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THE MACRAE INVARIANT AND THE FIRST LOCAL CHERN CHARACTER

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ABSTRACT. The first local Chern character of a bounded complex of locally free sheaves on a scheme Y is given by intersection with a Cartier divisor. In the case of the resolution of a module of finite projective dimension, this is the invariant defined by MacRae.

Let R be a commutative Noetherian ring, and let M be a finitely generated module of finite projective dimension. In his investigation of invariants of these modules, MacRae [6] constructed an invertible ideal $G(M)$ associated to M which describes the part of the support of M of codimension 1. The fact that $G(M)$ is invertible implies many properties of the support of M, and it has recently been used by Foxby [3] to prove some conjectures on intersection multiplicities of modules of finite projective dimension with modules of Krull dimension one.

In this paper we generalize the construction of MacRae to a bounded complex E_* of locally free sheaves on a Noetherian scheme Y and show that this can be used to describe the first local Chern character of E_*. A bounded complex of locally free sheaves of finite rank will be called a *perfect* complex. For technical reasons, we assume that Y is connected and quasi-projective over an affine scheme. Let X be the support of E_*, denoted $\operatorname{Supp}(E_*)$; this can be defined as the set of points of Y where E_* is not exact, or, equivalently, as the union of the supports of the homology modules $H_i(E_*)$. We assume that X is contained in some Cartier divisor. Locally, this means that the ideal defining X contains a non-zero-divisor, and if E_* is a resolution of a module, our assumption follows whenever X is a proper subset of Y. An equivalent formulation of this condition is that X contains no points y of Y such that the local ring $\mathcal{O}_y$ has depth zero; such a point will be called a point of depth zero. In this situation we construct a Cartier divisor $G(E_*)$ on Y generalizing the MacRae invariant.

If Y is quasi-projective over a regular local ring (this includes, among others, the case where $Y = \operatorname{Spec} R$ and R is a complete local ring), there is a theory of local Chern characters defined for perfect complexes on Y. For a complex E_* with support X as above, for any scheme Y' together with a map of finite type $f\colon Y' \to Y$, and for integers n and j, the jth local Chern character $\operatorname{ch}_j(E_*)$ defines an

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"intersection operator"

$$\mathrm{ch}_j(E_*)\colon A_n(Y') \to A_{n-j}(f^{-1}(X)),$$

where, for any scheme Z, $A_*(Z) = A_*(Z) \otimes Q$ is the rational Chow group of cycles on Z modulo rational equivalence. We refer to Fulton [**4**, Chapter 18] for definitions and properties of these operators. We show below that in our situation $\mathrm{ch}_1(E_*)$ is the operator defined by intersecting with the Cartier divisor $G(E_*)$. This implies that if the codimension of X in Y is at least 2, then $\mathrm{ch}_1(E_*) = 0$, a result which is used in Roberts [**7**] to prove a vanishing conjecture on multiplicities for rings with singular locus of dimension 1. It should be remarked that in higher codimensions the corresponding statement is not true; Dutta, Hochster, and McLaughlin [**2**] have constructed an example in which the codimension of the support of E_* is 3, but $\mathrm{ch}_2(E_*) \neq 0$.

An invariant similar to the one described here has been constructed for complexes by Iversen [**8**]; his is an element of the local cohomology group $H^1_Z(\mu)$, where Z contains the support of E_* and μ is the sheaf of units. He also outlines a proof that this gives the first Chern class in étale cohomology; in this context, it appears to be possible to reduce the "global" case; a technique which has not been successfully carried out for the Chern characters defined as operators in the Chern group which we use here.

Before constructing $G(E_*)$, it is necessary to deal with one technical point. We need to know that the following two constructions are possible:

1. If K is a coherent sheaf on Y, then there is a locally free sheaf F and a (locally) surjective map: $F \to K$.

2. Suppose we are given a finite subset T of Y containing all points of depth zero. If we have a diagram of maps of locally free sheaves:

$$\begin{array}{ccccc} & & F_0 & & \\ & & \downarrow & & \\ E_{i+1} & \to & E_i & \xrightarrow{d_i} & E_{i-1} \end{array}$$

such that $\mathrm{Im}(F_0) \subseteq \mathrm{Ker}\, d_i$, F_0 has rank r, and the support of $H_i(E_*)$ contains no points of T, then there is a locally free sheaf F_1, also of rank r, together with maps

$$\begin{array}{ccccc} F_1 & \xrightarrow{\phi} & F_0 & & \\ \downarrow & & \downarrow & & \\ E_{i+1} & \to & E_i & \xrightarrow{d_i} & E_{i-1} \end{array}$$

such that the diagram commutes and Supp(Coker ϕ) contains no points of T.

If Y is affine, the first construction is accomplished by mapping generators of a free module onto generators of K, and the second by choosing elements of F_0 which map to the image of d_{i+1} and which avoid the prime ideals corresponding to points of T. The first construction is in fact standard in more generality; a proof can be found in Borelli [**1**]. The method of proof is to find a line bundle L on Y and a global section s of L such that $Y_s = \{ y \in Y \mid s \text{ does not generate } L \text{ at } y \}$ is affine. One makes the construction on Y_s using $\mathcal{O}_{Y_s}^k$ as above, then, using EGA I (9.3.1) (reference [**5**]), one deduces the construction on Y using $(L^{\otimes(-m)})^k$ for some integer m, at least up to the support of s. The same method works for the second construction, provided that L and s can always be chosen so that the support of s contains no point of T; if Y is quasi-projective over an affine scheme, this can always be done.

We now define $G(E_*)$ when E_* is a perfect complex of length 1. In this case E_* is just a map between locally free sheaves of the same rank (recall that $\operatorname{Supp}(E_*)$ is always assumed to contain no points of depth zero). Let

$$E_* = \cdots \to 0 \to E_{i+1} \xrightarrow{d_{i+1}} E_i \to 0 \to \cdots$$

with $\operatorname{rank}(E_i) = \operatorname{rank}(E_{i+1}) = r$. We then have a map

$$\begin{array}{ccc} \bigwedge^r E_{i+1} & \to & \bigwedge^r E_i \\ \| & & \| \\ L_{i+1} & \to & L_i \end{array}$$

where L_{i+1} and L_i are locally free sheaves of rank one. These give a map $L_{i+1} \otimes L_i^{-1} \to \mathcal{O}_Y$.

The image of this map is a Cartier divisor D, with $L_{i+1} \otimes L_i^{-1} \cong \mathcal{O}_Y(-D)$, and we define $G(E_*)$ to be $(-1)^i D$ (using additive notation for divisors). Locally, the map on rth exterior powers is given by the determinant of a matrix defining d_{i+1}, so this is the same as the MacRae invariant for a module of projective dimension 1.

PROPOSITION 1. *Let E_* be a perfect complex of length* 1. *If α is a cycle in $A_n Y'$, where $f\colon Y' \to Y$ is a map of schemes, then*

$$\operatorname{ch}_1(E_*)(\alpha) = G(E_*) \cap \alpha \quad \textit{in } A_{n-1}(f^{-1}(X)).$$

PROOF. Since all of the operations used in defining $G(E_*)$ are compatible with pullbacks, we can assume that $Y' = Y$, and, replacing Y' by a component of α, we can assume that Y is an integral scheme of dimension n and $\alpha = [Y]$. The proof is divided into two cases.

Case 1. $X = Y$. In this case,

$$\operatorname{ch}_1(E_*) = (-1)^i(\operatorname{ch}_1(E_i) - \operatorname{ch}_1(E_{i+1})).$$

We use the equality

$$\operatorname{ch}_1(E_j) = c_1(E_j) = c_1\left(\bigwedge^r E_j\right) = \operatorname{ch}_1\left(\bigwedge^r E_j\right),$$

where $c_1(E)$ denotes the first Chern class of E (see Fulton [**4**, Remark 3.2.3]). Thus

$$\begin{aligned} \operatorname{ch}_1(E_*)(\alpha) &= (-1)^i(c_1(L_i) - c_1(L_{i+1}))(\alpha) = (-1)^i c_1(L_i \otimes L_{i+1}^{-1})(\alpha) \\ &= (-1)^i c_1(\mathcal{O}(D))(\alpha) = G(E_*) \cap \alpha. \end{aligned}$$

Case 2. $X \neq Y$. In this case X is a proper subset of Y, and, since $\mathrm{ch}_1(E_*)[Y]$ is a cycle of codimension 1, we can localize and assume that $Y = \operatorname{Spec} R$, where R is a local domain of dimension 1 and X is the closed point of Y. By normalizing, we can then assume that R is a discrete valuation ring; let t generate its maximal ideal. Then the complex E_* decomposes into a sum of complexes $\cdots 0 \to R \xrightarrow{t^m} R \to 0$, and it suffices to prove the result in this case. However, in this case both sides can easily be computed and we obtain

$$\mathrm{ch}_1(E_*)([Y]) = m[X] = G(E_*) \cap [Y].$$

PROPOSITION 2. *Let* $0 \to E'_* \to E_* \to E''_* \to 0$ *be a short exact sequence of perfect complexes of length* 1 *with* $E'_j = E_j = E''_j = 0$ *for* $j \neq i,\ i+1$. *Then* $G(E_*) = G(E'_*) + G(E''_*)$.

PROOF. Let r', r, and r'' be the ranks of E'_i, E_i, and E''_i respectively. We then have $r = r' + r''$, and

$$\wedge^r E_j \cong \wedge^{r'} E'_j \otimes \wedge^{r''} E''_j$$

or

$$L_j \cong L'_j \otimes L''_j \quad \text{for } j = i,\ i+1.$$

Furthermore, the embedding $L_{i+1} \to L_i$ factors as follows:

$$L_{i+1} \cong L'_{i+1} \otimes L''_{i+1} \to L'_{i+1} \otimes L''_i \to L'_i \otimes L''_i \cong L_i.$$

The corresponding Cartier divisors of these factors are $G(E''_*)$ and $G(E'_*)$ respectively, so the assertion follows.

Now that $G(E_*)$ has been defined for complexes of length 1, the general case can be defined by approximating a general complex by complexes of length 1. More precisely, let $E_* = 0 \to E_k \to \cdots \to E_m \to 0$, so that k is the largest integer j for which $E_j \neq 0$, and let i be the smallest integer j for which $H_j(E_*) \neq 0$. We define $G(E_*)$ by induction on $k - i$.

It is impossible for $k - i$ to equal zero, since the support of E_* contains no points of depth zero.

If $k - i = 1$, we map a free module F_{k-1} to the kernel of d_{k-1} so that the map from F_{k-1} to $H_{k-1}(E_*)$ is surjective. We then have a diagram

$$\begin{array}{ccccc} & & F_{k-1} & & \\ & & \downarrow \phi & & \\ 0 \to E_k & \xrightarrow{d_k} & E_{k-1} & \to E_{k-2} \to & \cdots . \end{array}$$

Let $F_k = \phi^{-1}(\mathrm{Im}(d_k))$. Since $H_{k-1}(E_*)$ has a resolution by locally free sheaves which stops in degree k, we deduce that F_* is locally free, and we define $G(E_*) = G(F_*)$, where F_* is the complex $0 \to F_k \to F_{k-1} \to 0$.

If $k - i > 1$, we map a locally free sheaf F_i to $\operatorname{Ker} d_i$ so that it induces a surjection onto $H_i(E_*)$ as above. If the rank of F_i is r, we can find another locally free sheaf F_{i+1} of rank r so that we have a diagram

$$\begin{array}{ccccc} 0 \to & F_{i+1} & \xrightarrow{f} & F_i & \\ & \downarrow \phi_{i+1} & & \downarrow \phi_i & \\ \cdots \to E_{i+2} \to & E_{i+1} & \to & E_i \to E_{i-1} \to \cdots . \end{array}$$

In addition, if T is any finite subset of Y disjoint from X and containing all points of depth zero, we can assume that $\mathrm{Supp}(\mathrm{Coker} f) \cap T$ is empty. Let C_* be the mapping cone of ϕ: $F_* \to E_*$. From the long exact sequence

$$\cdots \to H_j(F_*) \to H_j(E_*) \to H_j(C_*) \to H_{j-1}(F_*) \to \cdots$$

and the surjectivity of $H_i(F_*) \to H_i(E_*)$ we deduce that $H_j(C_*) = 0$ for $j \leqslant i$. Furthermore, since $i - k \geqslant 2$, $C_j = 0$ for $j > k$. Hence, by induction, we have defined $G(C_*)$. Let $G(E_*) = G(C_*) + G(F_*)$.

There are a number of choices in this definition, but the next theorem shows that the result is independent of these choices.

THEOREM 1. (a) *$G(E_*)$ is independent of the choices of complexes $F_{i+1} \to F_i$ and maps into E_* used in the construction.*

(b) *If $0 \to E'_* \to E_* \to E''_* \to 0$ is a short exact sequence of perfect complexes, then $G(E_*) = G(E'_*) + G(E''_*)$.*

PROOF. We prove both of these assertions by induction on $k - i$, where k and i are defined as above for (a), while for (b) we let $k = \sup\{ j \mid E_j, E'_j, \text{ or } E''_j \neq 0\}$ and $i = \inf\{ j \mid H_j(E_*), H_j(E'_*), \text{ or } H_j(E''_*) \neq 0\}$.

As before, the lowest value of $k - i$ which must be considered is $k - i = 1$.

To prove (a) in this case, suppose that F_{k-1} and G_{k-1} are surjective maps of locally free sheaves into $\mathrm{Ker}(d_{k-1})$ which induce surjections onto $H_{k-1}(E_*)$. Replacing F_{k-1} with $F_{k-1} \oplus G_{k-1}$, we can assume that there is a map f_{k-1} from F_{k-1} onto G_{k-1}. The map f_{k-1} will induce a map f_* on kernels, and we have

$$\begin{array}{ccccccc} & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \\ 0 \to \mathrm{Ker}(f_k) & \to & F_k & \xrightarrow{f_k} & G_k & \to 0 \\ \wr\downarrow & & \downarrow & & \downarrow & & \\ 0 \to \mathrm{Ker}(f_{k-1}) & \to & F_{k-1} & \xrightarrow{f_{k-1}} & G_{k-1} & \to 0 \\ & & \downarrow & & \downarrow & & \\ & & H_{k-1}(E_*) & = & H_{k-1}(E_*) & & \\ & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & \end{array}$$

with exact rows and columns. Statement (a) now follows from Proposition 2 applied to the short exact sequence of complexes in the top two rows.

To prove (b) in this case, we note that we have a short exact sequence

$$0 \to H_{k-1}(E'_*) \to H_{k-1}(E_*) \to H_{k-1}(E''_*) \to 0.$$

Let F'_{k-1} be a locally free sheaf mapping onto $H_{k-1}(E'_*)$, and let F''_{k-1} map onto $H_{k-1}(E_*)$. We form the diagram

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & F'_k & \to & F_k & \to & F''_k & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & F'_{k-1} & \to & F'_{k-1} \oplus F''_{k-1} & \to & F''_{k-1} & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & H_{k-1}(E'_*) & \to & H_{k-1}(E_*) & \to & H_{k-1}(E''_*) & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

where the top row consists of the kernels of the vertical maps. From (a), we can use these complexes to compute $G(E'_*)$, $G(E_*)$, and $G(E''_*)$, and (b) now follows from Proposition 2.

We now prove (a) by induction, assuming that $k - i = s \geqslant 2$ and that (a) and (b) hold whenever $k - i < s$. Let $F_* \to E_*$ and $G_* \to E_*$ be two maps of complexes of length 1 to E_* as in the above construction. We then have a map $F_* \oplus G_* \to E_*$ of the same type, and it suffices to compare the definition of $G(E_*)$ by means of $F_* \oplus G_*$ with that by means of G_*. Denote C_*^F the mapping cone of $F_* \to E_*$ and $C_*^{F\oplus G}$ the mapping cone of $F_* \oplus G_* \to E_*$. We have exact sequences

$$0 \to F_* \to F_* \oplus G_* \to G_* \to 0$$

and

$$0 \to C_*^G \to C_*^{F\oplus G} \to F_*[-1] \to 0.$$

We thus have

$$\begin{aligned}
G(E_*) \text{ (computed from } G_*) &= G(C_*^G) + G(G_*) \\
&= \big(G(C_*^{F\oplus G}) - G(F_*[-1])\big) + \big(G(F_* \oplus G_*) - G(F_*)\big) \\
&\qquad\qquad \text{(by induction and Proposition 2)} \\
&= G(C_*^{F\oplus G}) + G(F_* \oplus G_*).
\end{aligned}$$

This completes the proof of (a).

We now prove (b), assuming that (a) holds for $k - i \leqslant s$ and that (b) holds for $k - i < s$. Take complexes $F'_* \to E'_*$ and $F''_* \to E_*$ as in the construction of $G(E'_*)$ and $G(E_*)$. Since $H_{i-1}(E'_*) = 0$, the map from $H_i(E_*)$ to $H_i(E''_*)$ is surjective, and the composition $F''_* \to E_* \to E''_*$ can be used to define $G(E''_*)$ (this uses assertion (a) for $k - i = s$). We now take the diagram of maps of complexes

$$\begin{array}{ccccccccc}
0 & \to & F'_* & \to & F'_* \oplus F''_* & \to & F''_* & \to & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \to & E'_* & \to & E_* & \to & E''_* & \to & 0.
\end{array}$$

If C'_*, C_*, and C''_* are the respective mapping cones, there is a short exact sequence $0 \to C'_* \to C_* \to C''_* \to 0$. The result now follows from the induction hypothesis applied to this sequence and from Proposition 2.

The uniqueness of $G(E_*)$ is needed in the next result.

THEOREM 2. *Let E_* be a perfect complex on Y with support X. If $f\colon Y' \to Y$ is any map of finite type, and if $\alpha \in A_n(Y')$, then* $\mathrm{ch}_1(E_*)(\alpha) = G(E_*) \cap \alpha$ *in* $A_{n-1}(f^{-1}(X))$.

PROOF. As before, we can assume that Y' is an integral scheme of dimension n and that $\alpha = [Y']$. The proof is by induction on $k - i$, where, as above, $k = \sup\{ j \mid E_j \neq 0\}$ and $i = \inf\{ j \mid H_j(E_*) \neq 0\}$. If $k - i = 1$, then the construction of $G(E_*)$ gives a quasi-isomorphism $F_* \to E_*$. Thus

$$\begin{aligned} \mathrm{ch}_1(E_*)(\alpha) &= \mathrm{ch}_1(F_*)(\alpha) \\ &= G(F_*) \cap \alpha \quad \text{by Proposition 1} \\ &= G(E_*) \cap \alpha \quad \text{by definition.} \end{aligned}$$

Assume now that $k - i > 1$.

Case 1. The function f maps Y' into X. Let $F_* \to E_*$ be as in the inductive construction of $G(E_*)$. The short exact sequence

$$0 \to f^*(E_*) \to f^*(C_*) \to f^*(F_*[-1]) \to 0$$

gives the equation

$$\mathrm{ch}_1(E_*)[Y'] = \mathrm{ch}_1(C_*)[Y'] + \mathrm{ch}_1(F_*)[Y']$$

in $A_{n-1}(Y')$. By definition, we have $G(E_*) = G(C_*) + G(F_*)$, so that

$$G(E_*) \cap [Y'] = G(C_*) \cap [Y'] + G(F_*) \cap [Y']$$

in $A_{n-1}(Y')$. By induction, the result is true for C_* and F_*, so it is true for E_*. Since f maps Y' into X, $Y' = f^{-1}(X)$, and this proves the result.

Case 2. Suppose now that $f(Y') \not\subseteq X$. Let y be the generic point of $\overline{f(Y')}$ in the scheme Y. Since $y \notin X$, in the inductive construction of $G(E_*)$, we can assume that $y \notin \mathrm{Supp}(F_*)$. Let $\overline{X} = \mathrm{Supp}(F_*) \cup X$. Then, using the short exact sequence

$$0 \to E_* \to C_* \to F_*[-1] \to 0,$$

we have that

$$\mathrm{ch}_1(E_*)[Y'] = \mathrm{ch}_1(C_*)[Y'] + \mathrm{ch}_1(F_*)[Y']$$

in $A_{n-1}(f^{-1}(\overline{X}))$. Since $y \notin \mathrm{Supp}(F_*)$, $f^{-1}(\overline{X})$ is a proper closed subset of Y', and any integral subscheme of $f^{-1}(\overline{X})$ of dimension $n-1$ is a component. Hence the coefficients of each component in $\mathrm{ch}_1(E_*)[Y']$ and $G(E_*) \cap [Y']$ must be equal, and, since we could exclude any component not lying in $f^{-1}(X)$ in the same way as we excluded Y', the coefficients of these components must be zero. Hence

$$\mathrm{ch}_1(E_*)[Y'] = G(E_*) \cap [Y'] \quad \text{in } A_{n-1}(f^{-1}(X))$$

as required.

We now use Theorem 2 to show that $\mathrm{ch}_1(E_*)$ vanishes if the codimension of X is greater than 1. However, we first show that $G(E_*)$ may itself not vanish in this case.

EXAMPLE. Let $Y = \operatorname{Spec} R$, where $R = K[[X, Y, S, T]]/(X, Y) \cap (S, T)$. Let E_* be the complex

$$R \xrightarrow{\binom{-Y-T}{X+S}} R^2 \xrightarrow{(X+\alpha S,\, Y+\alpha T)} R$$

where $\alpha \neq 0, 1$ is an element of K. Then $G(E_*)$ is defined by $(X + \alpha S)/(X + S)$, which is not a unit in R. However, the support of E_* has codimension two. It can be verified directly that $\operatorname{ch}_1(E_*) = 0$ in this case; it also follows from the third case of the following theorem.

THEOREM 3. *Let E_* be a perfect complex on Y with support X. Assume that one of the following three conditions holds*:

(a) $Y = \operatorname{Spec} R$, *where R is a local ring, E_* is the resolution of an R-module M, and* $\dim R - \dim M \geqslant 2$.

(b) *Y is normal, and* $\dim Y - \dim X \geqslant 2$.

(c) *For every component Z of Y,* $\dim Z - \dim(X \cap Z) \geqslant 2$.

Then $\operatorname{ch}_1(E_*) = 0$.

PROOF. In case (a), MacRae [**6**, Proposition 5.2] shows that $G(E_*)$ is defined by a principal ideal of R. Hence, if $G(E_*)$ is a proper ideal, the codimension of the support of M is at most one. Thus $G(E_*)$ is trivial, so, by Theorem 2, $\operatorname{ch}_1(E_*) = 0$.

In case (b), $G(E_*)$ is given locally by a quotient x/y, for x and y in R, where R is an integrally closed domain, and $\operatorname{Spec} R$ is part of an affine cover of Y. Since R is integrally closed and x/y is not a unit in R, then the support of x/y has codimension at most one, and the conclusion follows as in part (a).

To prove (c), let $f\colon Y' \to Y$ be a map of schemes, where, as before, we can assume that Y' is an integral scheme of dimension n and we wish to find $\operatorname{ch}_1(E_*)[Y']$. Since Y' is integral, it must map into a component of Y, and, replacing Y by this component, we can assume that Y is integral as well. Let $\tilde{Y}$ be the normalization of Y, and consider the diagram

$$\begin{array}{ccc} \tilde{Y} \times_Y Y' & \to & \tilde{Y} \\ g\downarrow & & \downarrow \\ Y' & \xrightarrow{f} & Y \end{array}$$

Since $\tilde{Y} \times_Y Y' \to Y'$ is finite and surjective, there is an integral subscheme $\tilde{Y}'$ of $\tilde{Y} \times_Y Y'$ of dimension n such that $g_*([\tilde{Y}']) = m[Y']$ for some positive integer m. Then

$$\begin{aligned} m(\operatorname{ch}_1(E_*)[Y']) &= \operatorname{ch}_1(E_*)(g_*([\tilde{Y}'])) = g_* \operatorname{ch}_1(E_*)([\tilde{Y}']) \\ &= 0 \quad \text{by part (b)}. \end{aligned}$$

Since we are using the Chow groups with rational coefficients, this implies that $\operatorname{ch}_1(E_*) = 0$, as was to be proven.

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