ERRATA TO "CONCERNING ARCWISE CONNECTEDNESS AND THE EXISTENCE OF SIMPLE CLOSED CURVES IN PLANE CONTINUA"

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Substitute the following for the last two paragraphs in the proof of Theorem 2 of my paper [1].

Assume without loss of generality that the sequence $F_1, F_2, F_3, \dots$ is such that for each positive integer n , there exist two arc-segments R_n and E_n such that (1) $R_n \subset \text{Bd } R$, (2) $E_n \subset \text{Bd } E$, and (3) each arc-segment meets $F_1, F_2, F_3, \dots$ only in F_{2n} and has one endpoint in F_{2n-1} and the other endpoint in F_{2n+1} . Let $p_1, p_2, p_3, \dots$ be a sequence of points converging to p such that for each positive integer n , the point p_n is in $F_{2n} \cap (R - \text{Cl } E)$. The sequence $R_1, R_2, R_3, \dots$ converges to a point w_1 of $M \cap \text{Bd } R$ and $E_1, E_2, E_3, \dots$ converges to a point w_2 of $M \cap \text{Bd } E$.

Since M is aposyndetic, for $n=1$ and 2, there exist a subcontinuum M_n of M and a circular region G_n ($\text{Cl } (G_1 \cup G_2) \cap \{y, z\} = \text{Cl } G_1 \cap \text{Cl } G_2 = \emptyset$) in S such that G_n contains w_n and meets only one component of $\text{Bd } (R - E)$, the point p is in the interior of M_n relative to M and $\text{Cl } G_n \cap M_n = \emptyset$. Let G denote a circular region in S containing p such that $\text{Cl } G \cap \text{Cl } (G_1 \cup G_2) = \emptyset$ and $G \cap M$ is contained in $M_1 \cap M_2$. Assume without loss of generality that for each positive integer i , $\text{Cl } R_i \subset G_1$, $\text{Cl } E_i \subset G_2$, and p_i belongs to G . Let k be a positive integer such that $\text{Cl } (U_k \cup V_k) \cap \text{Cl } (G_1 \cup G_2) = \emptyset$ and $F_1, F_2, \dots, F_7$ all lie in Y_k . Let j be a positive integer such that $U_k \cup V_k \supset \text{Cl } (U_j \cup V_j)$. Let P_1 be a circular region in G centered on p_1 such that $\text{Cl } P_1$ does not meet $F_1 \cup F_3 \cup R_1 \cup E_1$. Since M is not aposyndetic at p_1 with respect to $\{y, z\}$, the component of $M - (U_j \cup V_j)$ which contains p_1 is not open relative to M at p_1 . Hence the boundary of P_1 contains an arc-segment S_1 whose endpoints a_1 and b_1 lie in different components of $M - (U_j \cup V_j)$ such that $M \cap S_1 = \emptyset$. There exists a simple closed curve C_1 which separates a_1 from b_1 in S and contains no point of $M - (U_j \cup V_j)$ such that $C_1 \cap S_1$ is connected. In C_1 there exists an arc-segment T_1 which crosses S_1 , contains no point of $M \cup \text{Cl } (U_j \cup V_j)$, and has its endpoints in $\text{Bd } (U_j \cup V_j)$. Let P_2 be a circular region in G centered on p_2 such that $\text{Cl } P_2$ does not meet $F_3 \cup F_5 \cup R_2 \cup E_2 \cup T_1$. The component of $(M \cup S_1 \cup \text{Cl } T_1) - (U_j \cup V_j)$ which contains p_2 is not open relative to $M \cup S_1 \cup \text{Cl } T_1$ at p_2 . Hence the boundary of P_2 contains

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an arc-segment S_2 whose endpoints a_2 and b_2 lie in different components of $(M \cup S_1 \cup \text{Cl } T_1) - (U_j \cup V_j)$ such that $M \cap S_2 = \emptyset$. There exists a simple closed curve C_2 which separates a_2 from b_2 in S and contains no point of $(M \cup S_1 \cup \text{Cl } T_1) - (U_j \cup V_j)$ such that $C_2 \cap S_2$ is connected. In C_2 there exists an arc T_2 which crosses S_2 , contains no point of $M \cup \text{Cl } (U_j \cup V_j)$, and has its endpoints in $\text{Bd } (U_j \cup V_j)$. Continue this process. There exist a circular region P_3 centered on p_3 , arc-segments S_3 and T_3 , and a simple closed curve C_3 such that (1) $\text{Cl } P_3$ does not meet $F_5 \cup F_7 \cup R_3 \cup E_3 \cup T_1 \cup T_2$, (2) S_3 has endpoints a_3 and b_3 in M and is contained in $(S - M) \cap \text{Bd } P_3$, (3) C_3 separates a_3 from b_3 and contains no point of $(M \cup \bigcup_{i=1}^3 (S_i \cup \text{Cl } T_i)) - (U_j \cup V_j)$, (4) $C_3 \cap S_3$ is connected, and (5) T_3 is contained in $C_3 - \text{Cl } (U_j \cup V_j)$, meets S_3 , and has its endpoints in $\text{Bd } (U_j \cup V_j)$.

For each i ($i=1, 2$, and 3), no component of $\text{Cl } (G_1 \cup G_2)$ contains both a point of T_i which precedes and a point of T_i which follows $T_i \cap S_i$ with respect to the order of T_i ; for otherwise, T_i union a component of $\text{Bd } (G_1 \cup G_2)$ would separate a_i from b_i in S which contradicts the existence of M_1 and M_2 [5, Theorem 32, p. 181]. For $m=1, 2$, and 3 , let v_m denote a point of $\text{Bd } (U_j \cup V_j) \cap \text{Cl } T_m$. For $m=1, 2$, and 3 , since $F_{2m-1} \cup F_{2m+1} \cup R_m \cup E_m$ separates v_m from $T_m \cap S_m$ in S and $F_{2m-1} \cup F_{2m+1}$ is in Y_k , there exists a component L_m of $R_m \cup E_m$ such that $L_m \cup Y_k$ separates v_m from $T_m \cap S_m$ in S [5, Theorem 20, p. 173]. For $m=1, 2$, and 3 , the set $L_m \cup Y_k$ separates v_m from the other endpoint z_m of T_m in S . To see this suppose that for some $m=1, 2$, or 3 , $L_m \cup Y_k$ does not separate v_m from z_m in S . It follows that $T_m \cap L_m$ contains a point which precedes and a point which follows $T_m \cap S_m$ with respect to the order of T_m . Since L_m is contained in a component of $G_1 \cup G_2$, this is impossible. Note that for $m=1, 2$, and 3 , z_m does not belong to the v_m -component of $\text{Bd } (U_j \cup V_j)$. Assume without loss of generality that for $m=1, 2$, and 3 , the point v_m belongs to $\text{Bd } U_j$.

Some two of L_1, L_2 , and L_3 , say L_1 and L_2 , are contained in the same component of $G_1 \cup G_2$. Assume that $L_1 \cup L_2$ is contained in G_1 . For $m=1$ and 2 , let D_m be the arc-segment $(R_m \cup E_m) - L_m$. Let A be an arc in $G_2 \cap \text{Bd } E$ which contains $D_1 \cup D_2$. There exists an arc-segment B in $E \cap G_2$ such that $\text{Cl } B$ contains the endpoints of A . The simple closed curve $A \cup B$ contains $D_1 \cup D_2$ and there exists a complementary domain K of $A \cup B$ which does not meet $F_1 \cup F_3 \cup F_5 \cup \text{Bd } R$. Assume that for $m=1$ and 2 , the order of the arc-segment T_m indicates that T_m goes from v_m to z_m . For $m=1$ and 2 , since $L_m \cup Y_k$ separates v_m from $T_m \cap S_m$, the arc-segment T_m must meet L_m before S_m ; and since $L_m \cup D_m \cup F_{2m-1} \cup F_{2m+1}$ separates $S_m \cap T_m$ from z_m and $L_m \cap T_m$ does not contain both a point which precedes and a point which follows $T_m \cap S_m$, $T_m \cap D_m$ must contain a point which follows $T_m \cap S_m$ with respect to the order of T_m . If for $m=1$ or 2 , $T_m \cap (A \cup B)$ contains a point which precedes and a point which follows $T_m \cap S_m$ with respect to the order of T_m , then $T_m \cup A \cup B$ separates a_m from b_m in S and does not meet M_2 . This contradicts the fact that M_2 is connected. Hence for $m=1$ and 2 , T_m must meet $A \cup B$ only at points which follow $T_m \cap S_m$ with respect to the order of T_m .

For $m=1$ and 2 , let c_m be the first point of $T_m \cap (A \cup B)$ with respect to the order of T_m , let K_m be the arc-segment in T_m from v_m to c_m , and let N_m denote the arc in $\text{Cl } T_m$ which goes from c_m to z_m . Note that for $m=1$ and 2 , the point c_m is in D_m . The set $K_1 \cup K_2 \cup \text{Bd } U_j$ is contained in $S - \text{Cl } K$. For $m=1$ and 2 , since T_m does not meet $\text{Bd } (U_j \cup V_j)$, $N_m \cap \text{Bd } U_j = K_m \cap \text{Bd } V_j = \emptyset$. Let U be an arc-segment in $K_1 \cup K_2 \cup \text{Bd } U_j$ which has endpoints c_1 and c_2 . The arc $\text{Cl } U$ separates F_1 from F_3 in $S - K$ [5, Theorem 28, p. 156]. Hence there exist arcs X_1 and X_2 in $\text{Cl } L_1$ such that (1) X_1 and X_2 abut on $\text{Cl } U$ from opposite sides with respect to a simple closed curve in $U \cup A \cup B$, (2) $X_1 \cap F_1 \neq \emptyset$, and (3) $X_2 \cap F_3 \neq \emptyset$. Since $\text{Cl } T_1 \cap \text{Cl } T_2 = \emptyset$, $(N_1 \cup N_2) \cap \text{Cl } (U_j \cup G_1) = \emptyset$, and $(K_1 \cup K_2) \cap \text{Bd } V_j = \emptyset$, there exists a simple closed curve J in $U \cup N_1 \cup N_2 \cup \text{Bd } V_j$ such that X_1 and X_2 abut on $\text{Cl } U$ from opposite sides with respect to J . It follows that J separates F_1 from F_3 in S . Since $J \cap (M - (U_k \cup V_k)) = \emptyset$ and $F_1 \cup F_3$ is contained in the x -component of $M - (U_k \cup V_k)$, this is a contradiction. Evidently $L_1 \cup L_2$ is not contained in G_1 . It can be shown by the same method that assuming $L_1 \cup L_2$ is in G_2 also involves a contradiction. It follows that M is not aposyndetic at p with respect to both w_1 and w_2 which contradicts the hypothesis of the theorem. Hence $L_{y_2}^x$ is locally connected.

REFERENCES

1. Charles L. Hagopian, *Concerning arcwise connectedness and the existence of simple closed curves in plane continua*, Trans. Amer. Math. Soc. **147** (1970), 389–402.

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