INERTIAL AUTOMORPHISMS OF A CLASS OF WILDLY RAMIFIED v-RINGS

BY
NICKOLAS HEEREMA[1]

I. **Introduction.** Let R_e be a ramified v-ring with ramification e. That is, R_e is a complete, discrete, rank one, valuation ring having characteristic zero with residue field k of characteristic p ($p \neq 0, 2$) and pR_e is the eth power of the maximal ideal M of R_e. Let $\mathfrak{G}$ represent the group of automorphisms of R_e, ε being the identity map. Then, for $i>0$, $\mathfrak{G}_i=\{\alpha \mid \alpha \in \mathfrak{G}; \alpha = \varepsilon, \bmod M^i\}$ and $\mathfrak{H}_i=\{\alpha \mid \alpha \in \mathfrak{G}_i, \alpha(m)-m \in M^{i+1}$ for $m \in M\}$. The ramification groups $\mathfrak{G}_i$ and $\mathfrak{H}_i$ are invariant in $\mathfrak{G}$. The object of this paper is to evaluate the factor groups of the series (1) of

$$\mathfrak{G}_i \supseteq \mathfrak{H}_1 \supseteq \mathfrak{G}_2 \supseteq \mathfrak{H}_2 \supseteq \cdots \tag{1}$$

ramification groups in that case in which $e=p$. A second objective is the determination of those automorphisms in $\mathfrak{G}_1$ which are derivation automorphisms (see below).

Neggers has shown [3, Theorem 6] that for any e and $i \geqq (e+p)/(p-1)$, $\mathfrak{G}_i/\mathfrak{G}_{i+1}$ is isomorphic to $\mathscr{D}(R_e)/\pi\mathscr{D}(R_e)$ where $\mathscr{D}(R_e)$ is the additive group of derivations on the ring R_e and $\pi\mathscr{D}(R_e)=\{\pi d \mid d \in \mathscr{D}(R_e)\}$ where π is a prime element in R_e. In addition he proved that $\mathfrak{H}_i/\mathfrak{G}_{i+1}$ is isomorphic to the additive group of those derivations on k which lift to R_e where again $i \geqq (e+p)/(p-1)$. The map used by Neggers to evaluate $\mathfrak{G}_i/\mathfrak{G}_{i+1}$ also shows that if $i \geqq (e+p)/(p-1)$, then $\mathfrak{G}_i/\mathfrak{H}_i$ is isomorphic to $\mathscr{D}(R_e)/\mathscr{D}^*(R_e)$ where $\mathscr{D}^*(R_e)=\{d \mid d \in \mathscr{D}(R_e), d(\pi) \in \pi R_e\}$ [3, proof of Theorem 6]. The principal tool of this investigation is the convergent higher derivation [2]. Let $D=\{D_i\}_{i=1}^{\infty}$ be a higher derivation on $R_e (D_i(R_e) \subset R_e$ for $i>0)$. D is convergent if, for $a \in R_e$, $\sum D_i(a)$ is a convergent series in the π-adic topology. If D converges the map $\alpha_D\colon a \to \sum_{i=0}^{\infty} D_i(a) (D_0(a)=a)$ is an inertial automorphism (see Theorem B). The group $\mathfrak{G}_D$ of all *derivation automorphism* α_D is an invariant subgroup of $\mathfrak{G}$.

Throughout this paper R will denote a v-ring in R_e such that $[R_e : R]=e$, and R is unramified. Thus R has the same residue field k as R_e. For a in R_e, $\bar{a}$ will denote the image of a under the natural map of R_e onto k, π always represents a prime element in R_e. We have

$$\pi^e + pu = 0, \qquad \bar{u} \neq 0. \tag{2}$$

If $e=p$ and $\bar{u} \in k^p$ then π can be chosen so that

$$\pi^p + p(1+\pi^t v) = 0, \qquad t > 0, \quad \bar{v} \neq 0, \tag{3}$$

or $\pi^p + p = 0$. We note that the conditions

$$\bar{u} \notin k^p;\ t = p \quad \text{and} \quad \bar{v} \notin k^p;\ \text{as well as } \bar{v} \text{ if } 1 \leqq t < p \tag{4}$$

Presented to the Society, September 1, 1967; received by the editors February 16, 1967.
[1] This research supported by NSF GP-4007.

are all independent of the choice of π, assuming (3) to be satisfied for all but the first listed. Throughout this paper the symbols u and v will represent the quantities given in (2) and (3). The considerations (4) determine, to a large extent, the structure of the groups (1) as is seen in the following two theorems which summarize the results of this study. $\mathfrak{G}(R_p; R)$ is the group of automorphism of R_p over R.

THEOREM 1. *Every inertial automorphism on R_p is a derivation automorphism, i.e. $\mathfrak{G}_1=\mathfrak{G}_D$, unless $\bar{u}\in k^p$ and $t=p-1$, in which case the following are equivalent.*

(a) *$\bar{v}$ is a* $(p-1)$th *root in k.*

(b) *R_p is Galois over R.*

(c) $\mathfrak{G}_2\neq\mathfrak{H}_2$.

(d) *$\mathfrak{G}_2|\mathfrak{H}_2$ is the group of order p.*

If R_p is Galois over R then $\mathfrak{G}(R_p; R)\subset\mathfrak{G}_D$ if and only if $\bar{v}\notin k^p$. In any case, $\mathfrak{G}_1=\mathfrak{G}_D\cdot\mathfrak{G}(R_p; R)$.

THEOREM 2. *If $\bar{u}\notin k^p$ then for $i\geqq 1$, $\mathfrak{H}_i/\mathfrak{G}_{i+1}$ is isomorphic to the subgroup $\overline{\mathscr{D}}$ of those $\delta\in\mathscr{D}(k)$ which lift to R_p. Also, $\overline{\mathscr{D}}=\{\delta\mid\delta\in\mathscr{D}(k),\ \delta(\bar{u})=0\}$. In this case $\mathfrak{G}_i/\mathfrak{H}_i$ is isomorphic to k^+, the additive group of k.*

If $\bar{u}\in k^p$, then for $i\geqq 1$, $\mathfrak{H}_i/\mathfrak{G}_{i+1}$ is isomorphic to $\mathscr{D}(k)$ unless $t=p$ and $i=1$. If $t=p$, $\mathfrak{H}_1/\mathfrak{G}_2$ is isomorphic to the subgroup of those $\delta\in\mathscr{D}(k)$ such that $\delta(\bar{v})=0$. Also, $\mathfrak{G}_i=\mathfrak{H}_i$, $i\geqq 1$, unless $t=p-1$, $i=2$ and one of the four equivalent conditions of Theorem 1 *holds.*

By Neggers' results referred to above [3, proof of Theorem 6] we have

COROLLARY. *$\mathscr{D}(R_p)/\mathscr{D}^*(R_p)$ is isomorphic to k^+ if $\bar{u}\notin k^p$ and is trivial if $\bar{u}\in k^p$.*

II. **Proofs.** For S a subring of R_e, the symbols $\mathscr{H}(S, R_e)$, $\mathscr{H}_c(S, R_e)$ and $\mathscr{H}_u(S, R_e)$ will stand for the set of all higher derivations, all convergent higher derivations, and all uniformly convergent higher derivations with domain S and range R_e. We quote the following two theorems which will be used repeatedly. Theorem A provides the necessary freedom in the construction of $D\in\mathscr{H}_u(R_e, R_e)$. Theorem B implies that if $D\in\mathscr{H}_c(R_e, R_e)$, then α_D is indeed an inertial automorphism.

THEOREM A [2, *Theorem* 4]. *Let $\overline{\mathscr{S}}$ be a p-basis for k and let $\mathscr{S}\subset R$ be a set of representatives of the elements of $\overline{\mathscr{S}}$. If I is the set of positive integers and f is a mapping from $\mathscr{S}\times I$ into R_e then there is one and only one $D\in\mathscr{H}(R, R_e)$ such that $D_i(s)=f(s, i)$ for all $s\in\mathscr{S}$ and $i\in I$. Moreover, D converges (uniformly) if and only if D converges (uniformly) on $\mathscr{S}$.*

THEOREM B [2, *Lemmas* 1 *and* 5]. *If D is in $\mathscr{H}_c(R_e, R_e)$ then $D_i(R_e)\subset\pi R_e$ and $D_i(\pi R_e)\subset\pi^2 R_e$ for $i>0$.*

Theorems 1 and 2 will be proved by means of a series of lemmas.

LEMMA 1. *If $\mathscr{S}$ is a set of representatives in R of a p-basis $\overline{\mathscr{S}}$ for k and D in $\mathscr{H}(R, R_e)$ is such that $D_j(\mathscr{S}) \subset \pi^{t_j} R_e \subset \pi R_e$, $j \geqq 1$, then $D_i(R) \subset \pi^{q_i} R_e$ where*

$$q_i = \min_{j_1 + \cdots + j_i = i;\, t_0 = 0} (t_{j_1} + \cdots + t_{j_i}), \qquad i \geqq 1.$$

Proof. For a given i we choose n sufficiently large so that $D_j(R^{p^n}) \subset \pi^{q_i} R_e$ for $j = 1, \ldots, i$ [2, Lemma 6] where R^{p^n} is the subring of R generated by the p^nth powers of elements in R. Since every element in k has a representative in $R^{p^n}[\mathscr{S}]$, it follows that $R = R^{p^n}[\mathscr{S}] + p^{q_i} R$. If $b = as_1, \ldots, s_r$ where $a \in R^{p^n}$ and $s_1, \ldots, s_r \in \mathscr{S}$ then

$$D_i(b) = \sum_{i_0 + \cdots + i_r = i} D_{i_0}(a) D_{i_1}(s_1), \ldots, D_r(s_r)$$

is seen to be in $\pi^{q_i} R$ and hence, since $D_j(p^{q_i}) = 0$ for all j, $D_i(R) \subset \pi^{q_i} R_e$.

Let α, an automorphism of R_e, be in $\mathfrak{H}_i$, $i \geqq 1$. Then $\alpha(a) = a + \pi^i \alpha^*(a)$ and the mapping α^* induces a derivation δ_α on k. The mapping

$$\phi_i : \alpha \to \delta_\alpha \tag{5}$$

is a homomorphism of $\mathfrak{H}_i$ into $\mathscr{D}(k)$ with kernel $\mathfrak{G}_{i+1}$.

LEMMA 2. *If an automorphism α of R_p is in $\mathfrak{H}_i$, then $\delta_\alpha(\bar{u}) = 0$. If $\pi^p + p(1 + \pi^p v) = 0$, then $\delta_\alpha(\bar{v}) = 0$ for α in $\mathfrak{H}_1$.*

Proof. Since $\alpha(\pi) - \pi \in \pi^{i+1} R_p$ it follows that $\alpha(\pi^p) - \pi^p \in \pi^{p+i+1} R_p$. Thus by (2) $\alpha(u) - u$ is in $\pi^{i+1} R_p$ or $\alpha^*(u) \in \pi R_p$ which implies $\delta_\alpha(\bar{u}) = 0$. In the remaining case let $\alpha(\pi) = \pi + \pi^2 b$. Then $\alpha(\pi^p) - \pi^p \equiv \pi^{2p} b^p$, mod $\pi^{2p+1} R_p$, and $\alpha(p(1 + \pi^p v)) - p(1 + \pi^p v) \equiv p\pi^{p+1} \alpha^*(v)$, mod $\pi^{2p+2} R_p$. Hence $b \in \pi R_p$ and it follows that $\alpha^*(v) \in \pi R_p$ or $\delta_\alpha(\bar{v}) = 0$.

Given D and H in $\mathscr{H}(R_e, R_e)$, $D \circ H$ in $\mathscr{H}(R_e, R_e)$ is given by

$$(D \circ H)_i = \sum_{j=0}^{i} D_j H_{i-j}.$$

$\mathscr{H}(R_e, R_e)$ is a group with respect to this composition and $\mathscr{H}_c(R_e, R_e)$, $\mathscr{H}_u(R_e, R_e)$ are subgroups [2, Theorems 1, 2]. Moreover, one can verify directly that, for D and H in $\mathscr{H}_c(R_e, R_e)$, $\alpha_{D \circ H} = \alpha_D \alpha_H$.

LEMMA 3. *Let $\{D^{(n)}\}_{n=1}^{\infty}$ where $D^{(n)} \in \mathscr{H}_c(R_e, R_e)$ be such that $D_i^{(n)}(R_e) \subset \pi^{s_n} R_e$, $i \geqq 1$, $n \geqq 1$, and $\lim_n s_n = \infty$. Let $\alpha_n = \alpha_{D^{(1)}}, \ldots, \alpha_{D^{(n)}}$ and $\bar{D}^{(n)} = D^{(1)} \circ \cdots \circ D^{(n)}$. Then $\lim_n \alpha_n(a)$ and $\lim_n \bar{D}_i^{(n)}(a)$ exist for all $i > 0$ and $a \in R_e$. Moreover, $\alpha : a \to \lim_n \alpha_n(a)$ is an automorphism, $D = \{D_i\}$ is in $\mathscr{H}_c(R_e, R_e)$ where $D_i(a) = \lim_n \bar{D}_i^{(n)}(a)$ and $\alpha = \alpha_D$. If $D^{(n)} \in \mathscr{H}_u(R_e, R_e) n \geqq 1$, then $D \in \mathscr{H}_u(R_e, R_e)$.*

Proof. By definition of product in $\mathscr{H}_c(R_e, R_e)$ we have

$$D_n^{(m+1)}(a) - D_n^{(m)}(a) = \sum_{i_1 + \cdots + i_{m+1} = n;\, i_{m+1} \neq 0} D_{i_1}^{(1)} \cdots D_{i_{m+1}}^{(m+1)}(a),$$

the right side of which is in $\pi^{s_{m+1}}R_e$ since for $D \in \mathscr{H}_c(R_e, R_e)$, $D_i(\pi^n R_e) \subset \pi^n R_e$ for all i and n by Theorem B. Thus, $\alpha_{m+1}(a) - \alpha_m(a) = \sum_{i=0}^{\infty} \bar{D}_i^{(m+1)}(a) - \sum_{i=0}^{\infty} \bar{D}_i^{(m)}(a) \in \pi^{s_{m+1}}R_e$. The rest of the lemma follows directly.

Since R_p is totally ramified over R and $[R_p : R] = p$ then $R_p = R[\pi]$ and the minimal polynomial $f(x)$ of π over R is an Eisenstein polynomial, that is,

(6) $$f(x) = x^p + pa_{p-1}x^{p-1} + \cdots + pa_1 x + pa_0$$

and a_0 is a unit. Clearly, $\bar{a}_0 = \bar{u}$ (see (2)). Also, if $\bar{u} \in k^p$ then $a_0 = b^p + pc$ where b and c are in R. By replacing π with $b^{-1}\pi$ we can assume that

(7) $$a_0 = 1 + pb_0.$$

We note next that every $D \in \mathscr{H}(R, R_e)$ extends uniquely to a higher derivation D of the quotient field of R_e. Also, $D(R_e) \subset R_e$ if and only if $D(\pi) \in R_e$. If D converges on R, D will converge on R_e if and only if D converges at π [2, Lemma 3].

Let (r, s) denote an ordered set of r nonnegative integers whose sum is s and let $|(r, s)|$ represent the largest integer in (r, s). We let $\sum_{(q,s)} D(a_1, \ldots, a_q)$ denote the sum of all products $D_{i_1}(a_1)D_{i_2}(a_2) \cdots D_{i_q}(a_q)$ such that $i_1 + \cdots + i_q = s$ and $i_j \geqq 0$. Also, $f'(x)$ and $f^{D_i}(x)$ represent respectively the ordinary derivative of f and the polynomial obtained by replacing each coefficient in f with its image under D_i. With these conventions it is useful to write the expression for $D_i(\pi)$ derived from $D_i(f(\pi)) = 0$ as follows:

(8, i) $$f'(\pi)D_i(\pi) = f^{D_i}(\pi) + \sum_{(p,i);\, |(p,i)| < i} D(\pi, \ldots, \pi) + \sum_{j=0}^{p-1} p \sum_{(j+1,i);\, |(j+1,i)| < i} D(a_j, \pi, \ldots, \pi).$$

Let v represent the exponential valuation on R_p. Note that $p \leqq v(f'(\pi)) \leqq 2p - 1$.

LEMMA 4. *A given $\delta \in \mathscr{D}(k)$ lifts to $d \in \mathscr{D}(R_p)$ if and only if $\delta(\bar{u}) = 0$.*

Proof. A derivation $d \in \mathscr{D}(R_p)$ induces a derivation on k under the natural map of R_p onto k only if $d(\pi) \subset \pi R_p$. But $d(f(\pi)) = 0$ means $d(\pi) = -f^d(\pi)/f'(\pi)$. Thus $f^d(\pi) \in \pi^{p+1}R_p$ which means $d(a_0) \in \pi R_p$ and, hence, if d induces δ on k, $\delta(\bar{a}_0) = \delta(\bar{u}) = 0$. Conversely, every $\delta \in \mathscr{D}(k)$ lifts to d' on R [1, Theorem 1]. If $\delta(\bar{u}) = 0$, $d'(a_0) \in pR$ which means that $f^{d'}(\pi)/f'(\pi) \in \pi R_p$. Thus the extension d, of d', to R_p is in $\mathscr{D}(R_p)$ and induces δ since d' does.

LEMMA 5. *Let $D \in \mathscr{H}_c(R, R_p)$ where $R_p = R[\pi]$ and $f(x) = x^p + \sum_{i=0}^{p-1} pa_i x^i$ is the minimum function of π over R. Let $q > 2$, $n \geqq 1$, and $m > p(n-1)$ be integers such that, using the same symbol for the extension of D to R_p,*

(9, 1) $$D_j(\pi) \in \pi^2 R_p \qquad \text{if } j < n,$$

(9, 2) $$D_j(\pi) \in \pi^q R_p \qquad \text{if } n \leqq j < m,$$

and, if $j \geqq n$

(9, 3) $$D_j(a_s) \in f'(\pi)\pi^{q-p-s}.$$

If $q=3$ *then* (9, 3) *is assumed to hold only for* $j \geqq m$. *Under these assumptions* $\sum D_j(\pi)$ *converges and*

$$\sum_{j=n}^{\infty} D_j(\pi) \in \pi^q R_p.$$

The proof of this lemma consists of checking the valuation v of the terms on the right side of (8). We show first that if $j \geqq m$,

$$D_j(\pi) \in \pi^q R_p. \tag{10}$$

Thus assuming (10) true for $j<r$ where $r \geqq m$ we consider (8, r). By (9, 3) $f^{D_r}(\pi)$ is in $f'(\pi)\pi^q R_p$. The term $D_{i_1}(\pi) \cdots D_{i_p}(\pi)$ of $A_r = \sum_{(p,r);|(p,r)|<r} D(\pi, \ldots, \pi)$ is in $\pi^{p+q}R_p$ in view of the fact that at least one $i_j > n$ and another is different from zero. The above term appears in A_r a multiple of p times unless $i_1 = i_2 = \cdots = i_p = r/p$ in which case it is in $\pi^{pq}R_p \subset f'(\pi)\pi^q R_p$, since $v(f'(\pi)) \leqq 2p-1$ and $p \neq 2$. Thus $A_r \in f'(\pi)\pi^q R_p$. A similar argument shows B_r, the remaining term on the right side of (8, r), to be in $f'(\pi)p^q R_p$. Thus by (8, r) $D_r(\pi) \in \pi^q R_p$.

Given $i \geqq 0$, we assume for some integer $s \geqq m$ that if $j>s$, then $D_j(\pi) \in \pi^{i+q}R_p$ and, for $h=0, \ldots, p-1$

$$D_j(a_h) \in f'(\pi)\pi^{i+q-p+1}R_p. \tag{11}$$

Let $s'=ps$ and let $j>s'$. Then $f^{D_j}(\pi) \in f'(\pi)\pi^{i+q+1}R_p$ and, by an analysis like that above, A_j and B_j are seen to be in $f'(\pi)\pi^{i+q+1}R_p$. Thus $D_j(\pi) \in \pi^{i+q+1}R_p$. Since D converges on R, given $i \geqq 0$, there is an s such that (11) holds for $j>s$. It follows that $\sum D_j(\pi)$ converges, and in view of (10) $\sum_{j=n}^{\infty} D_j(\pi) \in \pi^q R_p$.

LEMMA 6. *If* $\alpha \in \mathfrak{H}_i$, $i \geqq 1$, *then there is a* $D \in \mathscr{H}_u(R_p, R_p)$ *such that* $\alpha^{-1}\alpha_D \in G_{i+1}$. *Moreover,* $\mathfrak{H}_i/\mathfrak{G}_{i+1}$ *is isomorphic to the subgroup of those* δ *in* $\mathscr{D}(k)$ *for which* $\delta(\bar{u})=0$ *with the following exception. If* $\bar{u} \in k^p$ *and, for suitable choice of* π *we have* $\pi^p = p(1+\pi^p v)$ *then* $\mathfrak{H}_1/\mathfrak{G}_2$ *is isomorphic to the subgroup of those*

$$\delta \in \mathscr{D}(k) \ni \delta(\bar{v}) = 0.$$

Proof. By Lemma 2, it will be sufficient to find $D \in \mathscr{H}_u(R_p, R_p)$ such that $\phi_i(\alpha_D)$ (see (5)) is a given δ for which $\delta(\bar{u})=0$, or, in the exceptional case, $\delta(\bar{v})=0$. Let (6) be the minimum function of π over R.

Case 1. $v(f'(\pi))<2p-1$, $i>1$. Let δ be any derivation on k for which $\delta(\bar{u}) = \delta(\bar{a}_0)=0$ and let $H=\{H_j\}$ be any higher derivation in $\mathscr{H}(R, R)$ satisfying the two conditions (a) H_1 induces δ, (b) $H_j(a_0) \in pR$, $j=1, \ldots, p-1$. Specifically, every derivation on k lifts to R [1, Theorem 1] which fact makes H_1 available. Let $H_j = H_1^j/j!$ for $j=2, \ldots, p-1$. By Theorem A, maps H_j, $j \geqq p$, can be defined so that $H=\{H_j\} \in \mathscr{H}(R, R)$. Let $D=\{D_j\}$ where

$$D_j = \pi^{ji}H_j. \tag{12}$$

Clearly, $D \in \mathscr{H}_u(R, R_p)$. We now show that

$$D_j(\pi) \in \pi^{i+1}R_p, \qquad j \geqq 1, \tag{13}$$

and

$$\sum D_j(\pi) \text{ converges,} \tag{14}$$

from which, with (12), it will follow that, using the same symbol for the extended higher derivation, $D \in \mathscr{H}_u(R_p, R_p)$, $\alpha_D \in \mathfrak{H}_i$ and $\Phi_i(\alpha_D)=\delta$.

Let $v(f'(\pi))=p+r-1$. Thus r is the least positive integer such that a_r is a unit. Looking to the conditions of Lemma 5 we note that $f'(\pi)\pi^{i+1-p-s}R_p=\pi^{i+r-s}R_p$. If $s \geqq r$, $D_j(a_s) \in \pi^{ji}R_p \subset \pi^{i+r-s}R_p$ for $j \geqq 1$. If $r>s>0$, $a_s \in pR$; hence, $H_j(a_s) \in pR$ and thus $D_j(a_s) \in \pi^{ij+p}R_p \subset \pi^{i+r-s}R_p$ for $j \geqq 1$. Finally, $D_j(a_0)=\pi^{ij}H_j(a_0)$. If $j<p$, $H_j(a_0) \in pR_p$ and $\pi^{ij}H_j(a_0) \in \pi^{ij+p}R_p \subset \pi^{i+r}R_p$. If $j \geqq p$, $ij \geqq i+r$. Hence, $D_j(a_0) \in \pi^{i+r}R_p$ for $j \geqq 1$. Thus conditions (9, 1) to (9, 3) are satisfied with $q=i+1$ and $n=m=1$. Hence (13) and (14) hold.

Case 2. $v(f'(\pi))<2p-2$, $i=1$. We define D as in Case 1 and note by inspection of (8, j) for $j=1,\ldots,p$ that $D_1(\pi) \in \pi^2R_p$, $D_j(\pi) \in \pi^3R_p$ for $j=2,\ldots,p$. Also, in this case, $f'(\pi)\pi^{3-p-s}R_p \supset \pi^{p-s}R_p \supset \pi^pR_p$ and, by (12) $D_j(a_s) \in \pi^pR_p \subset f'(\pi)\pi^{3-p-s}R_p$ for $j \geqq p$ and $s=0,\ldots,p-1$. Thus conditions (9, 1) to (9, 3) are satisfied for $n=2$, $m=p+1$ and $q=3$. Hence $\sum D_i(\pi)$ converges and is in π^2R_p. Thus, $\alpha_D \in \mathfrak{H}_1$ and $\phi_1(\alpha_D)=\delta$.

Case 3. $v(f'(\pi))=2p-2$, $i=1$. We consider a number of subcases. In each case D is constructed by the method of Theorem A.

(3, i). $\bar{a}_{p-1} \notin k^p$, $\bar{a}_0 \notin k^p$, *$\bar{a}_{p-1}$ and $\bar{a}_0$ p-independent.* As before, we initiate the construction of $D \in \mathscr{H}_u(R, R_p)$ by letting $D_j=\pi^jH_j$, $j=1,\ldots,p-1$, where $\{H_j\}_1^{p-1}$ are chosen so that H_1, a derivation on R_p induces a given $\delta \in \mathscr{D}(k)$ such that $\delta(\bar{a}_0)=0$ and $H_j(a_0) \in pR$ for $j=2,\ldots,p-1$. Let $\mathscr{S}$ be a set of representatives in R of a p-basis $\overline{\mathscr{S}}$ of k. We may assume both a_0 and a_{p-1} in $\mathscr{S}$. By inspection of (8, 1) to (8, $p-1$) we have $D_1(\pi) \in \pi^2R_p$ and $D_j(\pi) \in \pi^3R_p, j=2,\ldots,p-1$. Considering (8, p), each summand of A_p and B_p is in $\pi^{2p+1}R_p$, except the term $[D_1(\pi)]^p$ which is in $\pi^{2p}R_p$ but not in $\pi^{2p+1}R_p$. Thus, we define D_p by $D_p(s)=0$ for $s \in \mathscr{S}-\{a_0\}$ and $D_p(a_0)$ is so chosen that $f^{D_p}(\pi)+A_p+B_p$ is in $f'(\pi)\pi^3R_p$. Thus, $D_p(a_0) \in \pi^pR_p$ and $D_p(\pi) \in \pi^3R_p$. For $j>p$ we let $D_j(s)=0$ for $s \in \mathscr{S}$. By Lemma 1 $D_j(a_s) \in \pi^{p+1}R_p$ for $j>p$. It follows that conditions (9) of Lemma 5 are fulfilled for $n=2$, $m=p+1$ and $q=3$. Thus by Theorem A the extension of D to R_p converges uniformly $\alpha_D \in \mathfrak{H}_1$ and $\Phi_1(\alpha_D)=\delta$.

(3, ii) $\bar{a}_{p-1} \notin k^p$, $\bar{a}_0 \notin k^p$, *$\bar{a}_{p-1}$ and $\bar{a}_0$ p-dependent.* Let $H_1 \in \mathscr{D}(R)$ induce $\delta \in \mathscr{D}_k$ where $\delta(\bar{a}_0)=0$ (Lemma 4) and let $\mathscr{S}$ be a set of representatives of a p-basis for k which contains a_0. $H \in \mathscr{H}_u(R, R)$ is defined by the conditions $H_j(s)=0$ for $j>1$ and $s \in \mathscr{S}$. Let $D_j=\pi^jH_j$ for $j \geqq 1$. Now $H_1(a_{p-1}) \in pR_p$ since $H_1(a_0) \in pR_p$ and the elements $\bar{a}_0$, $\bar{a}_{p-1}$ are p-dependent. Thus by (8, 1), $D_1(\pi) \in \pi^3R_p$. Also $D_j(a_s) \in f'(\pi)\pi^{3-p-s}R_p=\pi^{p+1-s}R_p$ for $j \geqq 1$, $s=0,\ldots,p-1$. Thus, conditions (9) of Lemma 5 are satisfied for $n=1$, $m=2$, $q=3$ and again $D \in \mathscr{H}_u(R_p, R_p)$, $\alpha_D \in \mathfrak{H}_1$ and $\Phi_1(\alpha_D)=\delta$.

(3, iii) $\bar{a}_{p-1} \in k^p$, $\bar{a}_0 \notin k^p$. A higher derivation D in $\mathscr{H}_u(R, R_p)$ is chosen as in (3, ii). Since $a_{p-1}=b_{p-1}^p+pc$, $H_1(a_{p-1}) \in pR_p$. Thus $D_1(\pi) \in \pi^3R_p$ and for the rest the argument of (3, ii) applies.

(3, iv) $\bar{a}_{p-1} \notin k^p$, $\bar{a}_0 \in k^p$. We choose π so that $a_0=1+pb_0$. Let $\mathscr{S}$ be a set of representatives of a p-basis for k. We can assume a_{p-1} in $\mathcal{S}$. Let H_1 in $\mathcal{D}(R)$ induce δ in $\mathcal{D}(k)$. For $j=2,\ldots,p-1$ and $s\in\mathcal{S}$ we let $H_j(s)=0$. For $j=1,\ldots,p-1$ let $D_j=\pi^j H_j$. By (8, 1), $D_1(\pi)\in\pi^2R_p$ ($D_1(\pi)\in\pi^3R_p$ unless $D_1(a_{p-1})\notin\pi^2R_p$). Also, $D_j(\pi)\in\pi^3R_p$ for $j=2,\ldots,p-1$. The terms A_p+B_p of (8, p) have $[D_1(\pi)]^p$ as the unique summand of minimum valuation, if $D_1(\pi)\notin\pi^3R_p$. In any case, D_p is defined by $D_p(s)=0$ for $s\in\mathcal{S}$, $s\neq a_{p-1}$ and $D_p(a_{p-1})\in\pi^2R_p$ is chosen so that $D_p(\pi)$ is in π^3R_p. Finally $D_j(s)=0$ for $s\in\mathcal{S}$ and $j>p$. Again by Theorem A these conditions determine D in $\mathcal{H}_u(R, R_p)$. By Lemma 1 $D_j(R)\subset\pi^3R_p$ for $j>p$. Again we invoke Lemma 5 with $n=2$, $m=p+1$ and $q=3$ to show that $D\in\mathcal{H}_u(R_p, R_p)$, $\alpha_D\in\mathfrak{H}_1$ and $\Phi_1(\alpha_D)=\delta$.

(3, v) $\bar{a}_0\in k^p$, $\bar{a}_{p-1}\in k^p$. Again it may be assumed that $a_0=1+pb_0$. We choose any $H\in\mathcal{H}(R, R)$ such that H_1 induces a given $\delta\in\mathcal{D}(k)$ and let $D_j=\pi^jH_j$, $j\geqq 1$. Lemma 5 applies with $n=1$, $m=2$ and $q=3$.

Case 4. $v(f'(\pi))=2p-1$, $\bar{a}_0\notin k^p$, $i>1$. Let $H\in\mathcal{H}(R, R)$ be chosen so that $H_j(a_0)\in pR_p$ for $j=1,\ldots,p-1$ and H_1 induces a given $\delta\in\mathcal{D}(k)$ for which $\delta(\bar{a}_0)=0$. Let $D=\{\pi^{ij}H_j\}$. Since by (8, 1) $D_1(\pi)\in\pi^{i+1}R_p$ and, by inspection, $D_j(a_s)\in f'(\pi)\pi^{i+1-p}$, $s=0,\ldots,p-1$, $j\geqq 1$, we see by Lemma 5 that $\sum D_j(\pi)\in\pi^{i+1}R_p$. Thus $\alpha_D\in\mathfrak{H}_i$ and $\Phi_i(\alpha_D)=\delta$. Clearly, $D\in\mathcal{H}_u(R_p, R_p)$.

Case 5. $v(f'(\pi))=2p-1$, $\bar{a}_0\in k^p$, $i>1$. We can assume that $a_0=1+pb_0$. Let $H\in\mathcal{H}(R, R)$ be such that H_1 induces a given $\delta\in\mathcal{D}_k$. Let $D=\{\pi^{ij}H_j\}$ and argue as above.

Case 6. $v(f'(\pi))=2p-1$, $\bar{a}_0\notin k^p$, $i=1$. Let $\delta\in\mathcal{D}(k)$, $\delta(\bar{a}_0)=0$, and let H_1 in $\mathcal{D}(R)$ induce δ. Let $\mathcal{S}$ be a set of representatives in R of a p-basis for k with a_0 in $\mathcal{S}$. We define $K_1\in\mathcal{D}(R)$ as follows: $K_1(a_0)=\pi^{-p}(H_1(a_0))$ and $K_1(s)=0$ for $s\in\mathcal{S}$, $s\neq a_0$. By Theorem A, these conditions determine a derivation on R. The derivation $D_1=\pi H_1-\pi^{p+1}K_1$ has the property $D_1(a_0)=0$ and is the first map of $D\in\mathcal{H}_u(R, R_p)$. For the rest, we define $D_j(s)=0$ for $s\in\mathcal{S}$ and $j>1$. By Theorem A, $D\in\mathcal{H}_u(R, R_p)$. By Lemma 1 $D_1(R)\subset\pi R_p$ and $D_j(R)\subset\pi^2R_p$ for $j\geqq 1$. The conditions of Lemma 5 are fulfilled for $n=1$, $m=1$ and $q=3$. Moreover, $\phi_1(\alpha_D)=\delta$.

Case 7. $v(f'(\pi))=2p-1$, $\bar{a}_0\in k^p$, $i=1$. Again, π is chosen so that $a_0=1+pb_0$. We have the situation (3) with $t\geqq p$ and $\bar{v}=\bar{b}_0$. Thus, in deference to Lemma 2, we choose $\delta\in\mathcal{D}(k)$ so that $\delta(\bar{b}_0)=0$ and let $H_1\in\mathcal{D}(R)$ induce δ. Let $H\in\mathcal{H}(R, R)$ be any higher derivation on R with the given H_1 as the first map. Let $D=\{D_j\}$ where $D_j=\pi^jH_j$, $j\geqq 1$. Let $n=m=1$, $q=3$ in Lemma 5 and we conclude that $\sum D_j(\pi)\in\pi^3R_p$. Again we have the desired conclusion and Lemma 6 is proved.

The next series of lemmas are concerned with automorphisms in the "gap" between $\mathfrak{G}_i$ and $\mathfrak{H}_i$.

LEMMA 7. *If π is a prime element of R_p and $\pi^p=-pu$ where $\bar{u}\notin k^p$, then, given $i\geqq 2$, there is a $D\in\mathcal{H}_u(R_p, R_p)$ such that $\alpha_D\in\mathfrak{G}_i$ and $\alpha_D(\pi)=\pi+\pi^ia$ where $\bar{a}$ is any given element of k. Hence $\mathfrak{G}_i/\mathfrak{H}_i$ is isomorphic to k^+.*

Proof. We assume (6) to be the minimum function of π over R and thus $\bar{a}_0\notin k^p$.

Let $\mathscr{S}$ be a set of representatives in R of a p-basis for k with $a_0 \in \mathscr{S}$. With a chosen arbitrarily in R_p we define a derivation D_1 mapping R into R_p by $D_1(a_0) = -p^{-1}f'(\pi)\pi^i a$ and $D_1(s)=0$ for $s \in \mathscr{S}$, $s \neq a_0$. Then $D_1(R) \subset f'(\pi)\pi^{i-p}R_p$ by Lemma 1 and by (8.1), $D_1(\pi) \equiv \pi^i a$, mod $\pi^{i+1}R_p$. Let $D_j(s)=0$, $s \in \mathscr{S}$, $j=2,\dots,p-1$. If $i=2$, the term $[D_1(\pi)]^p$ in A_p of (8, p) makes it necessary to consider cases.

Case 1. $i>2$ or $v(f'(\pi))<2p-2$. In this case we let $D_j(s)=0$, $s \in \mathscr{S}$, $j>p-1$. Thus, by Lemma 1, $D_j(R) \subset [f'(\pi)\pi^{i-p}]^j R_p$, $j \geqq 1$, and if $j>1$, $D_j(R) \subset f'(\pi)\pi^{i+1-p}$ since $f'(\pi) \in \pi^p R_p$. The conditions of Lemma 5 are fulfilled for $n=2$, $m=p+1$ and $q=i+1$. Thus D extends to R_p, is uniformly convergent on R_p and $\sum_{j=2}^{\infty} D_j(\pi) \in \pi^{i+1}R_p$. In particular then, $\sum_{j=1}^{\infty} D_j(\pi) \equiv \pi^i a$, mod $\pi^{i+1}R_p$.

Case 2. $i=2$, $v(f'(\pi)) \geqq 2p-2$. In this case we choose $D_p(s)=0$, $s \in \mathscr{S}$, $s \neq a_0$ and $D_p(a_0) \in \pi^p R_p$ so that $D_p(\pi)$ will be in $\pi^3 R_p$. Again, we let $D_j(s)=0$ for $j>p$, $s \in \mathscr{S}$ and apply Lemma 5 with $n=2$, $m=p+1$ and $q=3$, obtaining the same conclusion as in Case 1.

The map τ_i: $\mathfrak{G}_i \to k^+$ given by $\tau_i(\alpha)=\bar{a}$ where $\alpha(\pi)=\pi+\pi^i a$, is a homomorphism with kernel $\mathfrak{H}_i$ and evidently maps onto k^+ if $i \geqq 2$.

LEMMA 8. *If π is a prime element of R_p, $\pi^p=-pu$, and $\bar{u} \in k^p$ then $\mathfrak{G}_i=\mathfrak{H}_i$ for $i>1$ unless $i=2$ and t of* (3) *is $p-1$. If $t=p-1$ the following are equivalent.*

(a) *$\bar{v}$ has a $(p-1)$th root in k.*

(b) *R_p is Galois over R.*

(c) $\mathfrak{G}_2 \neq \mathfrak{H}_2$.

(d) *$\mathfrak{G}_2/\mathfrak{H}_2$ is the group of order p.*

Proof. Let α be in $\mathfrak{G}_i$. Then $\alpha=\varepsilon+\pi^i\alpha^*$. The relation

$$[\alpha(\pi)]^p-\pi^p = p[1+[\alpha(\pi)]^t\alpha(v)]-p(1+\pi^t v)$$

becomes

$$\begin{aligned} (15)\qquad & p\pi^{i+p-1}\alpha^*(\pi)+\cdots+\pi^{ip}[\alpha^*(\pi)]^p \\ &= p[\pi^t+t\pi^{t-1+i}\alpha^*(\pi)+\cdots+\pi^{ti}(\alpha^*(\pi))^t]\pi^i\alpha^*(v) \\ &\quad +p[t\pi^{t-1+i}\alpha^*(\pi)+\cdots+\pi^{ti}(\alpha^*(\pi))^t]v. \end{aligned}$$

If $i>2$ the unique term having minimal valuation on the left side of (15) is $p\pi^{i+p-1}\alpha^*(\pi)$. If $p \nmid t$ the unique term of minimal valuation on the right is $pt\pi^{t-1+i}v\alpha^*(\pi)$, unless $\alpha^*(\pi)$ is in πR_p. Thus, either $\alpha^*(\pi) \in \pi R_p$ or $t+i-1=p+i-1$, which cannot be. Thus, if $i>2$ and $p \nmid t$, then $\alpha \in \mathfrak{H}_i$ or $\mathfrak{G}_i=\mathfrak{H}_i$. If $p|t$ and $i \geqq 2$ the left side of (15) has valuation less than the right side unless $\alpha^*(\pi) \in \pi R_p$. Thus again $\mathfrak{G}_i=\mathfrak{H}_i$.

If $i=2$ and $p \nmid t$, the unique term of minimal valuation on the left side of (15) is $\pi^{2p}[\alpha^*(\pi)]^p$, assuming $\alpha^*(\pi)$ to be a unit. The corresponding term on the right is $pt\pi^{t+1}\alpha^*(\pi)v$. Thus, $2p=p+t+1$ or $t=p-1$. So, if $t \neq p-1$, $\mathfrak{G}_2=\mathfrak{H}_2$. If $t=p-1$, then by (15), $\pi^{2p}[\alpha^*(\pi)]^p \equiv p(p-1)\pi^p\alpha^*(\pi)v$, mod $\pi^{2p+1}R_p$, or, using (3), $[\alpha^*(\pi)]^{p-1} \equiv (p-1)v$, mod πR_p. Thus, $(p-1)\bar{v}$, or $\bar{v}$, is a $(p-1)$th root in k and the residue of $\alpha^*(\pi)$ is a $(p-1)$th root of $(p-1)\bar{v}$. We have shown that (c) $\to$ (d) $\to$ (a). A theorem of Wishart [4, Theorem 4.15] asserts that (a) $\to$ (b).

Suppose, finally, that α in $\mathfrak{G}_1$ leaves R element-wise fixed. Then, if $\alpha(\pi)=\pi+\pi^i b$, $\alpha\in\mathfrak{G}_i$. Thus, if $\alpha\neq\varepsilon$, then $\alpha\in\mathfrak{G}_r$, $\alpha\notin\mathfrak{H}_r$ for some $r>1$. Evidently, $r=2$ and (b) $\to$ (c). This fact was also observed by Wishart [4, Corollary 4.16] who noted that if $\bar{u}\in k^p$ then R_p is Galois over R if and only if $\mathfrak{G}_2\neq\mathfrak{H}_2$. It follows from Lemma 7 that if $\bar{u}\notin k^p$, then $\mathfrak{G}_2$ can be different from $\mathfrak{H}_2$ without R_p being Galois over R.

LEMMA 9. *If $\mathfrak{G}_2\neq\mathfrak{H}_2$, then, for each $\alpha\in\mathfrak{G}_2$, there is a D in $\mathscr{H}_u(R_p, R_p)$ such that $\alpha\alpha_D^{-1}\in\mathfrak{H}_2$ if and only if, in* (3), $\bar{v}\notin k^p$.

Proof. Assuming first that $\bar{v}\notin k^p$ it follows from Lemma 8 that in (3), $t=p-1$ and $\bar{v}$ is a $(p-1)$th root in k. Assuming (6) to be the minimal polynomial of π over R, relation (3) with $t=p-1$ implies that $a_1,\dots,a_{p-2}$ are in pR, $\bar{a}_{p-1}(=-\bar{v})$ is a $(p-1)$th root in k, $v(f'(\pi))=2p-2$ and $a_0=1+pb_0$.

Let w be a unit in R_p such that $\bar{w}$ is a $(p-1)$th root of $\bar{a}_{p-1}(p-1)$. We wish to construct $D\in\mathscr{H}_u(R_p, R_p)$ such that $\alpha_D\in\mathfrak{G}_2$ and $\alpha_D(\pi)\equiv\pi^2 w$, mod $\pi^3 R_p$.

Let $\mathscr{S}$ be a set of representatives in R for a p-basis of k chosen to include a_{p-1}. Then D_1 is defined by $D_1(a_{p-1})=-f'(\pi)\pi^2 w/p\pi^{p-1}$, $D_1(s)=0$ for $s\in\mathscr{S}$, $s\neq a_{p-1}$. By Lemma 1 $D_1(R)\subset\pi^2 R_p$ and by (8.1) $D_1(\pi)\equiv\pi^2 w$, mod $\pi^3 R_p$. For $j=2,\dots,p-1$ and $s\in\mathscr{S}$, $D_j(s)=0$. By (8, 2) to $(8, p-1)$, $D_j(\pi)\in\pi^3 R_p$ for $j=2,\dots,p-1$. The term $[D_1(\pi)]^p$ in (8, p) leads us to define D_p by $D_p(a_{p-1})=-\pi^{2p}w^p/p(\pi^{p-1})$ and $D_p(s)=0$, $s\in\mathscr{S}$, $s\neq a_{p-1}$. Since each term of (8, p) in A_p+B_p is in $\pi^{2p+1}R_p$ save $[D_1(\pi)]^p$ and $[D_1(\pi)]^p\equiv\pi^{2p}w^p$, mod $\pi^{2p+1}R_p$, we have $D_p(\pi)\in\pi^3 R_p$. Finally, we let $D_j(s)=0$ for $s\in\mathscr{S}$ and $j>p$. Then $D_j(R)\subset\pi^2 R_p$ for $j>p$ and by Lemma 5 with $n=2$, $m=p+1$ and $q=3$, we conclude that $\sum D_j(\pi)$ converges and $\sum_{j=2}^{\infty} D_j(\pi)\in\pi^3 R_p$.

It remains to show that α_D is in $\mathfrak{G}_2$. We have shown that $\alpha_D(\pi)-\pi$ is in $\pi^2 R_p$ and it is shown below that

$$\alpha_D(s)-s\in\pi^2 R_p,\qquad s\in\mathscr{S}. \tag{16}$$

If $s\in\mathscr{S}$, $s\neq a_{p-1}$ then $\alpha_D(s)=s$ by definition of D. Since $D_j(a_{p-1})=0$ for $j\neq 1$, p it is sufficient to show that $D_1(a_{p-1})+D_p(a_{p-1})\in\pi^2 R_p$. Now, $f'(\pi)\equiv(p-1)a_{p-1}\pi^{p-2}$, mod $\pi^{p-1}R_p$. Also $w^{p-1}\equiv(p-1)a_{p-1}$, mod πR_p, by choice of w. Using these facts as well as the congruence $\pi^p\equiv -p$, mod $\pi^{p+1}R_p$, leads to the conclusion $D_1(a_{p-1})+D_p(a_{p-1})=-f'(\pi)\pi^2 w/p\pi^{p-1}-\pi^{2p}w^p/p\pi^{p-1}\in\pi^2 R_p$.

Since α_D is inertial, $\alpha_D(a^p)-a^p\in\pi^p R_p$ and every unit in R is, mod pR, a polynomial in elements of $\mathscr{S}$ with coefficients in R^p. It follows that $\alpha_D(a)-a\in\pi^2 R_p$ for a in R. Thus α is in $\mathfrak{G}_2$.

It was shown in the proof of Lemma 8 that if $\alpha\in\mathfrak{G}_2$ then $\alpha=\varepsilon+\pi^2\alpha^*$ and $\alpha\in\mathfrak{H}_2$ or the residue of $\alpha^*(\pi)$ is a $(p-1)$th root of $(p-1)\bar{v}=\bar{a}_{p-1}(p-1)$. Thus if we choose w, in the construction of D, to be $\alpha^*(\pi)$, then $\alpha\alpha_D^{-1}\in\mathfrak{H}_2$.

If $\bar{v}\in k^p$ then $v=v_0^p+\pi v_1$. Thus $\bar{a}_{p-1}=\bar{v}_0^p$ and $\bar{b}_0=\bar{v}_1$ where, again, $a_0=1+pb_0$. We choose c_0 and c_1 in R so that $a_{p-1}=c_0^p+pc_1$. Let $D\in\mathscr{H}_c(R_p, R_p)$ be such that

$\alpha_D \in \mathfrak{G}_2$, $\alpha_D \notin \mathfrak{H}_2$. There is, then, a first index j such that $f^{D_j}(\pi) \in f'(\pi)\pi^2 R_p$ and $f^{D_j}(\pi) \notin f'(\pi)\pi^3 R_p$. This requires that $D_j(a_{p-1}) = D_j(c_0^p + pc_1) \in \pi^2 R_p$ and $D_j(a_{p-1}) \notin \pi^3 R_p$. However, $D_j(R) \subset \pi R_p$ and hence $D_j(c_0^p + pc_1) \in \pi^p R_p$. We have a contradiction. Thus $\mathfrak{G}_2 \cap \mathfrak{G}_D \subset \mathfrak{H}_2$, and Lemma 9 is proved.

For $i > 1$ and $\alpha \in \mathfrak{H}_i$ there is a $D \in \mathscr{H}_u(R_p, R_p)$ such that $D(R_p) \subset \pi^i R_p$ (see (12)) and $\alpha\alpha_D \in \mathfrak{G}_{i+1}$. Also if $i > 2$ and $\alpha \in \mathfrak{G}_i$ then $\alpha \in \mathfrak{H}_i$ or there is a $D \in \mathscr{H}_u(R_p, R_p)$ such that $D(R_p) \subset \pi^i R_p$ and $\alpha\alpha_D \in \mathfrak{H}_i$. This follows from Lemma 7, Case 1 of the proof of Lemma 7 and Lemma 8. Thus, given $\alpha \in \mathfrak{H}_2$ there is a sequence $\{D^{(n)}\}$, $D^{(n)} \in \mathscr{H}_u(R_p, R_p)$ such that, $D^{(n)}(R_e) \subset \pi^{s_n} R_p$ where $\lim_n s_n = \infty$, and $\alpha = \alpha_{D_1}\alpha_{D_2} \cdots \alpha_{D_{2n}}$, mod $\mathfrak{H}_{n+2}$. By Lemma 3, there is a $D \in \mathscr{H}_u(R_p, R_p)$ such that $\alpha = \alpha_D$.

By Lemma 6 and Lemma 9, we conclude that $\mathfrak{G}_D$ and $\mathfrak{G}(R_p, R)$ together generate $\mathfrak{G}$. If β is an automorphism on R and $D \in \mathscr{H}_c(R_e, R_e)$ then $H = \{H_i\}$ where $H_i = \beta^{-1} D_i \beta$ is also in $\mathscr{H}_c(R_e, R_e)$. If D converges uniformly so does H. Thus $\mathfrak{G}_D$ is an invariant subgroup of G the automorphism group of R_p. Hence $\mathfrak{G}_1 = \mathfrak{G}_D \cdot \mathfrak{G}(R_p, R)$. These observations along with Lemmas 8 and 9 prove Theorem 1. Theorem 2 follows directly from Lemmas 2, 4, 6, 7 and 8.

REFERENCES

1. N. Heerema, *Derivations on p-adic fields*, Trans. Amer. Math. Soc. **102** (1962), 346–351.

2. ———, *Convergent higher derivations on local rings*, Trans Amer. Math. Soc. **132** (1968), 31–44.

3. J. Neggers, *Derivations on $\bar{p}$-adic fields*, Trans. Amer. Math. Soc. **115** (1965), 496–504.

4. E. Wishart, *Higher derivations on $\bar{p}$-adic fields*, Dissertation, Florida State Univ., Tallahassee, 1965.

FLORIDA STATE UNIVERSITY,
TALLAHASSEE, FLORIDA