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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2+b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos\frac{x}{2b}}{\cos\frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a+bx)\cos t \text{ is preferable to } \cos t(a+bx).$$

Figures: Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Stromung zaher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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0033-569X(199506)53:2;1-F

Parametric Optimization: Singularities, Pathfollowing and Jumps. By J. Guddat and F. Guerra Vasquez, with H. Th. Jongen. John Wiley & Sons, 1990. viii+191 pp., $85.00.

The subject of this book is concerned with optimization problems in which some or all of the individual data involved depend on one parameter. Applications of solution algorithms are considered mainly in the following fields: (i) globally convergent algorithms for nonlinear, in particular non-convex, optimization problems; (ii) global optimization, (iii) multi-objective optimization. The main tools for a solution algorithm are the so-called path following methods, also known as continuation or homotopy methods. Classical methods in the set of stationary points are extended to the set of all generalized critical points. Chapter headings: 1. Introduction; 2. Theoretical background; 3. Pathfollowing of curves of local minimizers; 4. Pathfollowing along a connected component in the Karush-Kuhn-Tucker set and in the critical set; 5. Pathfollowing with jumps in the set of local minimizers and in the set of generalized critical points; 6. Applications.

Introduction to Shannon Sampling and Interpolation Theory. By Robert J. Marks II. Springer-Verlag, 1991. xiii+324 pp., $59.00.

This is a volume in the series *Springer Texts in Electrical Engineering.* It is written for second-year graduate students who have a foundation in Fourier analysis and stochastic processes. Chapter headings: 1. Introduction; 2. Fundamentals of Fourier analysis and stochastic processes; 3. The cardinal series; 4. Generalizations of the sampling theorem; 5. Sources of error; 6. The sampling theorem in higher dimensions; 7. Continuous sampling.

The Analysis of Linear Partial Differential Operators I—*Distribution Theory and Fourier Analysis.* Second Edition. By Lars Hörmander. Springer-Verlag, 1990. xi+440 pp., $39.50 (paper).

This is a Springer Study Edition of the classic work, first published as volume 256 in the series Grundlehren der Mathematischen Wissenschaften. The main change in this new edition is the inclusion of exercises with answers and hints, so that the volume can serve as a general course in modern analysis on a graduate level, and also as an introduction to harmonic analysis.

Developmental Systems—At the Crossroads of System Theory, Computer Science, and Genetic Engineering. By S. Wegrzyn, J.-C. Gille, and P. Vidal. Springer-Verlag, 1990. vi+120 pp., $39.50.

This monograph presents a model for developmental systems, based on six elementary operations grouped in a generating word. The elements of the system are the analogues of the cells of an organism. The generating word is the analogue of the computer problem and of the genetic code of DNA in the cells of living organisms. The model accounts for many properties of living organisms: their intended hierarchy, their ability to regenerate after a trauma, their sensitivity to mutation, the possibility of cloning or of grafting one organism onto another, and their growth, decay, and reproduction. It is the authors' ambition that their work will help system and computer engineers become acquainted with these problems and will suggest some hypotheses to biologists.

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Mathematical Control Theory—Deterministic Finite Dimensional Systems. By E. D. Sontag. Springer-Verlag, 1990. xiii+396 pp., $39.00.

This is volume 6 in the series *Texts in Applied Mathematics.* It introduces the basic concepts and results of mathematical control and system theory, presenting the subject in a self-contained and elementary fashion, geared to mathematically mature advanced undergraduates or beginning graduate students. It covers what constitutes the common core of control theory: the algebraic theory of linear systems, including controllability, observability, feedback equivalence, and minimality; stability via Lyapunov, as well as input/output methods; ideas of optimal control; observers and dynamic feedback; parameterization of stabilizing controllers; and some basic facts about frequency domain such as the Nyquist criterion. Kalman filtering is introduced briefly through a deterministic version of "optimal observation".

Ordinary and Partial Differential Equations. By R. P. Gilbert and H. C. Howard. Prentice-Hall, 1990. viii+613 pp., $76.50.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. It was developed as a two-semester course which integrated concepts from intermediate calculus with a first course in differential equations, teaching the students not only differential equations but also analysis. Chapter headings: 1. Ordinary differential equations; 2. Methods based on infinite series; 3. Existence and numerical methods; 4. Sturm-Liouville theory; 5. Modelling with partial differential equations; 6. Fourier series; 7. Integral transforms; 8. Integral operators.

The Geometry of Biological Time. By A. T. Winfree. Springer-Verlag, 1990. xiii+530 pp., $32.50.

This is a Springer Study Edition (a corrected printing) of volume 8 in the series *Biomathematics* published by Springer-Verlag in 1980. This original and fascinating monograph describes periodic processes in living systems and their nonliving analogues in the abstract terms of systems theory. Emphasis is given to phase singularities, waves, and mutual synchronization in tissues composed of many clocklike units. Also provided are descriptions of the best-studied experimental systems such as chemical oscillators, pacemaker neurons, circadian clocks, the cell division cycle, and severed limbs regenerating in clocklike patterns. No theoretical background is assumed. The author's contributions have—since the first edition—been recognized by a MacArthur Prize.

Acanthaster and the Coral Reef:— A Theoretical Perspective. Edited by R. H. Bradbury. Springer-Verlag, 1990. vi+337 pp., $39.00.

This is volume 88 in the series *Lecture Notes in Biomathematics* and constitutes the Proceedings of a Workshop held at the Australian Institute of Marine Science, Townsville, August 6–7, 1988. It was organized in conjunction with the Sixth International Coral Reef Symposium to enable mathematicians who had been modelling the phenomenon of the outbreaks of the crown-of-thorns starfish, *Acanthaster planci*, to meet with those scientists who had been actively working on the phenomenon in the field. There is a plenary address by J. Farrands entitled *On Modelling*, and the 18 papers are divided into three groups: Global spatial models, Local spatial models, and Nonspatial models.

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Fourier Methods for Mathematicians, Scientists and Engineers. By Mark Cartwright. Prentice-Hall, 1990. 326 pp., $49.95.

This is a volume in the Ellis Horwood Series in Mathematics and its Applications. The first part of this book (chapters 1–9) introduces the basic aspects of Fourier theory, including various series and continuous and discrete transforms, convolution, filtering and correlation. The second part consists of chapters on applications in, respectively, mathematics, physics, chemistry, the life sciences, and miscellaneous fields such as turbulence, seismology, glaciology, and economics. There are many exercises with solutions.

Dependence with Complete Connections and its Applications. By Marius Iosifescu and Serban Grigorescu. Cambridge University Press, 1990. xiv+340 pp., $69.50.

Dependence with complete connections is a more general type of stochastic process Markovian dependence, accounting for a complete, rather than recent, history of a stochastic evolution. The co-founders of the subject were Octav Onicescu and Gheorghe Mihoc, to whose memory this monograph is dedicated. It is a survey of the current state of knowledge of the subject, dealing with the basic theoretical understanding, and with applications to stochastic models of learning, branching processes in random environments, continued fractions, and dynamical systems.

An Introduction to Harmonic Analysis on Semisimple Lie Groups. By V. S. Varadarajan. Cambridge University Press, 1989. x+316 pp., $69.50.

The aim of the course on which this monograph is based was to introduce graduate students to the representation theory of semisimple groups, a vast and active subject that brings together at a deep level algebra, geometry, analysis, and arithmetic. Chapter headings: 1. Introduction; 2. Compact groups: the work of Weyl; 3. Unitary representations of locally compact groups; 4. Parabolic induction, principal series representations, and their characters; 5. Representation of Lie groups; 6. The Plancherel formula: character form; 7. Invariant eigendistributions; 8. Harmonic analysis of the Schwartz space. There are appendices on background material: functional analysis, topological groups, Lie groups and algebras.

An Introduction to General Relativity. By L. P. Hughston and K. P. Tod. Cambridge University Press, 1991. 183 pp., $49.50 (Cloth), $19.95 (Paper).

This is volume 5 in the series *London Mathematical Society Student Texts*, in which the authors' aim has been to present a text suitable for a course in general relativity theory on the undergraduate level. The text is thus made very readable, and the material and exercises chosen so as to be suitable for undergraduate examinations—the latter, alas, given without solutions. To this end, also, more emphasis is placed on an intuitive grasp of the subject and a calculational facility than on a rigorous mathematical exposition. After a review of Cartesian tensor notation and special relativity, the concepts of Riemannian geometry are introduced and general relativity presented as a relativistic theory of gravity. The Schwarzschild solution is derived and the gravitational redshift, time dilatation and classic tests of general relativity discussed. There is a brief account of gravitational collapse and black holes based on the extended Schwarzschild solution. Other vacuum solutions are described, motivated by their counterparts in linearised general relativity. The text ends with chapters on cosmological solutions to the field equations.

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Huygens & Barrow, Newton & Hooke—Pioneers in Mathematical Analysis and Catastrophe Theory for Evolvents to Quasicrystals. By V. I. Arnol'd. Springer-Verlag, 1990. 118 pp.

This is a translation, by Eric J. F. Primrose, of the Russian original published by Nauka FMI, Moscow, in 1989. The author retraces the beginnings of mathematical analysis and theoretical physics in the works of these 17th century scientists. He traces the link between some of Huygens' and Newton's ideas on the one hand and certain developments in contemporary mathematics on the other. He provides generalisations of Newton's theorems on the elliptical shape of orbits and on the transcendence of Abelian integrals; he offers a brief review of the theory of regular and chaotic movement in celestial mechanics, including the problem of ports in the distribution of smaller planets and a discussion of the structure of planetary rings. Chapter headings: 1. The law of universal gravitation; 2. Mathematical analysis; 3. From evolvents to quasicrystals; 4. Celestial mechanics; 5. Kepler's second law and the topology of Abelian integrals.

Numerical Methods for Conservation Laws. By Randall J. LeVeque. Birkhäuser-Verlag, Basel, 1990. x+214 pp.

This is a volume in the series *Lectures in Mathematics ETH Zürich.* The overall emphasis is on studying the mathematical tools that are essential in developing, analyzing, and successfully using numerical methods for nonlinear systems of conservation laws, particularly for problems involving shock waves. Part I (chapters 1–9) deals with the mathematical structure of these equations and their solution. Part II (chapters 10–18) deals more directly with numerical methods, again with the emphasis on general tools that are of broad use. The underlying ideas used in the various classes of methods are stressed rather than the most sophisticated methods presented in great detail. Chapter headings: 1. Introduction; 2. The derivation of conservation laws; 3. Scalar conservation laws; 4. Some scalar examples; 5. Some nonlinear systems; 6. Linear hyperbolic systems; 7. Shocks and the Hugoniot locus; 8. Rarefaction waves and integral curves; 9. The Riemann problem for the Euler equation; 10. Numerical methods for linear equations; 11. Computing discontinuous solutions; 12. Conservation methods for nonlinear problems; 13. Godunov's method; 14. Approximate Riemann solvers; 15. Nonlinear stability; 16. High resolution methods; 17. Semi-discrete methods; 18. Multidimensional problems.

An Introduction to the Theory of Numbers. Fifth Edition. By Ivan Niven, Herbert S. Zuckerman, and Hugh L. Montgomery. John Wiley & Sons, 1991. xiii+529 pp., $50.95.

This text is intended for use in a first course in number theory. Large collections of problems of varying difficulty are included. New in this edition are accounts of the binomial theorem, public-key cryptography, simultaneous systems of linear Diophantine equations, rational points on curves, description of Falting's theorem, Dirichlet series, asymptotic estimates of arithmetic functions, and other revisions. A number of calculational issues are also addressed.

Optimization. Edited by S. Dolecki. Springer-Verlag, 1989. 220 pages. $20.00.

This is volume 1405 of Lecture Notes in Mathematics. It is the Proceedings of the Fifth French-German Conference held in Castel-Novel (Varetz), France, October 3–8, 1988. There are 15 papers.

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Singular Perturbations I—*Singular Perturbations on Manifolds without Boundaries.* By Leonid S. Frank. North-Holland, 1990. xxiv+556 pp., $89.75.

This is volume 23 in the series *Studies in Mathematics and its Applications.* This first volume of a set of two deals with linear singular perturbations (on smooth manifolds without boundary) considered as equicontinuous linear mappings between corresponding families of Sobolev-Slobodetski's type spaces of vectorial order. The second volume will deal with singular perturbations in elasticity theory and, more generally, coercive singular perturbations, as well as with singular perturbations of dissipative and dispersive type. There are three chapters in this volume: 1. Manifolds, functional analysis, distributions: this chapter provides the necessary background; 2. Sobolev spaces of vectorial order; 3. Singular perturbations on smooth manifolds without boundary: this chapter deals essentially with elliptic and hyperbolic singular perturbations and their finite difference counterparts, as well as the classical asymptotic stationary phase, Laplace and saddle point methods.

Mathematical Ecology of Plant Species Competition. By Anthony G. Pakes and R. A. Maller. Cambridge University Press, 1990. xiii+193 pp., $49.50.

This is a volume in the series *Cambridge Studies in Mathematical Biology.* It is a graduate/research level case study of mathematical modeling, as opposed to computer modeling, of the dynamics of two competing plant species or strains, or genotypes. The process modeled is a binary mixture of two strains of the annual legume subterranean clover, which propagate by means of "soft" or germinable seeds each year, whilst retaining a store of "hard" dormant seeds for germination in future years. The interaction between the two types is described by the de Wit replacement model. The models were developed in response to four major questions on the long-term outcome of binary mixtures: (i) is ultimate coexistence possible?; (ii) if not, which strain will win?; (iii) does the mixture approach an equilibrium?; (iv) if so, then how long does the mixture take to attain it? The book gives a detailed account of model construction, analysis, and application to field data.

Introduction to Statistics—Concepts and Applications. Second Edition. By David R. Anderson, Dennis J. Sweeney, and Thomas A. Williams. West Publishing Company, 1991. xxi+714 pp.

The purpose of this text is to provide a comprehensive treatment of introductory statistics for students from a wide variety of academic backgrounds. It is applications oriented and has been written with the needs of the nonmathematician in mind. The mathematical prerequisite is a course in college algebra. The second edition is a major revision, providing a much more complete and detailed topical coverage than the first edition.

An Introduction to Dynamical Systems. By D. K. Arrowsmith and C. M. Place. Cambridge University Press, 1990. 423 pp., $79.50 (cloth), $29.95 (paper).

This text is aimed at the interface between undergraduate and (post)graduate studies, and one of its major features is its extensive set of exercises (more than 300 in all). Chapter headings: 1. Diffeomorphisms and flows; 2. Local properties of flows and diffeomorphisms; 3. Structural stability, hyperbolicity and homoclinic points; 4. Local bifurcations I: planar vector fields and diffeomorphisms on $\mathbb{R}$; 5. Local bifurcations II: diffeomorphisms on $\mathbb{R}^2$; 6. Area-preserving maps and their perturbations.

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Mapping Crime in its Community Setting—Event Geography Analysis. By Michael D. Maltz, Andrew C. Gordon, and Warren Friedman. Springer-Verlag, 1990. $55.00.

This volume is concerned with the use of statistics in a law enforcement context.

Lukasiewicz-Moisil Algebras. By V. Boicescu, A. Filipoiu, G. Georgescu, and S. Rudeanu. North-Holland, 1991. xv+583 pp., $143.00.

This is volume 49 in the series *Annals of Discrete Mathematics.* It is a detailed presentation, with full proofs, of the algebraic side of Lukasiewicz-Moisil Algebras, the algebraic counterpart of systems of many-valued logic.

Viscous Vortical Flows. By L. Ting and R. Klein. Springer-Verlag, 1991. iv+222 pp., $30.00.

This is volume 374 of the series *Lecture Notes in Physics.* The lectures are based on a series of ten lectures given by L. Ting in commemoration of the 75th anniversary of the founding of the Aerodynamisches Institut, Rheinisch-Westfälische Technische Hochschule, Aachen, in April 1988. The notes are composed of four chapters: 1. Vortex dominated flows and general theory: here, the mathematical problem is formulated as an initial value problem for the incompressible Navier-Stokes equations in an unbounded domain with an initial velocity field; 2. Motion and decay of vortex filaments: this motion is studied using the method of matched asymptotic expansions; 3. Numerical solutions of viscous vortical flows: the merging of the core structures of vortices is studied by numerical solutions of the N.-S. equations; 4. Closing remarks: several new problem areas are mentioned here.

Introduction to Mathematical Physics—Methods and Concepts. By Chun Wa Wong. Oxford University Press, 1991. x+386 pp., $49.95.

This book is based on lecture notes for two undergraduate courses given at UCLA. There are six chapters: 1. Vectors and fields in space; 2. Transformations, matrices, and operators; 3. Fourier series and Fourier transforms; 4. Differential equations in physics; 5. Special functions; 6. Functions of a complex variable.

The Geometry and Physics of Knots. By Sir Michael Atiyah. Cambridge University Press, 1990. x+78 pp., $39.50.

These lecture notes are an expanded version of the series of lectures given by the author under the auspices of the Accademia Nazionale dei Lincei at the University of Florence in November 1988. They deal with an area that lies at the crossroads of mathematics and physics and the material presented here rests primarily on the pioneering work of Vaughan Jones and Edward Witten relating polynomial invariants of knots to a topological quantum field theory in 2+1 dimensions. The author presents an introduction to Witten's ideas from a mathematical point of view. The chapter headings are: 1. History and background; 2. Topological quantum field theories; 3. Non-abelian moduli spaces; 4. Symplectic quotients; 5. The infinite-dimensional case; 6. Projective flatness; 7. The Feynman integral formulation; 8. Final comments.

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Existence and Optimality of Competitive Equilibria. By C. D. Aliprantis, D. J. Brown, and O. Burkinshaw. Springer-Verlag, 1990. xii+284 pp., $35.00.

This is a corrected softcover edition of the monograph first published in 1988. It is a systematic exposition of the authors' research on general equilibrium models with an infinite number of commodities. It is intended to serve both as a graduate text on aspects of general equilibrium theory and as an introduction, for economists and mathematicians working in mathematical economics, to current research in a frontier area of general equilibrium theory. Two introductory chapters on the basic economic model and the mathematical framework are provided. Chapter headings: 1. The Arrow-Debreu model; 2. Riesz spaces of commodities and prices; 3. Markets with infinitely many commodities; 4. The overlapping generations model.

Module Theory—An Approach to Linear Algebra. Second Edition. By T. S. Blyth. Oxford University Press, 1990. vi+360 pp.

The aim of this text is to develop the basic properties of modules and to show their importance, mainly in the theory of linear algebra. In the first eleven chapters all the basic material on modules and vector spaces required for advanced courses is covered. The remainder of the text (nine chapters) can be used as an advanced course. Here, the foundations of multilinear and exterior algebra are developed. In particular, it is shown how exterior powers lead to determinants. In this edition, some results of a ring-theoretic nature directly related to modules and linear algebra are included. In particular, the Wedderburn-Artin theorem, that every simple ring is isomorphic to the ring of endomorphisms of a finite-dimensional module over a division ring, is established. Also, the text discusses in detail the structure of finitely generated modules over a principal ideal domain. Exercises are provided at the end of each chapter.

Twistors in Mathematics and Physics. Edited by T. N. Bailey and R. J. Baston. Cambridge University Press, 1991. 382 pp., $34.50.

This is volume 156 in the London Mathematical Society Lecture Notes Series. It collects together review articles that reflect the wide diversity of ideas and techniques that constitute modern twistor theory. The main objective of the "twistor program" for fundamental physics is a theory that unites Einstein's general relativity and the world of quantum physics—a theory in which the role of complex holomorphic geometry is fundamental. In this volume, R. Penrose reviews its current status. Other contributors cover the advances that have occurred since the major successes of Penrose's nonlinear graviton and Ward's construction of the Yang-Mills instantons. In twistor mathematics, articles cover differential geometry, integrable systems, and several topics related to representation theory.

Minimum Entropy H_∞ Control. By D. Mustafa and K. Glover. Springer-Verlag, 1990. viii+144 pp., $30.00.

This is volume 146 in the series *Lecture Notes in Control and Information Sciences.* The central question addressed in this monograph is to investigate which closed-loop transfer functions to choose if one wishes them to have H_∞-norm less than some prescribed tolerance. One possible choice is to take that which minimizes an entropy integral, and the derivation and properties of this solution occupy most of this monograph. The approach taken here exploits recent progress in the state-space solution to the H_∞ control problem and explicit formulae can be obtained for the solution.

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A Course in Mathematics for Students of Physics: Volume 2. By Paul Bamberg and Shlomo Sternberg. Cambridge University Press, 1990. xvii, 407–847 pp., $54.50.

This book represents the text of a course that has been taught at Harvard for the past eight years, aimed at students with an interest in physics who have a good grounding in one-variable calculus. The chapters in volume 1 were: 1. Linear transformations of the plane; 2. Eigenvectors and eigenvalues; 3. Linear differential equations in the plane; 4. Scalar products; 5. Calculus in the plane; 6. Theorems of the differential calculus; 7. Differential forms and line integrals; 8. Double integrals; 9. Gaussian optics; 10. Vector spaces and linear transformations; 11. Determinants; and volume 2 contains: 12. The theory of electrical networks; 13. The method of orthogonal projection; 14. Higher-dimensional complexes; 15. Complexes situated in $\mathbb{R}^n$; 16. Electrostatics in $\mathbb{R}^3$; 17. Currents, flows and magnetostatics; 18. The star operators; 19. Maxwell's equations; 20. Complex analysis; 21. Asymptotic evaluation of integrals; 22. Thermodynamics.

H^∞-Optimal Control and Related Minimax Design Problems—A Dynamic Game Approach. By Tamer Basar and Pierre Bernhard. Birkhäuser, Boston, 1991. vii+224 pp., $49.50.

This is a volume in the series *Systems & Control: Foundations & Applications*. It uses the framework of dynamic (differential) game theory to develop a complete theory of H^∞-optimal control that encompasses continuous-time as well as discrete-time systems, finite as well as infinite horizons, and several different measurement schemes, including closed-loop imperfect state, delayed imperfect state, and sampled imperfect state information patterns. The only prerequisite for the book is a basic knowledge of linear control theory, and the authors intend the book to be suitable for a second-level graduate course in a control curriculum.

Advanced Matrix Theory for Scientists and Engineers. Second Edition. By A. S. Deif, Gordon and Breach Science Publishers, 1991. x+302 pp.

In this revised edition of a text first published in 1981, the theory of nth-order differential equations based upon properties of matrix polynomials is studied, and a new chapter is also added on the theory of interval matrices. A new appendix is devoted to the discussion of some well-known matrix packages.

Modeling One-dimensional Pattern Formation by Anti-diffusion. By J. F. Kaashoek. Stichting Mathematisch Centrum, Amsterdam, 1990. vi+271 pp., Dfl. 67.00.

This is CWI Tract 72. It is the result of research carried out by an interdisciplinary study group on pattern formation and bifurcation formed at the Erasmus University, Rotterdam, in 1982. The work is based on Prigogine's theory and Haken's synergetic approach, which are described before several pattern formation processes are explored. The scope of the book is indicated by its table of contents: 1. Pattern formation; 2. Thom's river basin model; 3. General properties of the anti-diffusion equation; 4. A stochastic model; 5. Existence and stability of stationary solutions; 6. Bifurcation analysis; 7. Non-constant stationary solutions; 8. Numerical solutions; 9. Pattern formation in chemico-physical systems; 10. Migration systems.