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NEARNESSES, PROXIMITIES, AND T_1-COMPACTIFICATIONS[1]

BY

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ABSTRACT. Gagrat, Naimpally, and Thron together have shown that separated Lodato proximities yield T_1-compactifications, and conversely. This correspondence is not 1–1, since nonequivalent compactifications can induce the same proximity. Herrlich has shown that if the concept of proximity is replaced by that of nearness then *all* principal (or strict) T_1-extensions can be accounted for. (In general there are many nearnesses compatible with a given proximity.) In this paper we obtain a 1-1 correspondence between principal T_1-extensions and *cluster-generated* nearnesses. This specializes to a 1-1 match between principal T_1-*compactifications* and *contigual* nearnesses.

These results are utilized to obtain a 1-1 correspondence between Lodato proximities and a subclass of T_1-compactifications. Each proximity has a largest compatible nearness, which is contigual. The extension induced by this nearness is the construction of Gagrat and Naimpally and is characterized by the property that the dual of each clan converges. Hence we obtain a 1-1 match between Lodato proximities and *clan-complete* principal T_1-compactifications. When restricted to *EF*-proximities, this correspondence yields the usual map between T_2-compactifications and *EF*-proximities.

0. Notation. Let $\mathcal{A}$ and $\mathcal{B}$ be families of subsets of a topological space X. Let λ be a collection of families of subsets of X, and let $A \subset X$.

(1) $\mathcal{A} < \mathcal{B}$ iff each set in $\mathcal{B}$ contains a set in $\mathcal{A}$;

(2) $\mathcal{A} \vee \mathcal{B} = \{A \cup B: A \in \mathcal{A} \text{ and } B \in \mathcal{B}\}$;

(3) $c_\lambda(A) = \{x \in X: \{A, \{x\}\} \in \lambda\}$;

(4) $c_\lambda(\mathcal{A}) = \{c_\lambda(A): A \in \mathcal{A}\}$;

(5) A^- is the topological closure of A;

(6) $\mathcal{A}^- = \{A^-: A \in \mathcal{A}\}$.

1. Nearnesses and extensions.

A. *Obtaining nearnesses from extensions*. In this part we will develop a map Tr from the T_1-extensions of a topological space to its compatible nearnesses.

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The main result states that principal T_1-extensions give rise (under Tr) to cluster-generated compatible Lodato nearnesses.

1.1 DEFINITION. A *nearness* on a set X is a collection ν of families of subsets of X such that

(N1) $\bigcap \mathcal{A} \neq \varnothing \Rightarrow \mathcal{A} \in \nu$;

(N2) If $\mathcal{A} \in \nu$ and $\mathcal{A} < \mathcal{B}$ then $\mathcal{B} \in \nu$;

(N3) If $\mathcal{A} \vee \mathcal{B} \in \nu$ then $\mathcal{A} \in \nu$ or $\mathcal{B} \in \nu$;

(N4) If $\mathcal{A} \in \nu$ then $\varnothing \notin \mathcal{A}$.

1.2 DEFINITION. A *grill* on a set X is any family $\mathcal{G}$ of subsets of X satisfying

(G1) $A \in \mathcal{G}$ and $A \subset B \Rightarrow B \in \mathcal{G}$;

(G2) $A \cup G \in \mathcal{G} \Rightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$;

(G3) $\varnothing \notin \mathcal{G}$.

1.3 REMARK. A grill is simply any union of ultrafilters. For more background, see Thron [9].

1.4 DEFINITION. Let ν be a nearness on a set X.

a. A *ν-clan* is a grill which is a member of ν.

b. A *ν-bunch* is a ν-clan σ satisfying

$$A^- \in \sigma \Rightarrow A \in \sigma.$$

c. A *ν-cluster* is a maximal element of ν.

1.5 DEFINITIONS–KINDS OF NEARNESSES. Let ν be a nearness on a set X and let $\mathcal{T}$ be a topology on X.

a. ν is *compatible* with $\mathcal{T}$ iff c_ν is the closure operator determined by $\mathcal{T}$; i.e. $c_\nu(A) = A^-$ for $A \subset X$.

b. ν is Lodato iff $\mathcal{A} \in \nu$ whenever $c_\nu(\mathcal{A}) \in \nu$.

c. Let $\chi \subset \nu$. We say ν is *χ-generated* iff every member of ν is contained in a member of χ. In particular then ν is *cluster*-generated iff each member of ν is contained in a ν-cluster.

1.6 PROPOSITION. *Let ν be a Lodato nearness on a set X. For $\mathcal{A} \subset \mathcal{P}(X)$, define*

$$b\mathcal{A} = \{A \subset X: c_\nu A \in \mathcal{A}\}.$$

Then

(1) *if σ is a ν-clan then b_σ is a ν-bunch containing σ;*

(2) *every maximal ν-clan is a ν-bunch.*

PROOF. (1) Let σ be a ν-clan. Since ν is Lodato, we have that c_ν is a Kuratowski closure operator. Clearly then $b\sigma$ is a grill. Note $c_\nu b\sigma \subset \sigma \subset b\sigma$. Since $\sigma \in \nu$, we have $c_\nu b\sigma \in \nu$. But ν is Lodato so $b\sigma \in \nu$. Thus $b\sigma$ is a ν-clan containing σ. Since c_ν is idempotent, $b\sigma$ is a bunch.

(2) Now let μ be a maximal ν-clan. Then by (1), b_μ is a ν-bunch containing μ. Since μ is maximal, $b\mu = \mu$, and so μ is a ν-bunch. □

1.7 PROPOSITION. *For any nearness ν, a ν-cluster is a maximal ν-clan. If ν is cluster-generated, then every maximal ν-clan is a ν-cluster.*

PROOF. Let ν be a nearness. Then every ν-cluster is a grill. This follows readily from the nearness axioms. Therefore each ν-cluster is a maximal ν-clan. (See Gagrat and Thron [4].)

Assume now that ν is cluster-generated. Let σ be a maximal ν-clan. Choose a ν-cluster $\mathfrak{M}$ such that $\sigma \subset \mathfrak{M}$. Then by the above remark, $\mathfrak{M}$ is a ν-clan, and so $\mathfrak{M} = \sigma$. □

The next theorem sets up a map Tr from T_1-extensions of a space X to bunch-generated Lodato nearnesses compatible with X. It is similar to a result of Bentley [2] which reaches the same conclusion under a slightly weaker hypothesis [2, Theorems 2.8 and 3.6]. The proof is given here for convenience.

1.8 THEOREM. *Let $\kappa = (e, Y)$ be a T_1-extension of a topological space X. Let*

$$\mathrm{Tr}(\kappa) = \nu_\kappa = \{\mathcal{A} : \cap\, e(\mathcal{A})^- \neq \varnothing\}.$$

Then ν_κ is a bunch-generated Lodato nearness compatible with the topology on X.

PROOF. Clearly (N1), (N2), and (N4) hold. Now suppose $\mathcal{A} \vee \mathcal{B} \in \nu_\kappa$ with $\mathcal{A} \notin \nu_\kappa$. Let $\mathcal{C} = \mathcal{A} \vee \mathcal{B}$. We claim $\cap\, e(\mathcal{C})^- \subset \cap\, e(\mathcal{B})^-$.

Let $y \in \cap\, e(\mathcal{C})^-$. Since $\mathcal{A} \notin \nu_\kappa$, choose $A \in \mathcal{A}$ such that $y \notin e(A)^-$. Now let $B \in \mathcal{B}$. Then $y \in e(A \cup B)^- = e(A)^- \cup e(B)^-$. Thus, $y \in e(B)^-$, as desired.

To see that ν_κ is compatible with the topology on X, note that for $A \subset X$ we have

$$e(A^-) = e(A)^- \cap e(X).$$

Therefore $x \in A^- \Leftrightarrow e(x) \in e(A)^- \Rightarrow \{e(x)\}^- \cap e(A)^- \neq \varnothing \Leftrightarrow \{\{x\}, A\} \in \nu_\kappa$. We obtain the missing implication from the fact that Y is T_1.

By virtue of the continuity of e we have $e(A)^- = e(A^-)^-$, for $A \subset X$. This is sufficient to establish that ν_κ is Lodato.

Finally, we claim that ν_κ is bunch-generated. For $y \in Y$ let $\tau(y) = \{A \subset X : y \in e(A)^-\}$. Note if $y \in \cap\, e(\mathcal{A})^-$ then $\mathcal{A} \subset \tau(y)$. Thus every member of ν_κ is a subset of some $\tau(y)$. From definitions it is easy to see that $\tau(y)$ is a ν_κ-clan. Since $e(A)^- = e(A^-)^-$ we have that $\tau(y)$ is a ν_κ-bunch. Hence ν_κ is bunch-generated. □

For completeness we will show that equivalent extensions induce the same nearness, so that Tr is defined on equivalence classes of T_1-extensions.

1.9 LEMMA. *If κ_1 and κ_2 are equivalent extensions of a space X then $\nu_{\kappa_1} = \nu_{\kappa_2}$.*

PROOF. Say $\kappa_i = (e_i, Y_i)$ and $f\colon Y_1 \to Y_2$ is an equivalence between κ_1 and κ_2. Let $g = f^{-1}$. Then for $A \subset X$ we have $e_2(A)^- = f(e_1(A)^-)$ and similarly $e_1(A)^- = g(e_2(A)^-)$. From this it is easy to see that $\nu_{\kappa_1} = \nu_{\kappa_2}$. □

Next we will show that in the case of a *principal* T_1-extension the associated nearness is *cluster*-generated. In fact in this case the trace of each point of the extension is a cluster; and the set of all these traces generates the nearness.

To proceed, we will develop several characterizations of a principal extension.

1.10 DEFINITIONS AND NOTATION. Let $\kappa = (e, Y)$ be an extension of a space X.

(a) For $y \in Y$ the *trace* of y in X is $\tau(y) = \{A \subset X\colon y \in e(A)^-\}$.

(b) For G an open set in Y, $G^+ = \{y \in Y\colon e^{-1}(G) \in e^{-1}(\mathfrak{N}_y)\}$.

1.11 THEOREM. *Let $\kappa = (e, Y)$ be an extension of a space X. The following conditions are equivalent*:

(1) $\{e(A)^-\colon A \subset X\}$ *is a base for the closed sets of Y*;

(2) $\{G^+\colon G$ *is open in* $Y\}$ *is a base for the open sets of Y.*

PROOF. The proof follows from the following facts.

(i) If G is open in Y then G^+ is open in Y.

(ii) If G is open in Y then $Y \setminus G^+ = ee^{-1}(X \setminus G)^-$.

(iii) For $A \subset X$, and $G = Y \setminus e(A)^-$ we have $G = G^+$. □

1.12 DEFINITION. An extension is *principal*, or *strict*, iff it satisfies one of the conditions of the preceding theorem.

Note that these are the same as the strict extensions of Banaschewski [1] and the principal extensions of Thron [8].

1.13 LEMMA. *Let $\kappa = (e, Y)$ be a principal extension of a space X.*

(1) *If $\tau(y) \subset \tau(z)$ then $\mathfrak{N}_z \subset \mathfrak{N}_y$ for all y, z in Y*;

(2) *if Y is T_1 then each $\tau(y)$ is a ν_κ-cluster.*

PROOF. (1) If $\tau(y) \subset \tau(z)$ then $\mathfrak{N}_z \subset \mathfrak{N}_y$.

Let G be open in Y with $z \in G$. Then $z \notin Y \setminus G$. Since κ is a principal extension, there is a set $A \subset X$ such that $Y \setminus G \subset e(A)^-$ and $z \notin e(A)^-$. Thus $A \notin \tau(z)$ and so $A \notin \tau(y)$. From this we obtain $y \in Y \setminus e(A)^- \subset G$.

(2) If Y is a T_1-space then each $\tau(y)$ is a ν_κ-cluster.

Note $\tau(y) \in \nu_\kappa$ since $y \in \bigcap e\tau(y)^-$. Now let $A \subset X$ and suppose $\mathcal{A} = \tau(y) \cup \{A\}$ is in ν_κ. We wish to show $A \in \tau(y)$. Let $z \in \bigcap e(\mathcal{A})^-$. Then $\tau(y) \subset \tau(z)$ and so by (1) we obtain $\mathfrak{N}_z \subset \mathfrak{N}_y$. Since Y is T_1, $z = y$ and so $\tau(z) = \tau(y)$. Note $A \in \tau(z)$ since $\mathcal{A} \subset \tau(z)$. Thus $A \in \tau(y)$ as desired. □

1.14 THEOREM. *If $\kappa = (e, Y)$ is a principal T_1-extension of a space X then ν_κ is a cluster-generated compatible Lodato nearness on X.*

PROOF. From Theorem 1.8 we know that ν_κ is a compatible Lodato nearness on X. From the preceding lemma we have that each $\tau(y)$ is a ν_κ-cluster. Since each member of ν_κ is contained in some $\tau(y)$ we conclude that ν_κ is cluster-generated. □

B. *Obtaining extensions from nearnesses.* In what follows we will construct a map Ext from the Lodato nearnesses on a space X to extensions of X. The main result states that if ν is a Lodato nearness compatible with a T_1-space X then Ext(ν) is a principal T_1-extension of X.

1.15 *Construction of* Ext. Let ν be a Lodato nearness compatible with a T_1-space X. Let Y_ν be the set of all ν-clusters. For $A \subset X$, let $A^\nu = \{\sigma \in Y_\nu: A \in \sigma\}$. Then $\{A^\nu: A \text{ is closed in } X\}$ is a base for the closed sets of a topology $\mathfrak{T}_\nu$ on Y_ν. Let $e_\nu(x) = \sigma_x = \{A \subset X: x \in A^-\}$. We define Ext($\nu$) = $\kappa_\nu = (e_\nu, (Y_\nu, \mathfrak{T}_\nu))$.

In the next theorem we will show that κ_ν is a principal T_1-extension of X. This theorem is actually a consequence of work of Herrlich [5], but a proof is given here, for completeness.

1.16 LEMMA. *Let ν be a Lodato nearness compatible with a T_1-space X. Then for $A \subset X$ we have*

$$A^\nu = (A^-)^\nu = e_\nu(A)^-.$$

In particular then the closed sets of κ_ν are generated by $\{A^\nu: A \subset X\}$.

PROOF. Note that each ν-cluster is a ν-bunch, by Propositions 1.6 and 1.7. Thus $A^\nu = (A^-)^\nu$. Now let $\sigma \in A^\nu$. We wish to show $\sigma \in e_\nu(A)^-$. Let $B \subset X$ such that $e_\nu(A) \subset B^\nu$. Then $A \subset B^-$ and so $\sigma \in (B^-)^\nu = B^\nu$. This establishes $\sigma \in e_\nu(A)^-$.

Now suppose $\sigma \in e_\nu(A)^-$. Then $e_\nu(A) \subset A^\nu$. Since A^ν is closed, we have $\sigma \in A^\nu$. □

1.17 THEOREM. *Let ν be a Lodato nearness compatible with a T_1-space X. Then κ_ν is a principal T_1-extension of X.*

PROOF. (1) e_ν is an injection from X to Y_ν.

Clearly each σ_x is a grill. Since ν is Lodato, $\sigma_x \in \nu$. Since ν is compatible with the topology on X, each σ_x is a maximal element of ν. Thus $e_\nu: X \to Y_\nu$. Since X is T_1, e_ν is 1-1.

(2) e_ν is a dense embedding of X into Y_ν.

Note that if K is closed in X then $e_\nu^{-1}(K^\nu) = K$. Hence e_ν is continuous.

Recall that for $A \subset X$, $A^\nu = e_\nu(A)^-$ (Lemma 1.16). Hence for K closed we have

$$e_\nu(K) = e_\nu(K)^- \cap e_\nu(X).$$

This establishes that $e_\nu: X \to e_\nu(X)$ is a closed map.

Finally, $e_\nu(X)$ is dense in Y_ν; for if K is closed in X and $e_\nu(X) \subset K^\nu$ then $X = K$.

(3) Y_ν is T_1. Let $\sigma \in Y_\nu$. We claim $\{\sigma\}^- = \{\sigma\}$. Let $\sigma_1 \in Y_\nu$ such that $\sigma_1 \neq \sigma$. Since σ is a ν-cluster we have $\sigma \not\subset \sigma_1$ and we choose $S \in \sigma \setminus \sigma_1$. Then $Y_\nu \setminus S^\nu$ is a neighborhood of σ_1 which misses $\{\sigma\}$.

(4) K_ν is a principal extension of X. This follows easily from the relation $A^\nu = e_\nu(A)^-$. □

C. *Correspondence between nearnesses and extensions*. In this section we will show that Ext and Tr are inverses if we restrict ourselves to cluster-generated nearnesses and principal extensions. Moreover, these maps yield a 1-1 correspondence between contigual nearnesses and T_1-compactifications. More precisely, Ext restricted to contigual nearnesses is a bijection to the set of principal T_1-compactifications.

1.18 THEOREM. *If ν is a cluster-generated Lodato nearness compatible with a T_1-space X, then $\nu_{K_\nu} = \nu$.*

PROOF. Let $\nu' = \nu_{K_\nu}$. In general we have $\nu' \subset \nu$. For let $\mathcal{Q} \in \nu'$. Then we can choose $\sigma \in \bigcap e_\nu(\mathcal{Q})^-$. From Lemma 1.16 we have $\mathcal{Q} \subset \sigma$. Since $\sigma \in \nu$ we have $\mathcal{Q} \in \nu$.

To get that $\nu \subset \nu'$ we will use the fact that ν is cluster-generated. Let $\mathcal{Q} \in \nu$ and let σ be a ν-cluster containing $\mathcal{Q}$. Then $\sigma \in \bigcap e_\nu(\mathcal{Q})^-$ (Lemma 1.16). Hence $\mathcal{Q} \in \nu'$. □

The next theorem is due to Herrlich [5], but a direct proof is given here for convenience.

1.19 THEOREM. *If κ is a princpal T_1-extension of a space X then κ is equivalent to κ_{ν_κ}.*

PROOF. Suppose $\kappa = (e, Y)$. Recall $\tau(y) = \{A \subset X: y \in e(A)^-\}$. We claim τ gives the desired equivalence.

Let κ_{ν_κ} be denoted by $\kappa' = (e', (Y', \mathfrak{T}'))$. We note first that κ' is a principal T_1-extension of X (Theorems 1.8 and 1.17).

(1) τ is a bijection from Y to Y'.

Note that each $\tau(y)$ is a ν_κ-cluster, by Lemma 1.13. Thus $\tau: Y \to Y'$. Now let σ be a ν_κ-cluster. Choose $y \in \bigcap e(\sigma)^-$. Clearly then $\sigma \subset \tau(y)$. Since σ is a maximal element of ν_κ, and $\tau(y) \in \nu_\kappa$, we have $\sigma = \tau(y)$. Thus τ is a surjection.

Now suppose $\tau(y) = \tau(z)$. From Lemma 1.13 it follows that $\mathfrak{N}_y = \mathfrak{N}_z$. Since Y is T_1, we have $y = z$. Thus τ is an injection.

(2) τ is a homeomorphism.

For $A \subset X$ let A^* denote A^{ν_κ}. Recall that $\{A^*: A \subset X\}$ is a base for the closed sets of Y'; and $\{e(A)^-: A \subset X\}$ is a base for the closed sets of Y.

The continuity of τ follows from the relation

$$\tau^{-1}(A^*) = e(A)^-.$$

To see that τ is a closed map, note $\tau(e(A)^- = A^*$.

(3) $\tau e = e'$.

We need to show that $\tau e(x) = \{A: x \in A^-\}$. Note $A \in \tau e(x) \Leftrightarrow e(x) \in e(A)^- \Leftrightarrow x \in A^-$. □

1.20 Corollary. *Let X be a fixed T_1-space. The map* Ext *is a bijection from the set of cluster-generated compatible Lodato nearnesses to the set of (equivalence classes of) principal T_1-extensions of X. The maps* Ext *and* Tr *are inverses on these two sets.*

Proof. Theorems 1.18 and 1.19. □

Next we will show that this correspondence extends to contigual nearnesses and T_1-compactifications. To proceed, we will define a contigual nearness and show its relation to other nearnesses.

1.21 Definition. A nearness ν is *contigual* iff for $\mathcal{A} \subset \mathcal{P}(X)$ we have $\mathcal{A} \in \nu$ whenever each finite subset of $\mathcal{A}$ is in ν.

1.22 Proposition. *Let ν be a nearness on a set X.*
Let

$$a\nu = \{\mathcal{A} \subset \mathcal{P}(X): \textit{every finite subset of } \mathcal{A} \textit{ is in } \nu\}.$$

Then $a\nu$ is a nearness which determines the same closure operator as ν. Moreover, $a\nu$ is the smallest contigual nearness larger than ν.

Proof. (1) $a\nu$ is a nearness which contains ν.

Clearly $\nu \subset a\nu$ and (N1), (N2), and (N4) hold.

Suppose $\mathcal{A} \vee \mathcal{B} \in a\nu$ and $\mathcal{A} \not\in a\nu$. Let $\mathcal{A}_0$ be a finite space of $\mathcal{A}$ such that $\mathcal{A}_0 \not\in \nu$. We claim $\mathcal{B} \in a\nu$. Let $\mathcal{F}$ be any finite subset of $\mathcal{B}$. Then $\mathcal{A}_0 \vee \mathcal{F}$ is a finite subset of $\mathcal{A} \vee \mathcal{B}$ and hence $\mathcal{A}_0 \vee \mathcal{F} \in \nu$. Since $\mathcal{A}_0 \not\in \nu$, we have $\mathcal{F} \in \nu$. Therefore $\mathcal{B} \in a\nu$.

(2) ν and a_ν determine the same closure operator.

Note for $x \in X$ and $A \subset X$ we have

$$\{\{x\}, A\} \in \nu \Leftrightarrow \{\{x\}, A\} \in a\nu.$$

(3) $a\nu$ is contigual, since every finite family $\mathcal{F}$ which is in $a\nu$ is also in ν.

(4) If ν_1 is a contigual nearness containing ν then ν_1 contains $a\nu$.

If $\mathcal{A} \in a\nu$ then every finite subset of $\mathcal{A}$ is in ν and hence in ν_1; consequently $\mathcal{A} \in \nu_1$. □

1.23 Lemma. *Every contigual nearness is cluster-generated.*

Proof. If ν is contigual then each nonempty chain in ν has its union in ν. Using Zorn's lemma then it is easy to prove that ν is cluster-generated. □

The next two results are due to Herrlich [5].

1.24 THEOREM. *If ν is a contigual Lodato nearness on a T_1-space X then κ_ν is compact.*

PROOF. Let $\mathcal{C}$ be any family of closed subsets of X. Suppose $\mathcal{C}^\nu = \{C^\nu: C \in \mathcal{C}\}$ has the f.i.p. To show κ_ν is compact it is sufficient to show $\mathcal{C}^\nu$ has nonempty intersection.

We claim that every finite subset of $\mathcal{C}$ is in ν. For if $\sigma \in C_1^\nu \cap \cdots \cap C_n^\nu$ then $\{C_1, \ldots, C_n\} \subset \sigma$ and hence $\{C_1, \ldots, C_n\} \in \nu$.

Since ν is contigual, $\mathcal{C} \in \nu$. But ν is cluster-generated (Lemma 1.23) and so we can choose a ν-cluster μ such that $\mathcal{C} \subset \mu$. Clearly then $\mu \in \bigcap \mathcal{C}^\nu$. □

1.25 THEOREM. *If $\kappa = (e, Y)$ is a T_1-compactification of a space X then ν_κ is contigual.*

PROOF. Let $\mathcal{A} \in a\nu_\kappa$. Then $e(\mathcal{A})^-$ has the f.i.p. Since Y is compact, $e(\mathcal{A})^-$ has nonempty intersection. By construction, $\mathcal{A} \in \nu_\kappa$. □

1.26 COROLLARY. *Let X be a fixed T_1-space. The map* Ext *is a bijection from the set of contigual compatible Lodato nearnesses to the set of (equivalence classes of) principal T_1-compactifications of X. The maps* Ext *and* Tr *are inverse maps on these two sets.*

PROOF. From Theorems 1.17 and 1.24 it follows that Ext maps contigual Lodato nearnesses compatible with X to principal T_1-compactifications of X. Similarly by Theorems 1.14 and 1.25 we have that Tr maps principal T_1-compactifications to contigual Lodato nearnesses on X. But Ext and Tr are inverses on these two sets, by Theorems 1.18 and 1.19 and Lemma 1.23. □

2. **Proximities and extensions.** Each proximity π carries with it a whole band of nearnesses. However there is always a largest one, $\nu_G(\pi)$, which is always contigual and is Lodato if π is Lodato. If π is an EF-proximity then $\text{Ext}(\nu_G(\pi))$ is the T_2-compactification corresponding to π. Moreover if π is Lodato then $\text{Ext}(\nu_G(\pi))$ is the compactification obtained by Gagrat and Naimpally [3]. These compactifications are characterized by the property that the dual of each clan converges. Using results of the previous section then we obtain a 1-1 correspondence between the Lodato proximities on a T_1-space and its clan-complete principal T_1-compactifications.

2.1 DEFINITION. A *proximity* on a set X is a family π of subsets of $\mathcal{P}(X)$ such that

(P0) If $\mathcal{A} \in \pi$ then $\mathcal{A}$ has at most two members;

(P1) $A \cap B \neq \varnothing \Rightarrow \{\{A, B\} \in \pi\}$;

(P2) For fixed $A \subset X$ the set $\pi(A) = \{B: \{A, B\} \in \pi\}$ is a grill.

A proximity π is *Lodato* iff for $\mathcal{A} \subset \mathcal{P}(X)$ we have $\mathcal{A} \in \pi$ whenever $c_\pi \mathcal{A} \in \pi$.

2.2 REMARK. Lodato proximities defined as above correspond to the usual definition as follows:

$A \,\delta\, B$ iff $\{A, B\} \in \pi$.

For a proof, see Thron [9].

2.3 THEOREM. *Let ν be a nearness on a set X and let π_ν be the set of all members of ν which have at most two members. Then π_ν is a proximity on X with the same closure operator as ν. If ν is Lodato then π_ν is Lodato.*

PROOF. See Gagrat and Thron [4]. □

2.4 DEFINITION. A proximity π on a set X is *compatible* with a nearness ν iff $\pi_\nu = \pi$. It is compatible with a topology $\mathcal{T}$ on X iff c_π is the closure operator determined by $\mathcal{T}$.

2.5. PROPOSITION. *Let ν be a cluster-generated Lodato nearness on a T_1-space X. Then*

$$\{A, B\} \in \pi_\nu \text{ iff } e_\nu(A)^- \cap e_\nu(B)^- \neq \varnothing.$$

PROOF. Let $\{A, B\} \in \pi_\nu$. Then $\{A, B\} \in \nu$ and there is a ν-cluster σ such that $A, B \in \sigma$. By Lemma 1.16, $\sigma \in e_\nu(A)^- \cap e_\nu(B)^-$.

Conversely, if $\sigma \in e_\nu(A)^- \cap e_\nu(B)^-$ then by Lemma 1.16 we have $A, B \in \sigma$. Since $\sigma \in \nu$ we have $\{A, B\} \in \pi_\nu$. □

2.6 DEFINITION. Let π be a proximity on a set X.

(a) A *π-clan* is a grill σ with the property that if $A, B \in \sigma$ then $\{A, B\} \in \pi$.

(b) A *π-bunch* is a π-clan σ such that $c_\pi A \in \sigma \Rightarrow A \in \sigma$.

2.7 THEOREM. *Let π be a proximity on a set X. Let*

$$\nu_G(\pi) = \{\mathcal{A} \subset \mathcal{P}(X) \colon \mathcal{A} \text{ is contained in a } \pi\text{-clan}\}.$$

Then $\nu_G(\pi)$ is the largest nearness compatible with π. Moreover, $\nu_G(\pi)$ is contigual. Finally, $\nu_G(\pi)$ is Lodato if π is Lodato.

PROOF. Let $\nu_G(\pi)$ be denoted by ν_G.

(1) ν_G is a nearness.

Let $\mathcal{A} \subset \mathcal{P}(X)$ and suppose $\bigcap \mathcal{A} \neq \varnothing$.

Let $x \in \bigcap \mathcal{A}$. Let $\dot{x} = \{S \subset X \colon x \in S\}$. Then $\dot{x}$ is a π-clan containing $\mathcal{A}$.

Suppose $\mathcal{A} \in \nu_G$ and $\mathcal{A} < \mathcal{B}$. Let σ be a π-clan such that $\mathcal{A} \subset \sigma$. Then $\mathcal{B} \subset \sigma$ and so $\mathcal{B} \in \nu_G$.

If $\mathcal{A} \vee \mathcal{B} \subset \sigma$, where σ is a π-clan, and if $\mathcal{A} \not\subset \sigma$ then $\mathcal{B} \subset \sigma$. Hence (N3) holds. It is clear that (N4) holds.

(2) ν_G is compatible with π.

Clearly if $\{A, B\} \in \nu_G$ then $\{A, B\} \in \pi$. The converse was actually proved by Thron [10]. However, the proof is short and will be given here.

Let $\{A, B\} \in \pi$. Then $B \in \pi(A)$, which is a grill. Let $\mathfrak{U}$ be an ultrafilter such that $B \in \mathfrak{U} \subset \pi(A)$. Now $\pi(\mathfrak{U})$ is also a grill.

$$\left(\pi(\mathfrak{U}) = \{D \subset X: \{D, U\} \in \pi \text{ for } U \in \mathfrak{U}\}.\right)$$

Since $A \in \pi(\mathfrak{U})$ we can choose an ultrafilter $\mathfrak{V}$ such that $A \in \mathfrak{V} \subset \pi(\mathfrak{U})$. Then $\mathfrak{U} \cup \mathfrak{V}$ is a π-clan containing $\{A, B\}$.

(3) ν_G is the largest nearness compatible with π.

Let ν be any nearness compatible with π. It is easy to see that $a\nu$ is also compatible with π. (See Proposition 1.22 for a definition of $a\nu$.) Now, $a\nu$ is contigual, and hence cluster-generated. (See Lemma 1.23.) But every grill in $a\nu$ is a π-clan, and so we have $\nu \subset a\nu \subset \nu_G$.

(4) ν_G is contigual.

Note $a\nu_G$ is compatible with π. Thus by (3) we have $a\nu_G \subset \nu_G$.

(5) If π is Lodato then ν_G is Lodato.

Let $A^- = c_\pi(A) = c_{\nu_G}(A)$. Suppose $\mathcal{Q} \subset \mathcal{P}(X)$ and $\mathcal{Q}^- \subset \sigma$ where σ is a π-clan. We wish to show $\mathcal{Q}$ is contained in some π-clan. Let

$$b\sigma = \{A \subset X: A^- \in \sigma\}.$$

Then $b\sigma$ is a grill containing $\mathcal{Q}$. Since π is Lodato, $b\sigma$ is a π-clan. □

2.8 Remark and Definition. If π is a Lodato proximity compatible with a T_1-space X then $\operatorname{Ext}(\nu_G(\pi))$ is a T_1-compactification of X (Corollary 1.26).

The next theorem verifies that this is the construction of Gagrat and Naimpally [3, Theorem 3.13]. For this reason we define a *GN-compactification* to be any compactification which is equivalent to some $\operatorname{Ext}(\nu_G(\pi))$, where π is a compatible Lodato proximity. We will denote $\operatorname{Ext}(\nu_G(\pi))$ by $\kappa_G(\pi)$, or κ_G.

The following theorem establishes that κ_G is indeed Gagrat-Naimpally construction.

2.9 Theorem. *Let π be a Lodato proximity compatible with a T_1-space X.*

(1) *The set of $\nu_G(\pi)$-clusters is the set of maximal π-bunches.*

(2) *The set of maximal π-bunches is the set of maximal π-clans.*

(3) *For $\omega \in Y_{\nu_G}$, we have $\omega^- = \bigcap\{\sigma: \bigcap\omega \subset \sigma\}$.*

Proof. (1) The set of ν_G-clusters is the set of maximal π-clans.

Since ν_G is cluster-generated, the ν_G-clusters are the maximal ν_G-clans (Proposition 1.7). But the ν_G-clans are just the π-clans. Thus ν_G-clusters are maximal π-clans, conversely.

(2) Every maximal π-clan is a maximal π-bunch. This follows from the fact π is Lodato. The proof is nearly identical with that of Proposition 1.6.

(3) Every maximal π-bunch is a maximal π-clan. Let σ be a maximal π-bunch. By a Zorn's lemma argument, σ is contained in a maximal π-clan μ. By (2), μ is a π-bunch, and hence $\mu = \sigma$. Thus σ is a maximal π-clan.

(4) For $\omega \subset Y_{\nu_G}$ we have $\omega^- = \{\sigma: \bigcap\omega \subset \sigma\}$. Let ν_G be denoted by ν. By Lemma 1.16, the closed sets of Y_ν are generated by $\{A^\nu: A \subset X\}$. Let $\sigma \in \omega^-$ and $A \in \bigcap\omega$. Then $\omega \subset A^\nu$ and hence $\omega^- \subset A^\nu$. Thus $A \in \sigma$, and $\bigcap\omega \subset \sigma$.

Conversely, suppose $\bigcap\omega \subset \sigma$. To show $\sigma \in \omega^-$ it is sufficient to show $\sigma \in A^\nu$ whenever $\omega \subset A^\nu$. If $\omega \subset A^\nu$ then $A \in \bigcap\omega \subset \sigma$. Thus $\sigma \in A^\nu$ as desired. □

2.10 DEFINITION. Let π be a proximity on a set X and let $\mathcal{Q} \subset \mathcal{P}(X)$. Let $A, B \subset X$.

(a) $A <_\pi B$ iff $\{A, X \setminus B\} \not\in \pi$;

(b) $r_\pi\mathcal{Q} = \{S: \exists A \in \mathcal{Q}$ such that $A <_\pi S\}$;

(c) $c\mathcal{Q} = \{S: X \setminus S \not\in \mathcal{Q}\}$.

We call $c\mathcal{Q}$ the *dual* of $\mathcal{Q}$.

2.11 REMARK. If σ is a grill then $c\sigma$ is a filter and if $\mathcal{F}$ is a filter, $c\mathcal{F}$ is a grill. Moreover, for any $\mathcal{Q} \subset \mathcal{P}(X)$, $cc\mathcal{Q} = \mathcal{Q}$. Finally, if $\mathcal{Q} \subset \mathcal{B}$ then $c\mathcal{B} \subset c\mathcal{Q}$. For proofs see Thron [9].

In general if $\mathcal{Q}$ is closed under supersets then $r_\pi\mathcal{Q} \subset \mathcal{Q}$. If $\mathcal{F}$ is a filter than $r_\pi\mathcal{F}$ is also a filter. However if σ is a grill then $r_\pi\sigma$ need not be a filter or a grill.

2.12 LEMMA. *Let ν be a Lodato nearness compatible with a T_1-space X, and let σ be a ν-cluster. Then*

$$c\sigma = e_\nu^{-1}(\mathfrak{N}_\sigma).$$

PROOF. Let $A \in c\sigma$. Then $Y \setminus (X \setminus A)^\nu$ is an open neighborhood G_σ of σ. But $e_\nu^{-1}(G_\sigma) \subset A$.

Conversely, suppose $A \supset e^{-1}(U_\sigma)$ for some open neighborhood U_σ of σ. Since $\sigma \not\in Y \setminus U_\sigma$ there is a closed set K in X such that $\sigma \not\in K^\nu$ and $Y \setminus U_\sigma \subset K^\nu$. Then $A \supset X \setminus K \in c\sigma$. □

2.13 LEMMA. *Let π be a Lodato proximity on a set X, and let σ be a grill on X. Then σ is a maximal π-clan iff $c\sigma$ is the filter generated by $r_\pi\sigma$.*

PROOF. (1) Assume σ is a maximal π-clan. Note first that since σ is a π-clan we have $r_\pi\sigma \subset c\sigma$. Since $c\sigma$ is a filter, this says the filter generated by $r_\pi\sigma$ is contained in $c\sigma$. Now suppose A is not in the filter generated by $r_\pi\sigma$. We wish to show $A \not\in c\sigma$. Note that $r_\pi\sigma \cup \{X \setminus A\}$ has the f.i.p. Let $\mathcal{U}$ be an ultrafilter containing $r_\pi\sigma \cup \{X \setminus A\}$. We claim that $\sigma \cup \mathcal{U}$ is a π-clan.

Note $\sigma \cup \mathcal{U}$ is a grill, since it is a union of ultrafilters. Clearly σ and $\mathcal{U}$ are separately π-compatible. Let $S \in \sigma$ and $U \in \mathcal{U}$ and suppose $\{S, U\} \not\in \pi$. Then $S <_\pi X \setminus U$. Hence $X \setminus U \in r_\pi\sigma \subset \mathcal{U}$, which violates $U \in \mathcal{U}$. Therefore $\{S, U\} \in \pi$ as desired.

Since σ is a maximal π-clan, we have $\mathfrak{U} \subset \sigma$. Hence $X \setminus A \in \sigma$, and $A \not\in c\sigma$.

(2) Conversely, assume $c\sigma$ is generated by $r_\pi\sigma$. Since $r_\pi\sigma \subset c\sigma$ we have that σ is a π-clan. To show that σ is a maximal π-clan it is sufficient to show that for every ultrafilter $\mathfrak{U}$, if $\sigma \cup \mathfrak{U}$ is a π-clan then $\mathfrak{U} \subset \sigma$.

Assume $\sigma \cup \mathfrak{U}$ is a π-clan. Suppose $A \notin \sigma$. Then $X \setminus A \not\subset c\sigma$, which is generated by $r_\pi\sigma$. Let $S_1, \dots, S_n \in \sigma$ such that $S_i <_\pi T_i$ and $\bigcap_i T_i \subset X \setminus A$. Since $\sigma \cup \mathfrak{U}$ is a π-clan, $X \setminus T_i \notin \mathfrak{U}$ for all i. Hence each T_i is in $\mathfrak{U}$, and $X \setminus A \in \mathfrak{U}$. Thus $A \notin \sigma \Rightarrow A \notin \mathfrak{U}$, and $\mathfrak{U} \subset \sigma$. □

2.14 THEOREM. *Let π be a Lodato proximity compatible with a T_1-space X. Then π is an EF-proximity iff $\kappa_G(\pi)$ is Hausdorff.*

PROOF. ($\Rightarrow$) Assume π is an *EF*-proximity.

(1) The dual of a maximal π-clan is a maximal round filter.

Let σ be a maximal π-clan. Then by Lemma 2.13, $c\sigma$ is generated by $r_\pi\sigma$. Let $A \in c\sigma$. We wish to find $B \in c\sigma$ such that $B <_\pi A$. Choose $S_1, \dots, S_n \in \sigma$ such that $S_i <_\pi T_i$ and $\bigcap_i T_i \subset A$. Since π is an *EF*-proximity, we can choose U_i such that $S_i <_\pi U_i <_\pi T_i$. Then $U = \bigcap_i U_i \in c\sigma$ and $U <_\pi A$.

To see that $c\sigma$ is maximal round, suppose $A <_\pi B$ with $B \not\in c\sigma$. We need $X \setminus A \in c\sigma$. Now $X \setminus B \in \sigma$ and $A <_\pi B \Rightarrow A \notin \sigma \Rightarrow X \setminus A \in c\sigma$.

(2) $\kappa_G(\pi)$ is Hausdorff.

Let σ_1, σ_2 be ν_G-clusters with $\sigma_1 \neq \sigma_2$. They are maximal π-clans, by Theorem 2.9. So each $c\sigma_i$ is a maximal round filter. Now $c\sigma_1 \neq c\sigma_2$. Let $A \in c\sigma_1 \setminus c\sigma_2$. Choose $B \in c\sigma_1$ such that $B <_\pi A$. Then $X \setminus B \in c\sigma_2$. Let e denote e_{ν_G}. Then $c\sigma_i = e^{-1}(\mathfrak{N}_{\sigma_i})$, by Lemma 2.12. Choose G_i, open sets in Y_{ν_G}, such that $\sigma_i \in G_i$ and $e^{-1}(G_1) \subset B$ and $e^{-1}(G_2) \subset X \setminus B$. Then $G_1 \cap G_2 = \varnothing$. For if $\sigma \in G_1 \cap G_2$ then both B and $X \setminus B$ are in $e^{-1}(\mathfrak{N}_\sigma)$.

($\Leftarrow$) Now assume $\kappa_G(\pi)$ is Hausdorff. To show π is an *EF*-proximity it is sufficient to show that $<_\pi$ is dense.

Suppose $A <_\pi B$. Then by Proposition 2.5, we have $e_G(A)^- \cap e_G(X \setminus B)^- = \varnothing$. Since every compact T_2-space is normal, we can choose disjoint open sets U and V such that $e_G(A)^- \subset U$ and $e_G(X \setminus B)^- \subset V$. Again from Proposition 2.5 we have $A <_\pi e_G^{-1}(U)$ and $X \setminus B <_\pi e_G^{-1}(V)$. Thus $A <_\pi e_G^{-1}(U) <_\pi B$. □

Next we wish to obtain a characterization of the *GN*-compactifications. These turn out to be the principal T_1-extensions for which the dual of every π^*-clan converges. Here π^* is the largest *LO*-proximity compatible with the topology, i.e., $\{A, B\} \in \pi^*$ iff $A^- \cap B^- \neq \varnothing$.

2.15 DEFINITION. A *clan* on a T_1-space X is a grill σ with the property that for $A, B \in \sigma$ we have $A^- \cap B^- \neq \varnothing$. A *bunch* is a clan μ such that $A \in \mu$ whenever $A^- \in \mu$.

2.16 REMARK. Let X be a T_1-space and let π^* be the Lodato proximity defined by

$$\{A, B\} \in \pi^* \quad \text{iff } A^- \cap B^- \neq \varnothing.$$

Then π^* is the largest Lodato proximity compatible with the topology on X. A *clan* on X is the same as a π^*-clan.

2.17 LEMMA. *Let $\kappa = (e, Y)$ be a T_1-extension of a space X. Let π be the proximity on X induced by ν_κ. Then $\nu_\kappa = \nu_G(\pi)$ iff for every clan σ on Y, if $eX \in c\sigma$ then $c\sigma$ converges.*

PROOF. ($\Leftarrow$) Assume that if σ is a clan and $eX \in c\sigma$ then $c\sigma$ converges. Note $\nu_\kappa \subset \nu_G(\pi)$ since $\nu_G(\pi)$ is the largest nearness compatible with π (Theorem 2.7).

Now suppose σ is a π-clan. We wish to show $\sigma \in \nu_\kappa$. Note first that $ce\sigma = ec\sigma$. Thus $ce\sigma$ is a filter, and $e\sigma$ is a grill. Since σ is a π-clan, clearly $e\sigma$ is a clan. Moreover, $eX \in ec\sigma = ce\sigma$. Thus by the assumption, $ce\sigma \to y$ for some $y \in Y$. We claim that $\sigma \subset \tau(y) \in \nu_\kappa$.

Let $S \in \sigma$. Then $e(S) \in e\sigma$ and so $Y \setminus e(S)^- \not\in ce\sigma$. Since $ce\sigma \to y$, this says $y \in e(S)^-$; i.e. $S \in \tau(y)$.

($\Rightarrow$) Now suppose $\nu_\kappa = \nu_G(\pi)$. Let σ be a clan on Y such that $eX \in c\sigma$. We need to show $c\sigma$ converges. We will show that $e^{-1}\sigma$ is a π-clan. Since $eX \in c\sigma$ we have that $S \in \sigma \Rightarrow S \cap eX \in \sigma$. Thus $A \in e^{-1}\sigma \Leftrightarrow e(A) \in \sigma$ and $A \in e^{-1}c\sigma$ iff $e(A) \in c\sigma$. From this it follows that $ce^{-1}\sigma = e^{-1}c\sigma$. Now $eX \in c\sigma \Rightarrow e^{-1}c\sigma$ is a proper filter. Thus $e^{-1}\sigma$ is a nonempty grill.

If $A, B \in e^{-1}\sigma$ then $e(A)$ and $e(B)$ are in σ; but σ is a clan, so $e(A)^- \cap e(B)^- \neq \varnothing$. Thus $\{A, B\} \in \nu_\kappa$, and $e^{-1}\sigma$ is a π-clan.

Now $\nu_G(\pi) = \nu_\kappa \Rightarrow e^{-1}\sigma \in \nu_\kappa$. Let $y \in \bigcap e(e^{-1}\sigma)^-$. Since $eX \in c\sigma$, $ee^{-1}\sigma = \sigma$. Thus $y \in \bigcap \sigma^-$. It is easy to check that $c\sigma \to y$. □

2.18 LEMMA. *Let $\kappa = (e, Y)$ be a principal T_1-extension of a space X. The following conditions are equivalent*:

(1) *the dual of every clan converges*;

(2) *if σ is a clan nad $eX \in c\sigma$ then $c\sigma$ converges.*

PROOF. Clearly (1) $\Rightarrow$ (2). Suppose (2) holds. Let σ be a clan. Then $b\sigma = \{A\colon A^- \in \sigma\}$ is a bunch, and $cb\sigma$ is an open filter. Let $\mathfrak{F}$ be the filter generated by $cb\sigma \cup \{eX\}$.

(1) $\mathfrak{F}$ converges.

Note that $c\mathfrak{F}$ is a grill. Moreover, since $cb\sigma \subset \mathfrak{F}$ we have $c\mathfrak{F} \subset b\sigma$ and so $c\mathfrak{F}$ is a clan. Since $eX \in cc\mathfrak{F}$ we have from (2) that $\mathfrak{F}$ converges.

(2) If $\mathfrak{F} \to y$ then $c\sigma \to y$.

Suppose $\mathfrak{F} \to y$. We claim that since κ is principal, we have that $cb\sigma \to y$. For let G be open such that $y \in G$. Let U be open such that $y \in U^+ \subset G$.

(See Definition 1.12.) Then $U^+ \in \mathfrak{F}$ and so we can choose $T \in cb\sigma$ such that $T \cap eX \subset U^+$. Since $cb\sigma$ is open, $T^i \in cb\sigma$. We claim $T^i \subset U^+ \subset G$. Let $z \in T^i$. Then $T \in \mathfrak{N}_z$ and $e^{-1}(T) \in e^{-1}(\mathfrak{N}_z)$. But $e^{-1}(T) \subset e^{-1}(U)$ and so $z \in U^+$.

Now since $\sigma \subset b\sigma$ we have $cb\sigma \subset c\sigma$. Since $cb\sigma \to y$, clearly $c\sigma \to y$. □

2.19 Definition. A topological space is *clan-complete* iff the dual of every clan converges.

2.20 Theorem. *Let $\kappa = (e, Y)$ be a principal T_1-extension of X. Then κ is a GN-compactification iff Y is clan-complete.*

Proof. Let κ be a principal T_1-extension of X. Suppose κ is equivalent to $\mathrm{Ext}(\nu_G(\pi)) = \kappa_G$, where π is a Lodato proximity compatible with X. Note that κ and κ_G induce the same nearness (Lemma 1.9). Since ν_G is cluster-generated, we have that ν_G is the nearness induced by κ_G (Theorem 1.18). Thus $\nu_G = \nu_\kappa$, and also ν_κ is compatible with π. Thus Lemma 2.17 applies, and every clan σ with $eX \in c\sigma$ has a convergent dual. Since κ is principal, this is equivalent to Y being clan-complete (Lemma 2.18).

Conversely, suppose Y is clan-complete. Let π be the proximity induced by ν_κ. Then by Lemma 2.17, $\nu_\kappa = \nu_G(\pi)$. Since κ is principal, κ is equivalent to $\mathrm{Ext}(\nu_G(\pi))$ (Theorem 1.19). Thus κ is a GN-compactification of X. □

2.21 Corollary. *Let X be a fixed T_1-space. The map $\pi \to \kappa_G(\pi)$ is a bijection from the Lodato proximities compatible with X to the clan-complete principal T_1-extensions of X.*

Proof. If π is a Lodato proximity compatible with X then $\kappa_G(\pi)$ is clan-complete, by the preceding theorem. Suppose now that $\kappa_G(\pi_1)$ and $\kappa_G(\pi_2)$ are equivalent. Then they induce the same nearness under Tr (Lemma 1.9). Since $\nu_G(\pi_i)$ is cluster-generated, it must be the nearness induced by $\kappa_G(\pi_i)$ (Theorem 1.18). Thus $\nu_G(\pi_1) = \nu G(\pi_2)$, and hence $\pi_1 = \pi_2$.

Finally, if κ is a clan-complete principal T_1-extension of X, then by the preceding theorem, κ is equivalent to $\kappa_G(\pi)$ for some Lodato proximity π compatible with X. □

The final result is a form of the correspondence obtained by Smirnov [7] between EF-proximities and T_2-compactifications.

2.22 Corollary. *Let X be a fixed completely regular T_2-space. Then the map $\pi \to \kappa_G(\pi)$ is a bijection from the compatible EF-proximities to the T_2-compactifications of X.*

Proof. Theorem 2.14 and Corollary 2.21. □

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