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PARTIAL REACHABILITY OF A THERMOELASTIC PLATE WITH MEMORY

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Abstract. We investigate the partial reachability of a thermoelastic plate with memory, a variant of a system studied earlier by Lagnese and Lions (1988) without memory. The well posedness of the system is established by transposition after having established the well posedness of the adjoint system by using Volterra equations and the Galerkin method. The partial reachability is deduced from classical theorems on Kirchhoff plates by a perturbation technique.

1. Introduction. The purpose of this paper is to investigate the reachability of the following system:

$$\begin{cases} w'' - \gamma\Delta w'' + \Delta^2 w + \beta_1\Delta\theta = 0 \quad \text{in} \quad Q, \\ \beta_2\theta' - \beta_0\Delta\theta - k * \Delta\theta - \beta_3\Delta w' = 0 \quad \text{in} \quad Q, \\ w = 0, \quad \frac{\partial w}{\partial \nu} = u \quad \text{and} \quad \theta = 0 \quad \text{on} \quad \Sigma, \\ w(0) = w'(0) = \theta(0) = 0 \quad \text{on} \quad \Omega. \end{cases} \tag{1.1}$$

Here Ω is a bounded open domain of $\mathbb{R}^n$ with a boundary Γ of class C^2, $T > 0$ is a given number, $Q = (0,T) \times \Omega$, $\Sigma = (0,T) \times \Gamma$, β_0, β_1, β_2, β_3, γ are positive constants, ν denotes the outward unit normal vector to Γ, the Laplacian operator Δ acts on the

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space variables, $k : [0,T] \to \mathbb{R}$ is a continuously differentiable function with $k(0) \neq 0$, and the convolution product is defined by the formula

$$(k * g)(x,t) := \int_0^t k(t-s)g(x,s)\,ds.$$

This system is a memory-containing version of a model formerly investigated by Lagnese and Lions [12].

We study the set of final states $(w(T), w'(T))$ when v runs over some natural set of controls.

Following the important work of Dafermos [5] on linear thermoelasticity systems, many models of thermoelastic plates have already been investigated by various methods: see, e.g., [1]–[4], [6]–[10], [12]–[13], [16] and their references. Our result, related to one of the first models, seems to be new.

2. Adjoint system. We will define the solutions of the system (1.1) by the transposition method. For this we introduce a dual system by the following formal computation. Let φ, ψ be two functions in Q, satisfying the boundary conditions

$$\varphi = \frac{\partial \varphi}{\partial \nu} = \psi = 0 \quad \text{on} \quad \Sigma.$$

If (w,θ) solves (1.1), then multiplying $(1.1)_1$ by φ, $(1.1)_2$ by ψ and integrating by parts their sum, we obtain the following identity:

$$\begin{aligned}
0 &= \int_Q \left(w'' - \gamma\Delta w'' + \Delta^2 w + \beta_1 \Delta\theta\right)\varphi + \left(\beta_2\theta' - \beta_0\Delta\theta - k * \Delta\theta - \beta_3\Delta w'\right)\psi\,dQ \\
&= \int_Q w\left(\varphi'' - \gamma\Delta\varphi'' + \Delta^2\varphi + \beta_3\Delta\psi'\right) + \theta\left(-\beta_2\psi' - \beta_0\Delta\psi - k \circ \Delta\psi + \beta_1\Delta\varphi\right)\,dQ \\
&\quad + \int_\Omega w'(\varphi - \gamma\Delta\varphi) + w(\varphi' - \gamma\Delta\varphi') + (\beta_2\theta - \beta_3\Delta w)\psi\,dx\,|_{t=T} \\
&\quad + \int_\Sigma \frac{\partial w}{\partial \nu}\Delta\varphi\,d\Sigma,
\end{aligned}$$

where we use the notation

$$(k \circ g)(x,t) := \int_t^T k(s-t)g(x,s)\,ds.$$

Hence, if φ, ψ solve the system

$$\begin{cases}
\varphi'' - \gamma\Delta\varphi'' + \Delta^2\varphi + \beta_3\Delta\psi' = 0 \quad \text{in} \quad Q, \\
-\beta_2\psi' - \beta_0\Delta\psi - k \circ \Delta\psi + \beta_1\Delta\varphi = 0 \quad \text{in} \quad Q, \\
\varphi = \frac{\partial\varphi}{\partial\nu} = \psi = 0 \quad \text{on} \quad \Sigma, \\
\varphi(T) = \varphi_0, \quad \varphi'(T) = \varphi_1 \quad \text{and} \quad \psi(T) = \psi_0 \quad \text{on} \quad \Omega,
\end{cases} \tag{2.1}$$

then we obtain the simple identity

$$\int_\Omega w'(\varphi - \gamma\Delta\varphi) + w(\varphi' - \gamma\Delta\varphi') + (\beta_2\theta - \beta_3\Delta w)\psi\,dx\,|_{t=T} = -\int_\Sigma v\Delta\varphi\,d\Sigma. \tag{2.2}$$

Moreover, this identity and its proof remain valid if we replace T by any $0 < T' < T$ and Q, Σ by $(0,T')\times\Omega$ and $(0,T')\times\Gamma$, respectively:

$$\int_\Omega w'(\varphi-\gamma\Delta\varphi)+w(\varphi'-\gamma\Delta\varphi')+(\beta_2\theta-\beta_3\Delta w)\psi\ dx\,|_{t=T'} \qquad (2.3)$$
$$= -\int_0^{T'}\int_\Gamma v\Delta\varphi\ d\Gamma\ dt.$$

Let us investigate the well posedness of the *adjoint system* (2.1). We are going to establish the following:

PROPOSITION 2.1. For any given φ_0, φ_1 and ψ_0 satisfying

$$\varphi_0\in H_0^2(\Omega),\quad \varphi_1\in H_0^1(\Omega)\quad\text{and}\quad \psi_0\in H_0^1(\Omega), \qquad (2.4)$$

the system (2.1) has a unique solution satisfying

$$(\varphi,\varphi',\psi,\psi')\in C([0,T];H_0^2(\Omega)\times H_0^1(\Omega)\times H_0^1(\Omega)\times L^2(\Omega))$$

and

$$\psi'\in L^2(0,T;H_0^1(\Omega)).$$

Furthermore, the linear map

$$(\varphi_0,\varphi_1,\psi_0)\mapsto(\varphi,\varphi',\psi,\psi')$$

is continuous for the indicated topologies.

For the proof of the Proposition 2.1 we will use the following classical lemma on the Volterra equation

$$r(t)-\int_0^t K(t,s)r(s)\ ds = f(t),\quad t\in[0,T]. \qquad (2.5)$$

LEMMA 2.2. If $K:[0,T]\times[0,T]\to\mathbb{R}$ and $f:[0,T]\to\mathbb{R}$ are continuous functions, then the equation (2.5) has a unique continuous solution $r:[0,T]\to\mathbb{R}$.

See, e.g., [17, pp. 165–169] or [18, 145–147] for a proof.

Proof of Proposition 2.1. It is convenient to reverse the time. Setting

$$\omega(t,x)=\varphi(T-t,x)\quad\text{and}\quad \eta(t,x)=\psi(T-t,x),$$

We find that (2.1) takes the following equivalent form:

$$\begin{cases}\omega''-\gamma\Delta\omega''+\Delta^2\omega-\beta_3\Delta\eta'=0 & \text{in}\quad Q,\\ \beta_2\eta'-\beta_0\Delta\eta-k*\Delta\eta+\beta_1\Delta\omega=0 & \text{in}\quad Q,\\ \omega=\frac{\partial\omega}{\partial\nu}=\eta=0 & \text{on}\quad \Sigma,\\ \omega(0)=\varphi_0,\quad \omega'(0)=-\varphi_1\quad\text{and}\quad \eta(0)=\psi_0 & \text{on}\quad\Omega.\end{cases} \qquad (2.6)$$

Differentiating $(2.6)_2$ we get

$$\beta_2\eta''-\beta_0\Delta\eta'-k(0)\Delta\eta-k'*\Delta\eta+\beta_1\Delta\omega'=0\quad\text{in}\quad Q, \qquad (2.7)$$

and we also deduce from $(2.6)_2$ that

$$\beta_2\eta'(0)=\beta_0\Delta\psi_0-\beta_1\Delta\varphi_0\quad\text{on}\quad\Omega. \qquad (2.8)$$

We simplify (2.7) by introducing the new unknown function

$$v := k(0)\eta + k' * \eta \tag{2.9}$$

instead of η. Hence

$$v(0) = k(0)\eta(0) = k(0)\psi_0,$$

and differentiating (2.9) we get

$$\begin{aligned} v'(0) &= k(0)\eta'(0) + k'(0)\eta(0) \\ &= \frac{k(0)}{\beta_2}(\beta_0\Delta\psi_0 - \beta_1\Delta\varphi_0) + k'(0)\psi_0. \end{aligned}$$

In order to obtain an inverse relation we introduce the solution r of the resolvent equation

$$k(0)r + k' * r = -\frac{k'}{k(0)}. \tag{2.10}$$

We apply Lemma 2.2 with $K(t,s) := -k'(t-s)/k(0)$ and $f(t) := -k'(t)/k(0)^2$.

We have

$$\begin{aligned} v + k(0)r * v &= k(0)\eta + k' * \eta + k(0)r * (k(0)\eta + k' * \eta) \\ &= k(0)\eta + k' * \eta + k(0)(k(0)r + k' * r) * \eta \\ &= k(0)\eta + k' * \eta - k' * \eta \\ &= k(0)\eta, \end{aligned}$$

so that

$$\eta = \frac{v}{k(0)} + r * v. \tag{2.11}$$

We deduce from (2.10) the equality

$$k(0)^2 r(0) + k'(0) = 0,$$

whence

$$v'(0) = \frac{k(0)}{\beta_2}(\beta_0\Delta\psi_0 - \beta_1\Delta\varphi_0) - k(0)^2 r(0)\psi_0.$$

Differentiating (2.11) we obtain

$$\eta' = \frac{v'}{k(0)} + r(0)v + r' * v \tag{2.12}$$

and

$$\eta'' = \frac{v''}{k(0)} + r(0)v' + r'(0)v + r'' * v. \tag{2.13}$$

Using (2.9), (2.12), (2.13) and the boundary and initial conditions in (2.6) and (2.8), we obtain from (2.6) and (2.7) the following system:

$$\begin{cases} \omega'' - \gamma\Delta\omega'' + \Delta^2\omega - \frac{\beta_3}{k(0)}\Delta v' = \beta_3\,(r(0)\Delta v + r' * \Delta v) \quad \text{in} \quad Q, \\ \frac{\beta_2}{k(0)}v'' - \frac{\beta_0}{k(0)}\Delta v' - \Delta v + \beta_1\Delta\omega' = \beta_0(r(0)\Delta v + r' * \Delta v) \\ \qquad\qquad -\beta_2\,(r(0)v' + r'(0)v + r'' * v) \quad \text{in} \quad Q, \\ \omega = \frac{\partial\omega}{\partial\nu} = v = 0 \quad \text{on} \quad \Sigma, \\ \omega(0) = \varphi_0, \quad \omega'(0) = -\varphi_1, \quad v(0) = k(0)\psi_0 \\ \qquad \text{and} \quad v'(0) = \frac{k(0)}{\beta_2}(\beta_0\Delta\psi_0 - \beta_1\Delta\varphi_0) - k(0)^2 r(0)\psi_0 \quad \text{on} \quad \Omega. \end{cases} \tag{2.14}$$

Multiplying $(2.14)_1$ by ω', $(2.14)_2$ by $\frac{\beta_3}{\beta_1 k(0)}v'$, and integrating by parts their sum in Q, we obtain the following identity:

$$\begin{aligned} E(T) - E(0) &+ \frac{\beta_0\beta_3}{\beta_2 k(0)^2}\int_Q |\nabla v'|^2 \; dQ \\ &= -\frac{\beta_0\beta_3}{\beta_2 k(0)}\int_Q (r(0)\nabla v + r' * \nabla v)\cdot\nabla v' \; dQ \\ &\quad - \beta_3\int_Q r(0)(\nabla v\cdot\nabla\omega') + (r' * \nabla v)\cdot\nabla\omega' \; dQ \\ &\quad - \frac{\beta_2\beta_3}{\beta_1 k(0)}\int_Q r(0)\,|v'|^2 + r'(0)vv' + (r'' * v)v' \; dQ, \end{aligned} \tag{2.15}$$

where we use the energy notation

$$E = \frac{1}{2}\int_\Omega |\omega'|^2 + \gamma\,|\nabla\omega'|^2 + |\Delta\omega|^2 + \frac{\beta_2\beta_3}{\beta_1 k(0)^2}\,|v'|^2 + \frac{\beta_3}{\beta_1 k(0)}\,|\nabla v|^2 \; dx.$$

Majorizing the right side by applying the Cauchy–Schwarz, Young and Poincaré inequalities, we deduce the estimate

$$E(T) + \frac{\beta_0\beta_3}{2\beta_2 k(0)^2}\int_Q |\nabla v'|^2 \; dQ \le E(0) + C_1\int_0^T E(t)\; dt \tag{2.16}$$

with some constant C_1.[1] We have used here the positivity of the coefficient of the integral on the left side of (2.15) in order to eliminate the same integral on the right side, coming from the application of the Young inequality.

The same computation, and hence the estimate (2.16), holds if we replace T by any $0 \le T' < T$. Applying the Gronwall inequality and the Galerkin method, using (2.4) and the Poincaré inequality in $H_0^1(\Omega)$, we deduce from these estimates that the system

[1]Here and in the sequel all constants C_i are independent of the particular initial data.

(2.14) has a unique solution (ω, v) satisfying the following regularity conditions:

$$\begin{aligned}
&\omega \in C([0,T]; H_0^2(\Omega)),\\
&\omega' \in C([0,T]; H_0^1(\Omega)),\\
&v \in C([0,T]; H_0^1(\Omega)),\\
&v' \in C([0,T]; L^2(\Omega)),\\
&v' \in L^2(0,T; H_0^1(\Omega)).
\end{aligned}$$

Since η and v have the same regularity by (2.9), the proposition follows. $\square$

3. Well posedness of the original system. In view of the considerations of the preceding section leading to the introduction of the dual system, we define the solutions of (1.1) as follows:

Definition 3.1. By a solution of (1.1) we mean a continuous function

$$(w, w', \theta) : [0,T] \to H_0^1(\Omega) \times L^2(\Omega) \times H^{-1}(\Omega)$$

satisfying the identity (2.3) for all $\varphi_0 \in H_0^2(\Omega)$, $\varphi_1 \in H_0^1(\Omega)$, $\psi_0 \in H_0^1(\Omega)$, and for all $0 \le T' \le T$, where (φ, ψ) denotes the corresponding solution of (2.2).

In order to justify this definition we need the following corollary of Proposition 2.1:

Corollary 3.2. Under the conditions of Proposition 2.1 the solutions of (2.1) satisfy $\Delta\varphi \in L^2(\Sigma)$, and the estimate

$$\|\Delta\varphi\|_{L^2(\Sigma)} \le C_2 \left(\|\varphi_0\|_{H_0^2(\Omega)} + \|\varphi_1\|_{H_0^1(\Omega)} + \|\psi_0\|_{H_0^1(\Omega)} \right)$$

holds with a constant C_2.

Proof. Since

$$f := -\beta_3 \Delta\psi' \in L^2(0,T; H^{-1}(\Omega))$$

by Proposition 2.1, the corollary follows from classical results on the system

$$\begin{cases}
\varphi'' - \gamma\Delta\varphi'' + \Delta^2\varphi = f \quad \text{in} \quad Q,\\
\varphi = \frac{\partial\varphi}{\partial\nu} = 0 \quad \text{on} \quad \Sigma,\\
\varphi(T) = \varphi_0 \quad \text{and} \quad \varphi'(T) = \varphi_1 \quad \text{on} \quad \Omega,
\end{cases}$$

as in [12, p. 157]. $\square$

Proposition 2.1 and Corollary 3.2 yield at once the following well posedness result:

Proposition 3.3. For any given $u \in L^2(0,T; L^2(\Gamma))$, the system (1.1) has a unique solution. Furthermore, the linear map $f \mapsto (w, w', \theta)$ is continuous for the indicated topologies.

4. Partial reachability. The reachable space for the w component of the system (1.1) is defined by

$$R := \left\{(w(T), w'(T)) \ : \ u \in L^2(0,T;L^2(\Gamma))\right\}.$$

It follows from Proposition 3.3 that

$$R \subset H_0^1(\Omega) \times L^2(\Omega).$$

We are going to prove that under some assumptions we have equality here.

THEOREM 4.1. If β_3 is sufficiently small, then

$$R = H_0^1(\Omega) \times L^2(\Omega).$$

First we establish a converse of Corollary 3.2. Fix an arbitrary point $x_0 \in \mathbb{R}^n$ and set

$$\Gamma_+ := \{x \in \Gamma \ : \ (x - x_0)\cdot \nu(x) > 0\}, \quad \Sigma_+ := \Gamma_+ \times (0,T)$$

and

$$T_0 := \max\{|x - x_0| \ : \ x \in \Gamma_+\}.$$

PROPOSITION 4.2. Let $T > T_0$. If β_3 is sufficiently small, then there exists a positive constant C_3 such that the solutions of (2.1) with $\psi_0 = 0$ satisfy the following inequality:

$$\|\Delta\varphi\|_{L^2(\Sigma_+)} \geq C_3 \left(\|\varphi_0\|_{H_0^2(\Omega)} + \|\varphi_1\|_{H_0^1(\Omega)}\right). \tag{4.1}$$

Proof. The component φ of the solution of (2.1) may be written in the form $\varphi = \chi + \beta_3\xi$ where χ and ξ solve respectively the problems

$$\begin{cases} \chi'' - \gamma\Delta\chi'' + \Delta^2\chi = 0 \quad \text{in} \quad Q, \\ \chi = \frac{\partial\chi}{\partial\nu} = 0 \quad \text{on} \quad \Sigma, \\ \chi(T) = \varphi_0 \quad \text{and} \quad \chi'(T) = \varphi_1 \quad \text{on} \quad \Omega \end{cases}$$

and

$$\begin{cases} \xi'' - \gamma\Delta\xi'' + \Delta^2\xi = -\Delta\psi' \quad \text{in} \quad Q, \\ \xi = \frac{\partial\xi}{\partial\nu} = 0 \quad \text{on} \quad \Sigma, \\ \xi(T) = \xi'(T) = 0 \quad \text{on} \quad \Omega. \end{cases}$$

By classical results on Kirchhoff plates (see, e.g., [12]) we have

$$\|\Delta\chi\|_{L^2(\Sigma_+)} \geq C_4 \left(\|\varphi_0\|_{H_0^2(\Omega)} + \|\varphi_1\|_{H_0^1(\Omega)}\right)$$

and

$$\|\Delta\xi\|_{L^2(\Sigma)} \leq C_5 \|\Delta\psi'\|_{L^2(\Sigma)}$$

with suitable positive constants C_4, C_5. Furthermore, Proposition 2.1 and Corollary 3.2 imply that

$$\|\Delta\psi'\|_{L^2(\Sigma)} \leq C_6 \left(\|\varphi_0\|_{H_0^2(\Omega)} + \|\varphi_1\|_{H_0^1(\Omega)}\right)$$

with a suitable constant C_6.

Combining these estimates we get

$$\begin{aligned}\|\Delta\varphi\|_{L^2(\Sigma_+)} &\geq \|\Delta\chi\|_{L^2(\Sigma_+)} - \beta_3 \|\Delta\xi\|_{L^2(\Sigma_+)} \\ &\geq (C_3 - \beta_3 C_5 C_6)\left(\|\varphi_0\|_{H_0^2(\Omega)} + \|\varphi_1\|_{H_0^1(\Omega)}\right).\end{aligned}$$

This yields (4.1) if β_3 is small enough such that $C_3 - \beta_3 C_5 C_6 > 0$. ☐

Now we are ready to prove the theorem.

Proof of Theorem 4.1. We are going to use controls of the form

$$u = \begin{cases} -\Delta\varphi & \text{on } \Sigma_+, \\ 0 & \text{on } \Sigma \setminus \Sigma_+, \end{cases} \tag{4.2}$$

where φ is the first component of the solution of the adjoint system with initial data φ_0, φ_1 satisfying (2.4) and $\psi_0 = 0$. By Propositions 2.1, 3.3 and Corollary 3.2 we obtain a bounded linear map

$$(\varphi_0, \varphi_1) \mapsto (w'(T) - \gamma\Delta w'(T), -w(T) + \gamma\Delta w(T)) \tag{4.3}$$

from $H_0^2(\Omega) \times H_0^1(\Omega)$ into its dual space $H^{-2}(\Omega) \times H^{-1}(\Omega)$.

Moreover, the identity (2.2) takes the form

$$\int_\Omega w'(\varphi - \gamma\Delta\varphi) + w(\varphi' - \gamma\Delta\varphi')\, dx\,|_{t=T} = \int_{\Sigma_+} |\Delta\varphi|^2 \, d\Sigma. \tag{4.4}$$

Let us choose $\beta_1\beta_3$ sufficiently small, so that the inequality (4.1) of Proposition 4.2 is satisfied. Then an application of the Lax–Milgram lemma implies that the linear map (4.3) is onto. Since, moreover, the operator $(I - \gamma\Delta)^{-1}$ maps $H^{-2}(\Omega)$ onto $L^2(\Omega)$ and $H^{-1}(\Omega)$ onto $H_0^1(\Omega)$, we will conclude that $H_0^1(\Omega) \times L^2(\Omega) \subset R$. Since the converse inclusion has already been established, the proof is complete. ☐

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