AN EXISTENCE AND UNIQUENESS THEOREM FOR INCREMENTAL VISCOELASTICITY*

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Introduction. The equations governing small deformations superposed on a large deformation for a body composed of viscoelastic material were established by Pipkin and Rivlin [1]. Recently Iesan [2] has presented equations for small thermoelastic deformations superposed on a general nonlinear thermomechanical deformation and an existence and uniqueness theorem for these equations was proven by Navarro and Quintanilla [3]. In this paper equations are developed which describe motions of a viscoelastic body which are incremental in the sense that they are close to a nonequilibrium nonlinear deformation of the body and an existence and uniqueness theorem for these equations with Dirichlet boundary conditions is proved. The paper is concluded with the remark that the Dirichlet problem of incremental viscoelasticity is well posed.

Basic theory. Let $\mathcal{B}$ be a body whose particles $\underline{X}$ are identified with their referential positions $\underline{X} \in B_r$, where B_r is some fixed reference configuration of $\mathcal{B}$. It is assumed that B_r is a bounded open subset of $\mathbb{R}^n$ ($n = 1, 2$ or 3) with smooth boundary ∂B_r. The motion of the body is given by the deformation function,

$$\underline{x} = \underline{x}(\underline{X}, t), \tag{1}$$

which gives the spatial position $\underline{x} \in B_t$ at time $t \leqslant t_0$ of the particle whose position in B_r is $\underline{X}$: B_t denotes the current configuration.

The deformation gradient $\underline{F}$ is defined by

$$\underline{F}(\underline{X}, t) = \frac{\partial \underline{x}(\underline{X}, t)}{\partial \underline{X}} = \frac{\partial x_i(\underline{X}, t)}{\partial X_A} \underline{e}_i \otimes \underline{E}_A, \tag{2}$$

where $\underline{e}_i$, $i = 1, 2, 3$, and $\underline{E}_A$, $A = 1, 2, 3$, are orthonormal bases representing spatial and referential cartesian coordinate systems. If the deformation (1) is to be possible in a real material then

$$\det \underline{F} > 0.$$

In this paper it is assumed that x_i is continuously differentiable with respect to each of the variables X_A, t as many times as required.

The law of balance of linear momentum for the body $\mathscr{B}$ may be expressed as the field equation

$$\rho \ddot{\underset{\sim}{x}} = \rho \underset{\sim}{b} + \operatorname{Div} \underset{\sim}{s}, \tag{3}$$

where $\rho(\underset{\sim}{X})$ is the referential mass density, $\underset{\sim}{b}(\underset{\sim}{X}, t)$ is, the body force per unit mass and $\underset{\sim}{s}(\underset{\sim}{X}, t)$ is the nominal stress tensor. Here $\operatorname{Div} \underset{\sim}{s}$ is given, in components, by $\partial s_{Ai}/\partial X_A$.

The history $\alpha^t(\cdot)$ of any function α up to time t is defined by

$$\alpha^t(s) = \alpha(t - s), \qquad 0 \leqslant s < \infty,$$

and it should be noted that $\alpha^t(\cdot)$ determines both the restricted history,

$$\alpha_r^t(s) = \alpha^t(s), \qquad 0 < s < \infty,$$

and also the present value $\alpha = \alpha(t)$.

In the purely mechanical theory of simple materials the stress tensor is completely determined by the history of the deformation gradient up to time t and is therefore given by the constitutive equation,

$$\underset{\sim}{s} = \underset{\sim}{S}(\underset{\sim}{F}^t) = \underset{\sim}{S}(\underset{\sim}{F}_r^t, \underset{\sim}{F}). \tag{4}$$

It·is implicit in equation (4) that the material is homogeneous although the analysis that follows remains valid for inhomogeneous materials.

Let $h(\cdot)$ be a fixed influence function which is positive, monotonic decreasing and continuous on $[0, \infty)$ which decays to zero fast enough to be square integrable: $h(\cdot)$ is referred to as the obliviator. It is clear that the space $\mathscr{H}$, consisting of all tensor-valued functions $\underset{\sim}{U}(\cdot)$ defined on $[0, \infty)$ and such that

$$\| \underset{\sim}{U}(\cdot) \| = \left\{ \int_0^\infty \underset{\sim}{U}(s) \cdot \underset{\sim}{U}(s) h(s)^2 \, ds \right\}^{1/2} < \infty,^1$$

is a Hilbert space. It is assumed henceforward that the material has fading memory and instantaneous elastic response in the sense that the response function $\underset{\sim}{S}(\underset{\sim}{F}_r^t, \underset{\sim}{F})$ is continuously differentiable with respect to $\underset{\sim}{F}$ as many times as necessary and continuously Fréchet differentiable (as many times as required) with respect to $\underset{\sim}{F}_r^t$ which must be an element of $\mathscr{H}$.

Following the generalisation by Williams [4] of the definitions of elastic moduli given by Chadwick and Ogden [5] the fourth-order tensor-valued function

$$\underset{\sim}{A}(\underset{\sim}{X}, t) = \frac{\partial \underset{\sim}{S}(\underset{\sim}{F}_r^t, \underset{\sim}{F})}{\partial \underset{\sim}{F}^T}, \qquad A_{AiBj} = \partial S_{Ai}/\partial F_{jB}, \tag{5}$$

is defined to be the tensor of first-order instantaneous elastic moduli and the tensor $\underset{\sim}{A}'(\underset{\sim}{X}, t, s)$ of first-order relaxation function slopes is implicitly defined by

$$\int_0^\infty \underset{\sim}{A}'(\underset{\sim}{X}, t, s)[\underset{\sim}{U}(s)] \, ds = \delta \underset{\sim}{S}(\underset{\sim}{F}_r^t, \underset{\sim}{F} | \underset{\sim}{U}^T) \quad \forall U(\cdot) \in \mathscr{H} \tag{6}$$

$^1 \underset{\sim}{U} \cdot \underset{\sim}{V} = \operatorname{tr}(\underset{\sim}{U} \underset{\sim}{V}^T) = U_{iA} V_{iA}.$

where δS denotes the Fréchet differential of S. It should be noted that the existence of A' is assured by the Riesz representation theorem for linear functionals and that $A'[U]$ has the component form $A'_{A_i B_j} U_{B_j}$.

Strictly speaking, the tensor $A'(X, t, \cdot)$ is only determined almost everywhere by Eq. (6). The uniqueness of A' is guaranteed, however, by the assumption that the relaxation function,

$$G(X, t, s) = A(X, t) + \int_0^s A'(X, t, u)\, du, \tag{7}$$

possesses the continuous partial derivative

$$\frac{\partial}{\partial s} G(X, t, s) = A'(X, t, s). \tag{8}$$

The relation

$$G(X, t, 0) = A(X, t),$$

follows by setting $s = 0$ in Eq. (7).

Incremental viscoelasticity. Suppose that the body force $b(X, t)$ and the boundary conditions,

$$s^T N = t(X, t) \quad \text{when } X \in \Omega, \tag{10}$$

$$x = d(X, t) \quad \text{when } X \in \partial B_r - \Omega, \tag{11}$$

are prescribed then the *primary state* [6] is defined to be a motion whose deformation function (1) satisfies the equation at motion (3) subject to the consitutive equation (4) and the boundary conditions (10) and (11). It should be noted that, in the conditions (10) and (11) ∂B_r denotes the boundary of B_r, Ω is some subset of ∂B_r and N is the outward unit normal on ∂B_r.

A secondary state [6] of the body $\mathcal{B}$ is a motion, defined by the deformation,

$$x^* = x^*(X, t).$$

In the secondary state the current configuration is denoted by B_t^* and dynamic quantities associated with this state will be denoted by an asterisk. Thus the secondary state satisfies the equation of motion,

$$\rho \ddot{x}^* = \rho b^* + \text{Div } s^*, \tag{12}$$

subject to the constitutive equation,

$$s^* = S(F^{*t}) = S(F^{*t}_r, F), \tag{13}$$

and the boundary conditions,

$$s^{*T} N = t^*(X, t) \quad \text{when } X \in \Omega, \tag{14}$$

$$x^* = d^*(X, t) \quad \text{when } X \in \partial B_r - \Omega. \tag{15}$$

In what follows the secondary state is regarded as a perturbation of the primary state. To this end the incremental quantities,

$$\underset{\sim}{u} = \underset{\sim}{x}^* - \underset{\sim}{x}, \tag{16}$$

$$\underset{\sim}{H} = \underset{\sim}{F}^* - \underset{\sim}{F} = \partial\underset{\sim}{u}/\partial\underset{\sim}{x}, \tag{17}$$

$$\underset{\sim}{f} = \underset{\sim}{b}^* - \underset{\sim}{b}, \tag{18}$$

and

$$\underset{\sim}{\sigma} = \underset{\sim}{s}^* - \underset{\sim}{s}, \tag{19}$$

are introduced. The incremental equation of motion,

$$\rho\ddot{\underset{\sim}{u}} = \rho\underset{\sim}{f} + \text{Div}\,\underset{\sim}{\sigma}, \tag{20}$$

now follows on the subtraction of the primary momentum Eq. (3) from the corresponding Eq. (12) in the secondary state.

The first approximation to the constitutive equation for the incremental stress $\underset{\sim}{\sigma}$ is given by

$$\underset{\sim}{\sigma} = \frac{\partial \mathbf{s}(\mathbf{F}^{t'}, \mathbf{F})}{\partial \mathbf{F}}[\underset{\sim}{H}] + \delta\underset{\sim}{S}(\underset{\sim}{F}_r^t, \underset{\sim}{F}|\underset{\sim}{H}_r^t) + o(\|\underset{\sim}{H}_r^t\| + \|\underset{\sim}{H}\|) \tag{21}$$

from which it follows that $\underset{\sim}{\sigma}$ satisfies the linear constitutive equation,

$$\underset{\sim}{\sigma} = \underset{\sim}{G}(t,0)[\underset{\sim}{H}^T] + \int_0^\infty \frac{\partial}{\partial S}\underset{\sim}{G}(t,s)[\{\underset{\sim}{H}_r^t(s)\}^T], \tag{22}$$

where the dependence of $\underset{\sim}{G}$ on $\underset{\sim}{X}$ has, for the sake of clarity, been suppressed and use has been made of Eq. (5) to (9). Therefore, substituting the relation (22) into Eq. (20), making use of the definition (17) and writing ∇ for $\partial/\partial\underset{\sim}{x}$ the linearised equation of motion,

$$\rho\ddot{\underset{\sim}{u}} = \rho\underset{\sim}{f} + \text{Div}\{\underset{\sim}{G}(t,0)[(\nabla\underset{\sim}{u})^T]\}$$

$$+ \text{Div}\int_0^\infty \frac{\partial}{\partial s}\underset{\sim}{G}(t,s)[\{\nabla\underset{\sim}{u}_r^t(s)\}^T]\,ds, \tag{23}$$

is arrived at. Equation (23) should be supplemented by the incremental boundary conditions,

$$\underset{\sim}{\sigma}^T\underset{\sim}{N} = \underset{\sim}{T}(\underset{\sim}{X}, t) \quad \text{when } \underset{\sim}{X} \in \Omega, \tag{24}$$

$$\underset{\sim}{u} = \underset{\sim}{D}(\underset{\sim}{X}, t) \quad \text{when } \underset{\sim}{X} \in \partial B_r - \Omega, \tag{25}$$

where $\underset{\sim}{T} = \underset{\sim}{t}^* - \underset{\sim}{t}$ and $\underset{\sim}{D} = \underset{\sim}{d}^* - \underset{\sim}{d}$. Equation (23) together with the conditions (24) and (25) represents the mixed boundary value problem of linearised incremental viscoelasticity.

The existence and uniqueness theorem. The theorem that is proved in this section is concerned only with the Dirichlet problem, that is when $\Omega = \varnothing$ and $\underset{\sim}{D} = \underset{\sim}{0}$, so that the

conditions (24) and (25) reduce to the Dirichlet condition

$$\underset{\sim}{u} = \underset{\sim}{0} \quad \text{when } \underset{\sim}{X} \in \partial B_r. \tag{26}$$

Further assumptions which are necessary preliminaries to the theorem are listed below.

(a) $\rho(\underset{\sim}{X}) \geqslant \rho_0 > 0, \underset{\sim}{X} \in B_r$.

(b) The relaxation function $\underset{\sim}{G}(\underset{\sim}{X}, t, s)$ is continuously differentiable with respect to $\underset{\sim}{X}$ and, for each fixed $\underset{\sim}{X} \in B_r$, is twice continuously differentiable on the domain $[0, t_0] \times [0, \infty)$.

(c) The relaxation function slope $\partial \underset{\sim}{G}(\underset{\sim}{X}, t, s)/\partial s$ is continuously differentiable with respect to $\underset{\sim}{X}$.

(d) The relaxation function is symmetric, that is

$$\underset{\sim}{P} \cdot \left\{ \underset{\sim}{G}(\underset{\sim}{X}, t, s)[\underset{\sim}{Q}] \right\} = \underset{\sim}{Q} \cdot \left\{ \underset{\sim}{G}(\underset{\sim}{X}, t, s)[\underset{\sim}{P}] \right\}, \qquad \underset{\sim}{X} \in B_r, t \in [0, t_0], s \in [0, \infty), \tag{27}$$

where $\underset{\sim}{P}$ and $\underset{\sim}{Q}$ are arbitrary second order tensors. (Equation (27) is equivalent to $G_{AiBj} = G_{BjAi}$.)

(e) The relaxation function satisfies the global monotonicity conditions

$$-\int_{B_r} (\nabla \underset{\sim}{u})^T \cdot \frac{\partial}{\partial s} \underset{\sim}{G}(\underset{\sim}{X}, t, s)\left[(\nabla \underset{\sim}{u})^T\right] dV \geqslant 0 \quad \text{on } t \in [0, t_0], s \in [0, \infty), \tag{28}$$

$$\int_{B_r} (\nabla \underset{\sim}{u})^T \cdot \frac{\partial^2}{\partial s^2} \underset{\sim}{G}(\underset{\sim}{X}, t, s)\left[(\nabla \underset{\sim}{u})^T\right] dV \geqslant 0, \quad \text{on } t \in [0, t_0], s \in [0, \infty),$$

for all $\underset{\sim}{u} \in C_0^\infty(B_r)$, where $C_0^\infty(B_r)$ denotes the set of n-dimensional vector fields which vanish on ∂B_r and possess continuous derivatives of all orders.

(f) There exists a positive constant δ such that

$$\int_{B_r} (\nabla \underset{\sim}{u})^T \cdot G(t, \infty)\left[(\nabla \underset{\sim}{u})^T\right] dV \geqslant \delta \int_{B_r} (\nabla \underset{\sim}{u})^T (\nabla \underset{\sim}{u}) \, dV \quad \text{on } t \in [0, t_0] \tag{29}$$

for all $\underset{\sim}{u} \in C_0^\infty(B_r)$.

(g) There exists $K > 0$ such that,

$$\int_{B_r} \int_0^\infty (\nabla \underset{\sim}{z}(s))^T \frac{\partial}{\partial t \partial s} \underset{\sim}{G}(s)\left[(\nabla \underset{\sim}{z}(s))^T\right] ds \, dV$$

$$\geqslant K \int_{B_r} \int_0^\infty (\nabla \underset{\sim}{z}(s))^T \frac{\partial}{\partial s} \underset{\sim}{G}(s)\left[(\nabla \underset{\sim}{z}(s))^T\right] ds \, dV, \quad \text{on } t \in [0, t_0] \tag{30}$$

for all $\underset{\sim}{z} \in C_0^\infty\{[0, \infty); C_0^\infty(B_r)\}$.

Consider the elements $w(t) = (\underset{\sim}{u}(\underset{\sim}{x}, t), \underset{\sim}{v}(\underset{\sim}{x}, t), \underset{\sim}{z}(x, t, s))$ which give the state of the body at time t by means of the incremental displacement $\underset{\sim}{u}$, the incremental velocity $\underset{\sim}{v}$ and the displacement history $\underset{\sim}{z}$, where

$$\underset{\sim}{v}(\underset{\sim}{x}, t) = \dot{\underset{\sim}{u}}(\underset{\sim}{x}, t), \qquad \underset{\sim}{z}(\underset{\sim}{x}, t, s) = \underset{\sim}{u}(\underset{\sim}{x}, t - s). \tag{31}$$

The Hilbert space obtained by the completion of the set $C_0^\infty(B_r) \times C_0^\infty(B_r) \times C_0^\infty\{[0, \infty);$ $C_0^\infty(B_r)\}$ under the norm induced by the inner product,

$$\langle (\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{z}), (\underset{\sim}{\tilde{u}}, \underset{\sim}{\tilde{v}}, \underset{\sim}{\tilde{z}}) \rangle_t = \int_{B_r} (\nabla \underset{\sim}{u})^T \cdot G(\infty) \left[(\nabla \underset{\sim}{\tilde{u}})^T \right] dV + \int_{B_r} \rho (\underset{\sim}{v} \cdot \underset{\sim}{\tilde{v}}) \, dV$$

$$- \int_{B_r} \int_0^\infty (\nabla \underset{\sim}{u} - \nabla \underset{\sim}{z}(s))^T \frac{\partial}{\partial s} G(s) \left[(\nabla \underset{\sim}{\tilde{u}} - \nabla \underset{\sim}{\tilde{z}}(s))^T \right] ds \, dV,$$

is denoted by X_t. It should be noted that the hypotheses (a) to (g) imply that the completions are equivalent for all t.

Defining the matrix operator,

$$A(t) = \begin{pmatrix} 0 & I & 0 \\ C & 0 & M \\ 0 & 0 & N \end{pmatrix}, \tag{33}$$

where

$$Cu = \frac{1}{\rho} \operatorname{Div}\left\{ G(0) \left[(\nabla \underset{\sim}{u})^T \right] \right\}, \tag{34}$$

$$Mz = \frac{1}{\rho} \operatorname{Div} \int_0^\infty \frac{\partial}{\partial s} G(s) \left[(\nabla \underset{\sim}{z}(s))^T \right] ds, \tag{35}$$

$$Nz = -\frac{\partial}{\partial s} z, \tag{36}$$

the abstract evolutionary equation on $0 \leqslant t \leqslant t_0$

$$\frac{d}{dt} \begin{pmatrix} \underset{\sim}{u}(t) \\ \underset{\sim}{v}(t) \\ \underset{\sim}{z}(t) \end{pmatrix} = A(t) \begin{pmatrix} \underset{\sim}{u}(t) \\ \underset{\sim}{v}(t) \\ \underset{\sim}{z}(t) \end{pmatrix} + \begin{pmatrix} 0 \\ f(t) \\ 0 \end{pmatrix}; \qquad \begin{pmatrix} \underset{\sim}{u}(0) \\ \underset{\sim}{v}(0) \\ \underset{\sim}{z}(0) \end{pmatrix} = \begin{pmatrix} \underset{\sim}{u}_0 \\ \underset{\sim}{v}_0 \\ \underset{\sim}{z}_0 \end{pmatrix} \tag{37}$$

follows from the incremental equation of motion (23). In equation (33) the domain D matrix operator $A(t)$ is given by

$$D = D[A(t)] = \left\{ \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \\ \underset{\sim}{z} \end{pmatrix} \in X \middle| A \begin{pmatrix} \underset{\sim}{u} \\ \underset{\sim}{v} \\ \underset{\sim}{z} \end{pmatrix} \in X \text{ and } \underset{\sim}{z}(\underset{\sim}{X}, t, 0) = \underset{\sim}{u}(\underset{\sim}{X}, t) \right\}.$$

The existence and uniqueness theorem may now be stated as follows.

THEOREM. If the conditions (a) to (g) on the viscoelastic coefficients hold and the source term satisfies $\underset{\sim}{f} \in C^1\{[0, t_0]; L_2(B_r)\}$ then, for any $(\underset{\sim}{u}_0, \underset{\sim}{v}_0, \underset{\sim}{z}_0) \in D$ there exists a unique solution

$$(\underset{\sim}{u}(t), \underset{\sim}{v}(t), \underset{\sim}{z}(t)) \in C^1([0, t_0]; \underset{\sim}{X}) \cap C^0([0, t_0]; D)$$

of the evolutionary Eq. (37). Furthermore, there exists $\gamma \geqslant 0$ such that

$$\|(\underset{\sim}{u}(t), \underset{\sim}{v}(t), \underset{\sim}{z}(t))\| \leqslant \gamma \left(\|(\underset{\sim}{u}_0, \underset{\sim}{v}_0, \underset{\sim}{z}_0)\| + \int_0^t \|\underset{\sim}{f}(s)\| \, ds \right), \qquad t \in [0, t_0]. \tag{38}$$

The proof of this theorem is via the following lemmas.

LEMMA 1 The conditions (a) to (f) on the coefficients imply that, for each fixed $t \in [0, t_0]$, $A(t)$ is the generator of a contractive semigroup.

The proof of this lemma can be found in Navarro [7] or Dafermos [8].

LEMMA 2. There exists a positive constant α such that

$$\|(\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{z})\|_t / \|(\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{z})\|_s \leq e^{\alpha|t-s|} \tag{39}$$

holds for each $(\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{z}) \in \underline{X}$ and each $s, t \in [0, t_0]$.

Proof. Let $\phi(t) = \|(\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{z})\|_t^2$ then, differentiating with respect to t, it follows that

$$\dot{\phi}(t) = \int_{B_r} (\nabla \underset{\sim}{u})^T \cdot \frac{\partial}{\partial t} \underline{G}(\underline{X}, t, \infty)\left[(\nabla \underset{\sim}{u})^T\right] dV$$

$$- \int_{B_r} \int_0^\infty (\nabla \underset{\sim}{u} - \nabla \underset{\sim}{z}(s))^T \cdot \frac{\partial^2}{\partial t \partial s} \underline{G}(\underline{X}, t, s)\left[(\nabla \underset{\sim}{u} - \nabla \underset{\sim}{z}(s))^T\right] ds \, dV$$

$$\leq \left| \int_{B_r} (\nabla \underset{\sim}{u})^T \cdot \frac{\partial}{\partial t} \underline{G}(\underline{X}, t, \infty)\left[(\nabla \underset{\sim}{u})^T\right] dV \right|$$

$$- \int_{B_r} \int_0^\infty (\nabla \underset{\sim}{u} - \nabla \underset{\sim}{z}(s))^T \frac{\partial^2}{\partial t \partial s} \underline{G}(\underline{X}, t, s)\left[(\nabla \underset{\sim}{u} - \nabla \underset{\sim}{z}(s))^T\right] ds \, dV. \tag{40}$$

The application of the Cauchy-Schwarz and Young inequalities together with the conditions (f) and (g) yields

$$\dot{\phi}(t) \leq 2\alpha\phi(t) \quad \text{where } \alpha \geq 0.$$

This inequality may be integrated between the limits s and t to obtain

$$\phi(t) \leq \phi(s)e^{\alpha|t-s|}$$

which is the result (39).

Proof of the theorem. It is clear that Lemmas 1 and 2 imply that the family of operators $\{A(t)|t \in [0, t_0]\}$ is stable in the sense of Kato [9] with stability constants $\gamma = e^{2\alpha t_0}$ and zero. Thus there exists an evolution operator $\underline{U}(t, s)$ defined on $\{0 \leq s \leq t \leq t_0\}$ which satisfies $\|\underline{U}(t, s)\| \leq \gamma$ and the solutions of the evolution equation can be written

$$(\underset{\sim}{u}(t), \underset{\sim}{v}(t), \underset{\sim}{z}(t)) = \underline{U}(t, 0)(\underset{\sim}{u}_0, \underset{\sim}{v}_0, \underset{\sim}{z}_0) + \int_0^t \underline{U}(t, s)(0, \underset{\sim}{f}(s), 0) \, ds. \tag{41}$$

The inequality (38) follows immediately.

REMARK 1. The inequality (38) implies the solution depends continuously on the initial conditions, that is the problem is well posed.

REMARK 2. A similar proof to that given by Chirita [10] leads to a theorem of uniqueness and continuous dependence for the mixed boundary value problem of incremental viscoelasticity.

REMARK 3. Arguments similar to those given by Navarro [7] show that the symmetry and monotonicity conditions (d) and (e) follow from the assumption that $A(t)$ is the generator of a contractive semigroup for all $t \in [0, t_0]$.

REFERENCES

[1] A. C. Pipkin and R. S. Rivlin, *Small deformations superposed on large deformations in materials with fading memory*, Arch. Rat. Mech. Anal. **8** 297–308 (1961)

[2] D. Iesan, *Incremental equations in thermoelasticity*, J. Therm. Stresses **3**, 41–56 (1980)

[3] C. B. Navarro and R. Quintanilla, *On existence and uniqueness in incremental thermoelasticity*, to appear in J App. Math. and Phys. (ZAMP)

[4] H. T. Williams, *Theory and applications of thermoviscoelasticity*, Ph.D. Thesis, University of East Anglia, 1983

[5] P. Chadwick and R. W. Ogden, *On the definition of elastic moduli*, Arch. Rat. Mech. Anal. **44** 41–53 (1971)

[6] R. J. Knops and E. W. Wilkes, *Theory of elastic stability*, Handbuch der Physik, vol. VIa/3, C. Truesdell (Ed.), Springer, Berlin, 1973

[7] C. B. Navarro, *On symmetry and monotinicity of relaxation functions in linear viscoelasticity*, J. App. Math. and Phys. (ZAMP) **30**, 541–546 (1979)

[8] C. M. Dafermos, *Contraction semigroups and trend to equilibrium in continuum mechanics*, IUTAM/IMU Symp. on Applications of Methods of Functional Analysis to Problems in Mechanics, P. Germain and B. Nayroles (Eds.), Lecture Notes in Math. 503, Springer, Berlin, pp. 295–306, 1976

[9] T. Kato, *Linear evolution equations of the hyperbolic type*, II, J. Math. Soc. Japan, **5** (1973)

[10] S. Chirita, *Uniqueness and continuous dependence results for the incremental thermoelasticity*, J. Therm. Stresses **5**, 161–172 (1982)