials equations and control, non-convex programming, and game theory.

Factorization theory of meromorphic functions. Edited by Chung-Chun Yang. (Pure and Applied Mathematics: A Series of Monographs and Textbooks, volume 78.) Marcel Dekker, Inc., New York and Basel, 1982. ix + 194 pp. $27.50.

This book contains fourteen original research papers in the subject.

Linear programming and applications: A course text. By W. Mc Lewin. Input-Output Publishing Co., London, England, 1981. xiv + 216 pp.

This book is intended as an undergraduate text for a course of about 40 classes. The presentation stresses the underlying mathematical structure.

Functions of several variables. By B. D. Craven. Chapman and Hall, London and New York, 1981. vii + 136 pp. $16.95 hardcover, $9.95 paperback.

This is a text aimed at mathematics students in their sophomore year.