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WHEN IS THE NATURAL MAP $X \to \Omega\Sigma X$ A COFIBRATION?

BY

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ABSTRACT. It is shown that a map $f\colon X \to F(A, W)$ is a cofibration if its adjoint $f\colon X \wedge A \to W$ is a cofibration and X and A are locally equiconnected (LEC) based spaces with A compact and nontrivial. Thus, the suspension map $\eta\colon X \to \Omega\Sigma X$ is a cofibration if X is LEC. Also included is a new, simpler proof that C.W. complexes are LEC. Equivariant generalizations of these results are described.

In answer to our title question, asked many years ago by John Moore, we show that $\eta\colon X \to \Omega\Sigma X$ is a cofibration if X is locally equiconnected (LEC)—that is, the inclusion of the diagonal in $X \times X$ is a cofibration [**2, 3**]. An equivariant extension of this result, applicable to actions by any compact Lie group and suspensions by an arbitrary finite-dimensional representation, is also given. Both of these results have important implications for stable homotopy theory where colimits over sequences of maps derived from η appear unbiquitously (e.g., [**1**]).

The force of our solution comes from the Dyer-Eilenberg adjunction theorem for LEC spaces [**3**] which implies that C.W. complexes are LEC. Via Corollary 2.4(b) below, this adjunction theorem also has some implications (exploited in [**1**]) for the geometry of the total spaces of the universal spherical fibrations of May [**6**]. We give a simpler, more conceptual proof of the Dyer-Eilenberg result which is equally applicable in the equivariant context and therefore gives force to our equivariant cofibration condition. The key to the simplified proof is Proposition 2.5, a result of independent interest on cofibrations joining a pair of pushout diagrams.

Our definitions and conventions on spaces and cofibrations are contained in §1. §2 contains the statements of our results and §3 is devoted to their proofs. §4 sketches the equivariant extensions of our theorems.

1. Preliminaries on spaces and cofibrations. We work in the category of compactly generated weak Hausdorff spaces (see [**4, 7, 8, 11, 13**]). Products, smash products, function spaces (written exponentially), and based function spaces (written $F(A, X)$) are given the topologies appropriate to this category. A map $f\colon X \to Y$ is called a

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cofibration if it has the homotopy extension property with respect to unbased homotopies and a fibration if it has the homotopy lifting property with respect to unbased homotopies. Note that, while we work mainly with based spaces and based maps, our results apply only to maps having the homotopy extension property with respect to unbased homotopies. This limitation arises from our use of Lillig's union theorem for cofibrations [5] and the NDR pair characterization of cofibrations [3, 7, 9], both of which are available only for cofibrations defined in terms of unbased homotopies. In practice, this is not really a limitation since any basepoint of an LEC space is nondegenerate [3] and, if X and Y are both nondegenerately based, then the based and unbased notions of a cofibration are equivalent [10].

2. The suspension map and adjunction theorem. Our principal result is

THEOREM 2.1. *Let A and X be LEC spaces with A compact and not the one point space. If $g\colon X \wedge A \to W$ is a based map and a cofibration, then its adjoint $\tilde{g}\colon X \to F(A, W)$ is also a cofibration. In particular, the adjoint $\eta\colon X \to F(A, X \wedge A)$ of the identity map of $X \wedge A$ is a cofibration.*

Since the circle S^1 is an LEC based space and the suspension map for X is the adjoint of the identity map of ΣX, our answer to Moore's question follows immediately.

COROLLARY 2.2. *If X is an LEC based space, then the suspension map $\eta\colon X \to \Omega\Sigma X$ is a cofibration.*

There is an obvious unbased analog of Theorem 2.1 (with a similar proof) giving a cofibration condition for the adjoint $\tilde{f}\colon Y \to Z^B$ of an unbased map $f\colon Y \times B \to Z$. In §4, we give an equivariant version of Theorem 2.1.

The force of Theorem 2.1 comes from the fact that metric ANR's and C.W. complexes are LEC [2, 3]. That C.W. complexes are LEC follows from the Dyer-Eilenberg adjunction theorem for LEC spaces [3] restated below. Their proof is unnecessarily complex, and unlike our new proof given in §3, does not obviously generalize to the equivariant case.

THEOREM 2.3 (SEE [3]). *If X and Y are LEC, $i\colon A \to X$ is a cofibration and $f\colon A \to Y$ is any map, then the adjunction space $Y \cup_f X$ is LEC.*

The following pair of results follow easily.

COROLLARY 2.4. (a) *C.W. complexes are LEC* (*see* [3]).

(b) *If $\underline{X} = \{X_n\}_{n \geq 0}$ is a simplicial space such that each X_n is LEC, then the geometric realization $|\underline{X}|$ of X is LEC.*

The key to our simpler proof of Theorem 2.3 is the following result on cofibrations joining a pair of pushout diagrams.

PROPOSITION 2.5. *If, in the commuting diagram*

$$\begin{tikzcd}
A \arrow[rrr, "\alpha"] \arrow[dr, "\iota"'] \arrow[dd, "f"'] & & & A' \arrow[dr, "\iota'"] \arrow[dd, "f'"] & \\
& B \arrow[rrr, "\beta"] \arrow[dd, "\tilde{f}"] & & & B' \arrow[dd, "\tilde{f}'"] \\
C \arrow[rrr, "\gamma"] \arrow[dr, "\tilde{\iota}"'] & & & C' \arrow[dr, "\tilde{\iota}'"] & \\
& D \arrow[rrr, "\delta"'] & & & D'
\end{tikzcd}$$

the right and left faces are pushouts, ι, ι', α, β, γ are cofibrations, and the top is a pullback (that is, A is $A' \cap B$ when the three are regarded as subspaces of B'), then δ is a cofibration.

Note. The assumption in 2.5 that the left face is a pushout can be weakened to the assumption that the map $B \amalg C \to D$, derived from $\tilde{f}$ and $\tilde{\iota}$, is an epimorphism. However, in our applications, the very existence of the map δ is obtained from the pushout hypothesis.

The assumption that the top is a pullback cannot be discarded. To see this, let X be a nondegenerately based space other than a point, and take $B = C = A' = B' = C' = D' = X$, $A = *$, and $D = X \vee X$. Take the maps from X to X in the diagram to be the identity, the maps from $*$ to X to be the inclusion of the basepoint, and the map δ: $X \vee X \to X$ to be the folding map. Clearly, all the hypotheses of the proposition, except the pullback condition, are satisfied; but the folding map is not a cofibration.

3. Proofs. We repeatedly use the following obvious corollary of Theorem II.7 of [3] which characterizes a halo retract of an LEC space as an LEC subspace whose inclusion is a cofibration.

LEMMA 3.1. (a) *If X is LEC and i: $A \to X$ is the inclusion of a retract, then i is a cofibration and A is LEC.*

(b) *If X is LEC and i: $A \to X$ is a cofibration, then A is LEC.*

For 3.1(a), the halo about A needed to apply Theorem II.7 of [3] can be obtained from the retraction and LEC data for X. For 3.1(b), a halo retraction for A can be constructed from NDR pair data for (X, A).

Now we begin our proofs.

PROOF OF 2.1. First we show that η: $X \to F(A, X \wedge A)$ is a cofibration. Select a point a_0 in A other than the basepoint (we denote basepoints generically by $*$). Let Z be the subspace of $F(A, X \wedge A)$ given by

$$Z = \{f \in F(A, X \wedge A) \mid f(a_0) \in X \wedge \{a_0, *\} \subset X \wedge A\}.$$

Clearly, the map η: $X \to F(A, X \wedge A)$ factors through Z and evaluation at a_0 provides a right inverse to η regarded as a map into Z. If we can show that $F(A, X \wedge A)$ is LEC and that the inclusion of Z into $F(A, X \wedge A)$ is a cofibration, then it follows from Lemma 3.1 that Z is LEC and that η: $X \to Z$ is a cofibration. The second part of the theorem follows since the composite

$$\eta\colon X \to Z \to F(A, X \wedge A)$$

must now be a cofibration. To see that $F(A, X \wedge A)$ is LEC, note that $X \times A$ is clearly LEC and the pair $(X \times A, \{*\} \times A \cup X \times \{*\})$ is an NDR pair since A and X are nondegenerately based **[9]**. Therefore, $X \wedge A$ is LEC by Theorem 2.3 and $F(A, X \wedge A)$ is LEC by Theorem II.6 of **[3]** which states that the space of based functions from a compact space to an LEC space is LEC. To see that the inclusion of Z into $F(A, X \wedge A)$ is a cofibration, note that the pair $(A, \{a_0, *\})$ is an NDR pair and therefore, the restriction map

$$(X \wedge A)^A \to (X \wedge A)^{\{a_0, *\}}$$

is a fibration (since $(X \wedge A)^?$ converts cofibrations to fibrations). Pulling back along the inclusion of $F(\{a_0, *\}, X \wedge A)$ in $(X \wedge A)^{\{a_0, *\}}$, we obtain that the restriction map

$$r\colon F(A, X \wedge A) \to F(\{a_0, *\}, X \wedge A) \cong X \wedge A$$

is a fibration. The pair $(F(A, X \wedge A), Z)$ is the inverse image under r of the NDR pair $(X \wedge A, X \wedge \{a_0, *\})$ and is therefore an NDR pair by Theorem 12 of **[9]**.

For the first part of Theorem 2.1, note that the adjoint of a map g: $X \wedge A \to W$ is the composite

$$g\colon X \xrightarrow{\eta} F(A, X \wedge A) \xrightarrow{g_*} F(A, W).$$

If g is a cofibration, then so is g_* by Lemma 4 of **[10]**. By the work above, η is a cofibration and we are done.

We turn now to

PROOF OF 2.3. Let X, Y, i: $A \to X$ and f: $A \to Y$ be as in the statement of the theorem. The idea of the proof is to describe $(Y \cup_f X) \times (Y \cup_f X)$ by a sequence of pushouts in such a way that Proposition 2.5 can be applied. We do this in the pushout diagrams below. Note that $(Y \cup_f X) \times (Y \cup_f X)$ is the pushout in the third diagram because products preserve colimits in the category of compactly generated weak Hausdorff spaces. Cofibrations in the diagrams below are denoted by arrows of the form "$\rightarrowtail$". We repeatedly use the fact that the pushout of a cofibration is a cofibration. The map

$$j_3\colon X \times A \underset{A \times A}{\cup} A \times X \rightarrowtail X \times X$$

below is a cofibration by Lillig's union theorem **[5]**.

$$\begin{tikzcd}
A \times A \amalg A \times A \arrow[r, "\nabla"] \arrow[d] & A \times A \arrow[d] & \\
A \times Y \amalg Y \times A \arrow[r] \arrow[d, tail] & A \times Y \underset{A \times A}{\cup} Y \times A \arrow[r, "\alpha"] \arrow[d, tail] & Y \times Y \arrow[d, tail, "j_1"] \\
X \times Y \amalg Y \times X \arrow[r] & X \times Y \underset{A \times A}{\cup} Y \times X \arrow[r, "\beta"] & X \times Y \underset{A \times A}{\cup} Y \times Y \underset{A \times A}{\cup} Y \times X
\end{tikzcd}$$

$$\begin{tikzcd}
A \times A \amalg A \times A \arrow[r, "\nabla"] \arrow[d, tail] & A \times A \arrow[d, tail, "j_2"] \\
X \times A \amalg A \times X \arrow[r] & X \times A \underset{A \times A}{\cup} A \times X
\end{tikzcd}$$

$$\begin{tikzcd}
X \times A \underset{A \times A}{\cup} A \times X \arrow[r, tail, "j_3"] \arrow[d, "(1 \times f) \cup (f \times 1)"] & X \times X \arrow[dd, "\gamma"] \\
X \times Y \underset{A \times A}{\cup} Y \times X \arrow[d, "\beta"] & \\
X \times Y \underset{A \times A}{\cup} Y \times Y \underset{A \times A}{\cup} Y \times X \arrow[r, tail, "j_4"] & (Y \cup_f X) \times (Y \cup_f X)
\end{tikzcd}$$

Above, ∇ denotes a folding map from a disjoint union and α is the obvious map induced by the pair

$$A \times A \xrightarrow{f \times f} Y \times Y,$$

$$A \times Y \amalg Y \times A \xrightarrow{f \times 1 \amalg 1 \times f} Y \times Y \amalg Y \times Y \xrightarrow{\nabla} Y \times Y.$$

The pushout diagram

$$\begin{array}{ccc}
A & \overset{i}{\rightarrowtail} & X \\
f\downarrow & & \downarrow \\
Y & \rightarrowtail & Y \underset{f}{\cup} X
\end{array}$$

maps into the third diagram above by the maps

$$A \overset{\Delta}{\rightarrowtail} A \times A \overset{j_2}{\rightarrowtail} X \times A \underset{A \times A}{\cup} A \times X,$$

$$X \overset{\Delta}{\rightarrowtail} X \times X,$$

$$Y \rightarrowtail Y \times Y \overset{j_1}{\rightarrowtail} X \times Y \underset{A \times A}{\cup} Y \times Y \underset{A \times A}{\cup} Y \times X,$$

$$Y \underset{f}{\cup} X \overset{\Delta}{\rightarrowtail} \left(Y \underset{f}{\cup} X\right) \times \left(Y \underset{f}{\cup} X\right).$$

Proposition 2.5 clearly applies to give that this last map is a cofibration so that $Y \cup_f X$ is LEC.

The key to proving Corollary 2.4 is the following lemma:

LEMMA 3.2. (a) *If* $\{\lambda_n\colon X_n \to X_{n+1}\}_{n \geqslant 0}$ *and* $\{\gamma_n\colon Y_n \to Y_{n+1}\}_{n\geqslant 0}$ *are sequences of spaces joined by cofibrations and* $\{f_n\colon X_n \to Y_n\}_{n\geqslant 0}$ *is a sequence of cofibrations such that the diagram*

$$\begin{array}{ccc} X_n & \overset{\lambda_n}{\to} & X_{n+1} \\ \downarrow f_n & & \downarrow f_{n+1} \\ Y_n & \overset{\gamma_n}{\to} & Y_{n+1} \end{array}$$

is a pullback for all $n \geqslant 0$, *then the induced map* $f\colon X \to Y$ *on the colimits*

$$X = \operatorname{colim} X_n, \qquad Y = \operatorname{colim} Y_n$$

is a cofibration.

(b) *If* $\{\lambda_n\colon X_n \to X_{n+1}\}_{n\geqslant 0}$ *is a sequence of LEC spaces joined by cofibrations, then* $X = \operatorname{colim} X_n$ *is also LEC.*

PROOF. For 3.2(a), given a commuting diagram like the solid arrow diagram below, we must construct a map H making the entire diagram commute.

$$\begin{tikzcd} X \arrow[r, "h"] \arrow[d, "f"'] & F(I^+, Z) \arrow[d, "e_0"] \\ Y \arrow[r, "g"'] \arrow[ur, dashed, "H"] & Z \end{tikzcd}$$

Here, I^+ denotes the unit interval with a disjoint basepoint and e_0 denotes evaluation at zero. We produce H by inductively defining maps

$$H_n\colon Y_n \to F(I^+, Z)$$

such that $H_{n+1}\gamma_n = H_n$ and the diagram

$$\begin{tikzcd} X_n \arrow[r, "i_n"] \arrow[d, "f_n"'] & X \arrow[r, "h"] & F(I^+, Z) \arrow[d, "e_0"] \\ Y_n \arrow[r, "j_n"'] \arrow[ur, dashed, "H_n"] & Y \arrow[r, "g"'] & Z \end{tikzcd} \tag{1}$$

commutes where i_n and j_n are the inclusions into the colimit. The map H_0 is obtained by applying the homotopy extension property for f_0 in diagram (1). Given H_n, H_{n+1} is obtained from the diagram

$$\begin{array}{ccc} Y_n \underset{X_n}{\cup} X_{n+1} & \xrightarrow{H_n \cup (h \circ i_{n+1})} & F(I^+, Z) \\ k \downarrow & \overset{H_{n+1}}{\nearrow} & \downarrow e_0 \\ Y_{n+1} & \xrightarrow{g \circ j_{n+1}} & Z \end{array}$$

by using Lillig's union theorem **[5]** to show that k is a cofibration. The required map H is obtained from the H_n by passage to colimits.

Lemma 3.2(b) follows immediately from 3.2(a) and the definition of LEC.

Corollary 2.4(a) follows directly from the lemma and Theorem 2.3 because any C.W. complex is the colimit of its skeleta which are LEC by inductively applying the theorem. The proof of Corollary 2.4(b) is more involved.

Proof of Corollary 2.4(b). The geometric realization $|\underline{X}|$ of a simplicial space X is the colimit of a sequence of spaces $F_n|\underline{X}|$ defined inductively by the equation $F_0|\underline{X}| = X_0$ and the pushout diagrams

$$\begin{array}{ccc} (X_{n+1} \times \partial\Delta_{n+1}) \cup (\sigma X_n \times \Delta_{n+1}) & \to & X_{n+1} \times \Delta_{n+1} \\ \downarrow & & \downarrow \\ F_n|\underline{X}| & \to & F_{n+1}|\underline{X}| \end{array}$$

where Δ_{n+1} is the standard $n+1$ simplex and σX_n is the degeneracy subspace of X_{n+1}. Thus, the inclusion of $F_n|\underline{X}|$ in $F_{n+1}|\underline{X}|$ is a cofibration if the inclusion of σX_n in X_{n+1} is a cofibration. Since it follows formally from the simplicial identities that

$$\begin{array}{ccc} X_{n-1} & \xrightarrow{s_i} & X_n \\ s_j \downarrow & & \downarrow s_{j+1} \\ X_n & \xrightarrow{s_i} & X_{n+1} \end{array}$$

is a pullback for $n \geqslant 1$ and $0 \leqslant i \leqslant j \leqslant n-1$, Lillig's union theorem **[5]** may be applied inductively to obtain that the inclusions

$$\sigma X_n \to X_{n+1}, \qquad n \geqslant 0,$$

are cofibrations if the degeneracy maps s_i: $X_n \to X_{n+1}$ are cofibrations for $n \geqslant 0$ and $0 \leqslant i \leqslant n$. If all of the X_n are LEC, then the s_i, being inclusions of retracts, are cofibrations by Lemma 3.1. Further, the $F_n|\underline{X}|$ are LEC by an inductive application of Theorem 2.3. Thus, by Lemma 3.2(b), the geometric realization $|\underline{X}|$ is LEC.

Finally, we have

PROOF OF PROPOSITION 2.5. Given a commuting diagram like the solid arrow diagram below, we must construct a map H such that the entire diagram commutes:

$$\begin{tikzcd} D \arrow[r, "h"] \arrow[d, "\delta"'] & F(I^+, Z) \arrow[d, "e_0"] \\ D' \arrow[r, "g"] \arrow[ur, dashed, "H"] & Z \end{tikzcd}$$

To obtain H, we first select a map $K\colon C' \to F(I^+, Z)$ using

$$C \xrightarrow{\tilde{\imath}} D \xrightarrow{h} F(I^+, Z), \qquad C' \xrightarrow{\tilde{\imath}'} D' \xrightarrow{g} Z$$

and the homotopy extension property for $\gamma\colon C \to C'$. Then, using the maps

$$A' \xrightarrow{f'} C' \xrightarrow{K} F(I^+, Z), \qquad B \xrightarrow{\tilde{f}} D \xrightarrow{h} F(I^+, Z)$$

which agree on A, we obtain a map

$$L\colon A' \underset{A}{\cup} B \to F(I^+, Z)$$

which agrees with $K \circ f'$ on A'. Applying the homotopy extension property for $A' \cup_A B \to B'$ (which is a cofibration by Lillig's union theorem **[5]**) to the maps

$$L\colon A' \underset{A}{\cup} B \to F(I^+, Z), \qquad B' \xrightarrow{\tilde{f}'} D' \xrightarrow{g} Z,$$

we obtain a map $J\colon B' \to F(I^+, Z)$ which agrees with $K \circ f'$ on A' and therefore determines H. Note that

$$H \circ \tilde{\imath}' = K\colon C' \to F(I^+, Z)$$

so that we can always pick our homotopy extension to agree with a preselected extension on C'.

4. Equivariant generalizations. If we switch from compactly generated weak Hausdorff spaces to compactly generated weak Hausdorff left G-spaces where G is a compact Lie group, then there are natural analogs of Theorem 2.3, Corollary 2.4 and Proposition 2.5. In these, we replace cofibrations by G-cofibrations and the LEC condition by the G-LEC condition (that the diagonal be a G-cofibration). The analog of a C.W. complex is a G-C.W. complex constructed from G-spheres of the form $G/H \times S^n$ for $n \geqslant 0$ and H a (closed) subgroup of G (see **[12]**). It is easy to see that Lillig's original proof of his union theorem **[5]** extends to the equivariant context. Thus, our proofs in §3 of 2.3, 2.4 and 2.5 extend to the equivariant context because, except for the use of Lillig's theorem and the fact that spheres are LEC, they are purely formal.

The equivariant extension of Theorem 2.1 and its corollary involves two minor technical points. First, by a based G-space, we mean a left G-space with a basepoint on which G acts trivially. Second, in the proof of Theorem 2.3, we had to select a nonbasepoint a_0 in A. It is easy to see that our proof extends to the equivariant context if we can select a_0 to be a G-fixed point. Thus, the equivariant analogs of Theorem 2.1 and its corollary are

THEOREM 4.1. *Let A and X be G-LEC based G-spaces with A compact and having a G-fixed point other than the basepoint. If $g\colon X \wedge A \to W$ is a based G-map and a G-cofibration, then its adjoint $\tilde{g}\colon X \to F(A, W)$ is also a G-cofibration. In particular, the adjoint $\eta\colon X \to F(A, X \wedge A)$ of the identity map of $X \wedge A$ is a G-cofibration.*

COROLLARY 4.2. *Let X be a G-LEC based G-space and V be a finite dimensional G-representation. If $g\colon \Sigma^V X \to W$ is a based G-map and a G-cofibration, then its adjoint $\tilde{g}\colon X \to \Omega^V W$ is also a G-cofibration. In particular, the adjoint $\eta\colon X \to \Omega^V \Sigma^V X$ of the identity map of $\Sigma^V X$ is a G-cofibration.*

In the corollary, we define Ω^V and Σ^V by

$$\Omega^V X = F(tV, X), \qquad \Sigma^V X = X \wedge tV$$

where tV is the one-point compactification of V with basepoint at infinity. Theorem 4.1 applies to give the corollary because 0 is a G-fixed point in tV.

There is an unbased analog of Theorem 4.1 giving a cofibration condition for the adjoint $\tilde{f}\colon Y \to Z^B$ of an unbased G-map $f\colon Y \times B \to Z$. As in the based case, we must require the presence of a G-fixed point in B.

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