ADDENDUM TO “SYMMETRIZATION, SYMMETRIC STABLE PROCESSES, AND RIESZ CAPACITIES”

DIMITRIOS BETSAKOS

In [1] we proved the following theorem.

Theorem 1. *Let $\alpha \in (0, 2)$. Let K be a compact set in $\mathbb{R}^n$ with $m_n(K) = V > 0$. Let $\mathbb{B}_V$ be a ball in $\mathbb{R}^n$ with $m_n(\mathbb{B}_V) = V$. Then*

$$C_\alpha(K) \geq C_\alpha(\mathbb{B}_V).$$

Here $m_n(K)$ is the n -dimensional Lebesgue measure and C_α is the α -capacity defined by M. Riesz (see e.g. [2]).

I, as well as several other people who work on such symmetrization results in potential theory, thought that this was an open problem. T. Watanabe informed me that in his paper [3] from 1983 he proved an isoperimetric inequality (Theorem 1) for certain Lévy processes that include the symmetric stable processes. Moreover, his Theorem 2 contains an equality statement for the isoperimetric inequality. In the introduction of [3], Watanabe points out that his Theorems 1 and 2 hold for Riesz capacities, i.e. Theorem 1 above follows from Theorem 1 in [3]. The proof in [3] uses Fukushima’s theory of Dirichlet forms and is completely different from the proof of Theorem 1 in [1].

REFERENCES

1. D. Betsakos, *Symmetrization, symmetric stable processes, and Riesz capacities*. Trans. Amer. Math. Soc. 356 (2004), 735-755.
2. N. S. Landkof, *Foundations of Modern Potential Theory*. Springer-Verlag, 1972. MR **50**:2520
3. T. Watanabe, *The isoperimetric inequality for isotropic unimodal Levy processes*. Z. Wahrsch. Verw. Gebiete 63 (1983), 487–499. MR **84m**:60093

DEPARTMENT OF MATHEMATICS, ARISTOTLE UNIVERSITY OF THESSALONIKI, 54124 THESSALONIKI, GREECE

E-mail address: `betsakos@auth.gr`

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