A person needing a kidney transplant may have a friend or relative who volun-
unteers to be a living donor, but whose kidney is incompatible, forcing the person
to wait for a transplant from a deceased donor. In the U.S. alone, thousands of
people die each year without ever finding a suitable kidney. A new technique
applies graph theory to groups of incompatible patient-donor pairs to create the
largest possible number of paired-donation exchanges. These exchanges, in which
a donor paired with Patient A gives a kidney to Patient B while a donor paired
with Patient B gives to Patient A, will dramatically increase transplants from living
donors. Since transplantation is less expensive than dialysis, this mathematical
algorithm, in addition to saving lives, will also save hundreds of millions of dollars
annually.

Naturally there can be more transplants if matches along longer patient-donor
cycles are considered (e.g., A’s donor to B, B’s donor to C, and C’s donor to
A). The problem is that the possible number of longer cycles grows so fast—
hundreds of millions of A→B→C→A matches in just 5000 donor-patient
pairs—that to search through all the possibilities is impossible. An ingenious
use of random walks and integer programming now makes searching through all
three-way matches feasible, even in a database large enough to include all incom-
patible patient-donor pairs.

For More Information:
“Matchmaking for Kidneys,” Dana
Mackenzie, SIAM News, December
2008.

Image of suboptimal two-way matching (in
purple) and an optimal matching (in green),
courtesy of Sommer Gentry.