CORRECTION TO THE PAPER

ON THE ZEROS OF POLYNOMIALS OVER DIVISION RINGS

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On p. 226 (**116** (1965), 218–226) it is stated that the curve $\xi_2^3 = (\xi_1 + 1)^3 - \xi_1^3$ has exactly nine points in $Q(\sqrt{-3})$. However, one of these is at infinity; hence $N(f) = 16$ rather than 18 for this example. An example of a quadratic polynomial over D with $16 < N(f) < \infty$ is

$$f(x) = (\xi_3 - \xi_2^2)e_1 + (\xi_1^2 - \xi_2\xi_3 - 1)e_2 + (\xi_4^2 - 1)e_3.$$

To find the roots of $f(x)$ we consider the system

$$\xi_3 = \xi_2^2,$$

$$\xi_1^2 = \xi_2\xi_3 + 1,$$

$$\xi_4^2 = 1.$$

Eliminating ξ_3 we obtain

$$\xi_1^2 = \xi_2^3 + 1. \tag{1}$$

Euler [1] proved in 1738 that the only solutions of (1) in Q are

$$(\xi_1, \xi_2) = (0, -1), (\pm 1, 0), \text{ and } (\pm 3, 2).$$

Hence by a theorem of Billing [2], there are only finitely many solutions of (1) in $Q(\sqrt{-3})$. These include the eleven pairs $(\xi_1, \xi_2) = (0, -\omega)$, $(\pm 1, 0)$, and $(\pm 3, 2\omega)$, where ω is any cube root of unity. Corresponding to each (ξ_1, ξ_2) there is a unique value of ξ_3, and two values for ξ_4. Thus $22 \leqq N(f) < \infty$.

REFERENCES

1. L. Euler, *Opera omnia*, Ser. 1, Vol. 2, Teubner, Leipzig, 1915; pp. 56–57.

2. G. Billing, *Über kubische diophantische Gleichungen mit endlich vielen Lösungen*, Comment. Math. Helv. **9** (1936), 161–165.

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